Evaluation of Relative Importance of Network Components by System-reliability-based Disaster Resilience Analysis

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Abstract

Disaster resilience is a recently emerging concept, which describes a system’s overall ability to handle risks caused by disastrous events. Recently, a system-reliability-based framework of disaster resilience analysis was proposed for appropriate and reliable resilience evaluations of civil infrastructure systems (Lim et al., 2022). The framework considers uncertainties in the post-disaster states of components and their interdependencies using a diagram of pairs of reliability index (β) and redundancy index (π), which respectively quantify the likelihood of each initial disruption scenario and the corresponding system failure probability. In this paper we apply the framework to lifeline networks to examine its application potential at the corresponding scale. In particular, a systematic methodology to evaluate the relative importance of the network components in the context of system-reliability-based disaster resilience is proposed. The method is demonstrated by applications to system reliability analysis of network connectivity, which is considered a top priority of many infrastructure networks to secure evacuations or material transmission routes in a post-disaster situation. The results confirm that the proposed method can evaluate relative component importance regarding the disaster resilience of the network with balanced considerations of network topology and component reliabilities. The proposed methodology can provide a theoretical basis for optimal decision-making strategies regarding disaster-resilience-based repairs and retrofits of network components.

Keywords: System reliability, Disaster resilience, Lifeline network, Component importance measures, Network connectivity

1. Introduction

Due to the rapid urbanization and globalization driven by the population growth worldwide, civil infrastructure systems become more complex than ever. While such complexity generally makes infrastructure systems more vulnerable to disasters, various unprecedented hazards, e.g., climate change, are increasing the risks all over the world.

For these reasons, research efforts to define and quantify the disaster resilience, i.e., the overall ability of an infrastructure system to cope with disasters, were initiated from the early 2000s. The concept of disaster resilience characterized by four criteria [1] – robustness, redundancy, resourcefulness, and rapidity – has been further studied and widely adopted. Based on this concept, researchers developed various methods to quantify disaster resilience in terms of system performance loss based on the uncertainties and variability in the system of interest and hazards [2].

In efforts to develop efficient methods to evaluate the disaster resilience, many researchers focused on system-performance-based resilience of lifeline networks, which are critical backbones of modern societies [3], [4]. In this approach, the system performance is defined in terms of the states of components and their interrelationship. Therefore, it is essential to compute the reliabilities of the components and their correlations.

In general, high computational cost is required to calculate the reliability of the network because the failure probabilities of most components far from the disaster source are low and spatial correlations between those components need to be computed. To address this issue, two approaches have been developed: probability-based approaches and scenario-based approaches, respectively focusing on the pre-disaster state the post-disaster state. In the former approach, the high computational cost required for system reliability can be reduced by using an alternative data structure [5] or an efficient system reliability method [6]. In the latter approach, an approximation has been proposed to identify component failure combinations (scenarios) that significantly affect system performance loss, under the assumption that all components are highly reliable and thus relative reliability comparisons are less important [7].

Researchers also suggested practical decision-making strategies to manage the disaster resilience of a system. In establishing such strategies, it is essential to prioritize and search for the components critical to system reliability. For this purpose, importance measures (IMs) serve as useful criteria, e.g., inspection priority [8], to quantify the relative contributions of components to the system performance. However, many IMs developed so far do not focus on the correlation between component failures, and the process of reflecting this correlation varies depending on the purpose of the analysis of the network problem, which often leads to ambiguity.

To clarify the previously mentioned issues, this paper applies a recently proposed disaster resilience analysis method [9] to lifeline network systems. In particular, considering the connectivity between major components after an earthquake as the target system performance, the relative importance of the network components is proposed based on the disaster resilience analysis.

After a brief review (Section 2), the new disaster resilience analysis framework is applied to lifeline networks in Section 3. A new causality-based component importance measure (CIM) is then proposed in Section 4 using the results of the disaster resilience analyses. The numerical examples in Section 5 demonstrate the validity of the proposed method in decision-making for system management. Finally, the paper concludes with a summary in Section 6.

2. Review of system-reliability-based disaster resilience analysis framework

Disaster resilience analysis methodologies in the literature show the following limitations: (1) the definition of the system performance is often subjective, e.g., the vertical coordinate of a restoration curve; (2) disaster
resilience assessment lacks integration between the system and its components; and (3) it is not clear how to incorporate the resilience assessment results into the decision-making process to ensure a desirable level of resilience. This section briefly reviews the comprehensive and intuitive framework by Lim et al. [9], recently proposed to overcome these limitations by introducing a system reliability perspective.

2.1 System reliability perspective on disaster resilience

Lim et al. [9] noted that proper analysis should be performed for each of the nine combinations made by three ‘scales’ and three ‘criteria,’ which were termed ‘3x3 resilience matrix.’ Tab. 1 shows a part of the matrix describing the three evaluation criteria at the infrastructure network scale, which is the focus of this paper.

Table 1. Three resilience criteria at the infrastructure network scale in the ‘3x3 resilience’ matrix [9]

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Scale</th>
<th>Infrastructure network</th>
</tr>
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<tbody>
<tr>
<td>Reliability</td>
<td>Avoiding initial disruption of network components</td>
<td></td>
</tr>
<tr>
<td>Redundancy</td>
<td>Preventing cascading failures causing degradation of network performance</td>
<td></td>
</tr>
<tr>
<td>Recoverability</td>
<td>Executing proper actions for loss of network functionality</td>
<td></td>
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</table>

Infrastructure networks, e.g., transportation networks, hydraulic systems, gas distribution systems, and electric power grids, are often modelled with various types of structural nodes and interconnecting links. Since network components are distributed in a large area, the variability and the spatial correlation of hazard demands on components should be properly considered for assessing the reliability. In evaluating the redundancy, the eventual system-level performance degradation due to sequential failures triggered by the initial component disruption should be considered.

The recoverability criterion represents an overall ability that includes not only recovering in the post-disaster stage but also securing essential resources in the pre-disaster stage. In the recovery process of an infrastructure network, it is critical to consider socioeconomic factors, which require extensive research. Therefore, this study focuses on reliability and redundancy, leaving the consideration of recoverability as a future study.

2.2 Reliability-redundancy (β-π) analysis

2.2.1 Reliability and redundancy index

Lim et al. [9] proposed to evaluate the disaster resilience of a system using a plot termed β-π diagram in which the reliability index (β) of components and the corresponding redundancy index (π) of the system are defined as follows. First, the reliability criterion is defined as the generalized reliability index $\beta_{i,j}$ for the probability of the $i$-th initial disruption scenario of network components, $F_i$, given the $j$-th external hazard $H_j$, i.e.,

$$\beta_{i,j} = -\Phi^{-1}\left(F_i\right)$$

where $\Phi^{-1}$ stands for the inverse of the standard normal cumulative distribution function (CDF).

Next, given the corresponding initial disruption $F_i$ and the hazard $H_j$ of the reliability criterion, the redundancy criterion $\pi_{i,j}$ is defined as the generalized reliability index for the probability of the final system-level degradation event $F_{sys}$ due to cascading component failure process induced by $F_i$, i.e.,

$$\pi_{i,j} = -\Phi^{-1}\left(F_{sys}\right)$$

2.2.2 Reliability and redundancy (β-π) diagram

β-π diagram is a two-dimensional plot in which the two axes represent the redundancy and reliability indices [9]. A point in a β-π diagram describes the indices in Eqs. 1 and 2 computed for an initial disruption under a given hazard scenario. In other words, a point with coordinates of $(\pi_{i,j}, \beta_{i,j})$ describe the two aspects of the disaster resilience of the system for the initial disruption scenario $F_i$ with the given hazard $H_j$. (Lim et al. [9] proposed to visualize the corresponding recoverability by colours in the diagram when available.)

In a risk management, it is crucial to define force majeure to set a realistic goal of disaster resilience from viewpoint of regulatory or legal concerns. To this end, “de minimis risk” concept was introduced to the β-π diagram. Its occurrence rate $R_{dm}$ was set as $10^{-7}$/yr [10]. Naturally, in various efforts to secure a proper level of disaster resilience, we aim to make sure that the occurrence rate of the system failure $F_{sys,j}$ originated from the initial disruption scenario $F_i$ given by the hazard $H_j$ is below $R_{dm}$, i.e.,

$$P(F_{sys,i,j}) = P(F_{sys}|F_i,H_j)P(F_i|H_j)P(H_j) < R_{dm}$$

where $\lambda_{H_j}$ denotes the annual mean occurrence rate of the hazard $H_j$. Dividing both sides of Eq. 3 by $\lambda_{H_j}$, the disaster resilience constraint can be expressed in terms of the reliability index $\beta_{i,j}$ and redundancy index $\pi_{i,j}$ in Eqs. 1 and 2 as

$$\Phi(-\pi_{i,j})\Phi(-\beta_{i,j}) < \frac{R_{dm}}{\lambda_{H_j}} = P_{dm,j}$$

The term $P_{dm,j}=P_{dm,j}$ was termed ‘per hazard de minimis risk’ representing the disaster resilience limit-state surface in the β-π diagram which will be visualized by contours in the diagrams below.

3. Application of β-π analysis to lifeline networks

Disaster resilience analysis using a β-π diagram is applicable to general civil infrastructure systems if a probabilistic model for the external hazard is available and the indices in Eqs. 1 and 2 can be computed. This study applies the β-π analysis to lifeline networks under seismic hazards.

3.1 Seismic hazard and fragility analysis

Ground-motion prediction equations (GMPEs), or attenuation relationships, provide a means of predicting the level of ground shaking based on earthquake magnitude,
source-to-site distance, local soil conditions, etc. GMPEs are often defined in terms of peak ground acceleration (PGA), peak ground velocity (PGV), and spectral acceleration (SA). In this study, PGA is set as a target intensity measure, which is not a function of the natural period of a structure and is mainly used for a reliability analysis for node-type components.

3.1.1 GMPE and spatial correlation
A general formulation of a GMPE [11] for PGA is given as

$$\ln \text{PGA}_{jm} = gm(M_{j}, R_{jm}) + \eta_{j} + \epsilon_{jm}, \quad (5)$$

where \( \text{PGA}_{jm} \) is the PGA at the \( m \)-th site for the \( j \)-th earthquake event, \( gm(\cdot) \) is the ground-motion functional model (GMM), \( M_{j} \) is the magnitude of the \( j \)-th earthquake, \( R_{jm} \) is the distance between the source of the \( j \)-th earthquake and the \( m \)-th site distance, and \( \eta_{j} \) and \( \epsilon_{jm} \) are inter- and intra-event residuals respectively. Therefore, the total standard deviation \( \sigma_{r} \) is derived as

$$\sigma_{r} = \sqrt{\sigma_{\eta}^{2} + \sigma_{\epsilon}^{2}.} \quad (6)$$

Based on Eqs. 5 and 6, the correlation coefficient between the PGAs at the \( m \)-th and \( n \)-th sites, separated by a distance \( \Delta \), is expressed as

$$\rho_{\ln \text{PGA}_{jm}, \ln \text{PGA}_{jn}}(\Delta) = \frac{\sigma_{\eta}^{2}}{\sigma_{\epsilon}^{2}} + \rho_{\epsilon_{m}, \epsilon_{n}}(\Delta) \frac{\sigma_{\eta}^{2}}{\sigma_{\epsilon}^{2}}, \quad (7)$$

where \( \rho_{\epsilon_{m}, \epsilon_{n}}(\Delta) \) denotes the spatial correlation model between the two sites.

3.1.2 Seismic fragility analysis
A seismic fragility is defined as the conditional probability that the structure will be in a certain damage state \( DS \) for a given intensity measure. Assuming the seismic capacity follows a lognormal distribution with a median of \( \alpha_{DS}^{l} \) and log-standard deviation \( \beta_{DS}^{l} \), the fragility is given as a function of PGA, i.e.,

$$P(DS|PGA) = \Phi\left(\frac{\ln \text{PGA} - \ln \alpha_{DS}^{l}}{\beta_{DS}^{l}}\right), \quad (8)$$

where \( \alpha_{DS}^{l} \) and \( \beta_{DS}^{l} \) are the lognormal distribution parameters of the target structure at the \( i \)-th site, and \( \Phi \) is the standard Gaussian CDF.

3.2 \( \beta \)-\( \pi \) analysis of a virtual lifeline network

Fig. 1 shows a virtual lifeline network which is subject to an earthquake event with a magnitude of 5.5. It is assumed that only the node-type components in the system can fail, whereas the link-type components never fail. The system failure event is defined as the disconnection between the critical nodes, i.e., components #2 and #13 in the network.

![Figure 1. Topology of a virtual lifeline network](image)

3.2.1 Reliability and Redundancy index
For a specific DS, the failure event of the \( i \)-th component given the \( j \)-th hazard \( H_{j} \) occurs when the measured \( \text{PGA}_{ij} \) at the component location is greater than \( \alpha_{DS}^{l} \). Therefore, the failure probability can be computed as

$$P(E_{i}|H_{j}) = P\left(\ln \alpha_{DS}^{l} \leq \ln \text{PGA}_{ij}\right) = P\left(Z_{i,j}^{l} \leq -\frac{\ln \alpha_{DS}^{l} - gm(M_{j}, R_{ji})}{\sqrt{\left(\beta_{DS}^{l}\right)^{2} + \sigma_{\eta}^{2}}}\right), \quad (9)$$

where \( Z_{i,j}^{l} \) denotes the standardized variable for \( E_{i} \) given \( H_{j} \), following a Gaussian distribution from Eq. 8. By Eq. 9, the reliability index for a single initial failure scenario \( \beta_{i,j}^{l} \) is written as

$$\beta_{i,j}^{l} = -\Phi^{-1}(P(E_{i}|H_{j})) = \frac{\ln \alpha_{DS}^{l} - gm(M_{j}, R_{ji})}{\sqrt{\left(\beta_{DS}^{l}\right)^{2} + \sigma_{\eta}^{2}}}. \quad (10)$$

In the \( \beta \)-\( \pi \) analysis, it is crucial to calculate the joint failure probability of the components, e.g., \( P(E_{1},E_{2},E_{3}) \) to compute the indices. A closed-form derivation [3] for the correlation between the failure events at the \( m \)-th and \( n \)-th sites is derived as,

$$\rho_{\alpha_{DS}^{l}, \beta_{DS}^{l}} = \left\{ \begin{array}{ll}
\frac{\sigma_{\eta}^{2} + \rho_{\epsilon_{m}, \epsilon_{n}}(\Delta) \sigma_{\eta}^{2}}{\sqrt{\left((\beta_{DS}^{l})^{2} + \sigma_{\eta}^{2}\right)^{2} + \sigma_{\epsilon}^{2}}}, & m = n \\n\end{array} \right., m \neq n. \quad (11)$$

To classify and further analyze the scenarios according to the number of initially disrupted components, we introduce \( F_{i}^{k} \), which denotes the \( i \)-th disruption scenario consisting of \( k \) initial disruption scenarios, i.e.,

$$F_{i}^{k} = \bigcap_{m=1}^{k} E_{aim}, \quad (12)$$

where \( t_{i}^{k} = \{a_{i1},a_{i2},\ldots,a_{il}\} \) is the subscript set of \( k \) interrupted components constituting the \( i \)-th scenario. By Eq. 12, \( F_{i}^{k} \) is a parallel system event of \( k \) component failure events. Using a multivariate Gaussian distribution, the reliability index \( \beta_{i,j}^{k} \) corresponding to the initial failure scenario \( F_{i}^{k} \) given the hazard \( H_{j} \) can be computed as

$$\beta_{i,j}^{k} = -\Phi^{-1}(P(F_{i}^{k}|H_{j})).$$
where $\Phi_k$ denotes a $k$-variate standard Gaussian joint CDF, $\beta_{l,i}^k$ is a $k$-by-$l$ reliability index vector in which the $m$-th entry ($\beta_{l,i}^{k_m}$) in Eq. 10, and $R_{l,i}^k$ is a $k$-by-$k$ correlation coefficient matrix with the $(m,n)$-th entry is determined as $R_{l,i}^{k_m} = \rho_{a_{lm}a_{in}}$ in Eq. 11 [12].

The redundancy index $\pi_{l,i}^k$ corresponding to the reliability index $\beta_{l,i}^k$ is defined as

$$\pi_{l,i}^k = -\Phi^{-1}\left(1 - \Phi_k(\beta_{l,i}^k, R_{l,i}^k)\right).$$  \hspace{1cm} (13)

where $\Phi_k$ denotes a $k$-variate standard Gaussian joint CDF, $\beta_{l,i}^k$ is a $k$-by-$l$ reliability index vector in which the $m$-th entry ($\beta_{l,i}^{k_m}$) in Eq. 10, and $R_{l,i}^k$ is a $k$-by-$k$ correlation coefficient matrix with the $(m,n)$-th entry is determined as $R_{l,i}^{k_m} = \rho_{a_{lm}a_{in}}$ in Eq. 11 [12].

The redundancy index $\pi_{l,i}^k$ corresponding to the reliability index $\beta_{l,i}^k$ is defined as

$$\pi_{l,i}^k = -\Phi^{-1}\left(P(F_{sys}, F_{i}^k, H_j)\right).$$  \hspace{1cm} (14)

For $P(F_{sys}|F_{i}^k, H_j)$ in Eq. 14, the minimum cut sets of the system are investigated in advance and the probability of the mutually exclusive and collectively exhaustive (MECE) events containing the minimum cut sets is computed and stored in the vector [13] for every scenario.

### 3.2.2 β-π diagram application

Among various GMMs in the literature, the fitted GMM for PGA in [14] is utilized in this study, that is,

$$gm(M, R) = \alpha + bM + \ln\left(\sqrt{R^2 + h^2}\right) + dR,$$  \hspace{1cm} (15)

where $M$ is the earthquake magnitude, $R$ is the epicentral distance measured in km, and $\alpha, b, c, d, h$ are the fitted coefficients. The GMM coefficients and the GMPE residuals used in this study are summarized in Tab. 2.

<table>
<thead>
<tr>
<th>Param.</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$h$</th>
<th>$\sigma_\eta$</th>
<th>$\sigma_\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>-3.07</td>
<td>0.73</td>
<td>0.76</td>
<td>-0.0029</td>
<td>1.7</td>
<td>0.17</td>
<td>0.34</td>
</tr>
</tbody>
</table>

This study adopts the spatial correlation model in [3] which is given as

$$\rho_{e_{2x}}(\Delta) = \exp(-0.27\Delta^{0.40}),$$  \hspace{1cm} (16)

where $\Delta$ is the distance between two sites, and values 0.27 and 0.40 are the fitted parameters of the model.

For the concise and intuitive analysis, all 14 components in Fig. 1 are set to have the same fragility properties; and $\sigma_\eta$ and $\sigma_\varepsilon$ are set at 0.77 and 0.65 respectively. Given the earthquake hazard $H_j$, the reliability and redundancy index in Eqs. 13 and 14 are computed for each initial disruption scenario $F_{k}^i$ based on $\beta_{l,i}^{k_j}$ in Eq. 10 and $\rho_{m,n}$ in Eq. 11. The computed resilience index pair $(\pi_{l,i}^k, \beta_{l,i}^k)$ are marked as a point in the $\beta$-$\pi$ diagram (Fig. 2) with different colours representing the number of initial disruptions in each scenario. For the scenarios where the redundancy index is negative infinity, for convenience, the point is located along the axis $\pi = -3$. In addition, the disaster limit-states discussed in Eqs. 3 and 4 are also shown according to the different annual disaster occurrence levels.

The $\beta$-$\pi$ diagram in Fig. 2 provides a compact visualization of the disaster resilience of a system. For most initial disruption scenarios, the virtual network exhibits a negative infinite redundancy index because the failure of a small number of components may lead to the system failure. In addition, for all three cases of initial disruptions ($k = 1, 2, 3$), it is observed that the redundancy index tends to decrease as the reliability index increases, i.e., an initial disruption scenario with low likelihoods tends to reduce the subsequent system's ability to resist the external disaster.

This is an intuitively understandable phenomenon because the reliability index $\beta_{l,i}^k$ handles the components in $F_{k,i}^i$, while the redundancy index $\pi_{l,i}^k$ only handles the remaining components except for those in the same $F_{k,i}^i$.

#### 4. Importance measure of relative contribution of network components

In decision-making processes about repair and retrofit of the network, importance measures (IMs) quantifying the contributions of components to the system performance provide an important basis. After a brief review on existing IMs based on the correlation coefficient, a new causality-based component importance measure (CIM) is proposed to overcome limitations of the existing IMs.

#### 4.1 Correlation-based existing IMs

This section reviews six IMs in the literature – conditional probability-based importance measure (CPIM), an inverse of conditional probability-based importance measure (ICPIM), and four IMs reviewed in [15], i.e., risk achievement worth (RAW), risk reduction worth (RRW), bound probability (BP), and Fussell-Vesely (FV).

First, $CPI_{Me}$ is defined as the conditional probability of the $e$-th component failure $E_e$ given the system failure $F_{sys}$, while $ICPIM_{Me}$ represents an inverse version, i.e.,

$$CPI_{Me} = P(E_e|F_{sys}) = \frac{P(F_{E,Fsys})}{P(F_{sys})}$$  \hspace{1cm} (17)

$$ICPIM_{Me} = P(F_{sys}|E_e) = \frac{P(F_{E,Fsys})}{P(E_e)}.$$  \hspace{1cm} (18)

On the other hand, the definitions of RAW, RRW, and BP hinges on the change in the probability of system failure. $RAW_e$ is the ratio of the failure probability of the system increased by the failure of the $e$-th component to the original system failure probability. $RRW_e$ is the ratio of the system failure probability to the failure probability decreased by the survival of the $e$-th component, i.e.,

$$RAW_e = \frac{P(F_{sys})}{P(F_{sys})}$$  \hspace{1cm} (19)
where $F_{sys}^{(e)}$ and $F_{sys}^{(e)}$ denote the system failure event with the $e$-th component guaranteed to unconditionally fail and survive respectively. BP for the $e$-th component, denoted by $BP_e$, is the absolute difference between the probability of $F_{sys}^{(e)}$ and $F_{sys}^{(e)}$ and expressed as

$$BP_e = P(F_{sys}^{(e)}) - P(F_{sys}^{(e)}) .$$

Finally, Fussell-Vesely measure of the $e$-th component, $FV_e$, is defined using the cut sets of the system, i.e.,

$$FV_e = \frac{P(\bigcup \{ \mathcal{C}_l : e \in \mathcal{E} \})}{P(F_{sys})} ,$$

where $\mathcal{C}_l$ is the $l$-th cut set of the system. The numerator probability term in Eq. 22 is based on the cut sets, which is not used in the other five IMs and sets $FV$ apart from the others. To estimate $FV$, it is necessary to identify the cut sets of the system in advance.

In this study, Monte Carlo simulation (MCS) is introduced to compute the IMs in Eqs. 17-22 approximately, and the correlation coefficient needs to be computed to generate multivariate normally distributed samples of the MCS. This computing process make sure that the effects of correlations are considered in evaluating the IMs.

### 4.2 Causality-based CIM

To reflecting causal effects of component failures on the breakdown of the system, a new CIM is formulated based on an operator calculating the causal effect of a variable on other variables.

#### 4.2.1 Do-operation

A causal model provides a mathematical description of the causal relationship within an individual system or population. It is often represented as a directed acyclic graph (DAG) consisting of nodes and arrows implying variables and the causality between the variables respectively [16]. For example, Fig. 3 shows the causal model diagram regarding the system connectivity loss. Each arrow in the diagram represents the causal tendency between two events. That is, the hazard $H$ induces the likelihood of component failures $E_1, E_2, ..., E_n$, which may cause the system failure $F_{sys}$.

![Figure 3. Causal diagram of system connectivity loss](image)

The model accounts for the system failure of a network due to external disasters, not requiring additional unobserved latent nodes. For example, an earthquake hazard is an obvious common cause, which is often termed a **confounder** in causality research. A confounder may introduce spurious associations between component events $E_e$ and $F_{sys}$, e.g., $E_1 \rightarrow H \rightarrow E_2 \rightarrow F_{sys}$. In other words, since the observation (conditional event) of a certain component failure does not cause another component to fail necessarily, the established joint probability density (JPD) for component failure (represented by the mean vector and the correlation coefficient matrix of a multivariate Gaussian distribution) cannot evaluate the causal effect accurately.

To address this issue, a **do-operator** is often used in causality research. If it is applied to the $e$-th component failure $E_e$, i.e., $do(E_e)$, all directed edges that headed into the node $E_e$ are eliminated to nullify all backdoor paths to $F_{sys}$ causing spurious associations. In this study, as the do-operator is applied to the failure and survival of the $e$-th component failure $E_e$, we propose to calculate the rate of change in the system failure probability for the given event $E$ as

$$\Delta P_e(F_{sys}|E) \equiv \begin{cases} 0, & \text{if } P(F_{sys}|do(E_e),E) = 0 \\ 1 - \frac{P(F_{sys}|do(E_e),E)}{P(F_{sys}|do(E_e),E)}, & \text{otherwise} \end{cases}$$

(23)

to consider the direct causal effect of the component failure on the system failure. Since the do-operator modifies the given causal model, an additional computation is required to obtain the JPD.

#### 4.2.2 CIM for system-reliability-based disaster resilience analysis

We propose a new causality-based CIM in the context of $\beta$-risk analysis. Its computation is initiated by exploring the critical scenarios $F_{ij}$ outside of the resilience constraint in the $\beta$-risk diagram. In the $\beta$-risk diagram, the resilience constraint $\mathcal{C}_j$ of the $j$-th hazard $H_j$ is rephrased as

$$C_j : \Phi( - \pi^k_{ij} ) \Phi( - \beta^k_{ij} ) < P_{dm,j} ,$$

(24)

where $\pi^k_{ij}$, $\beta^k_{ij}$, and $P_{dm,j}$ are the same as the terms introduced in Eqs. 13, 14, and 4, respectively. For the scenarios $F^k_{ij}$ satisfying Eq. 24, the components in $I^k_{ie}$ have no meaningful impacts on system failure for the given hazard occurrence rate and *de minimis* risk level.

For a concise expression, the set $I^k_{ie}$ for $e \in I^k_{i}$, is defined as $I^k_{i} \setminus \{e\}$ consisting of $(k - 1)$ components. Then, the causal weights $\Delta_p P(F_{sys}|E)$ in Eq. 23 are computed for all the conditional events of $2^{k-1}$ MECE component failure events. To organize this process into an equation, a set $S^k_{i,e}$ having the MECE events as elements, i.e., all the intersections of every event or its complementary event in $\{E_m|m \in I^k_{i,e}\}$ is defined as

$$S^k_{i,e} = \left\{ \bigcap_{m \in I^k_{i,e}} C_m \bigg| C_m \in \{E_m,F_m\} \right\} , \quad k \geq 2$$

(25)

where $U$ is the universal set for the given sample space. $S^k_{i,e}$ is separately introduced since $I^k_{i,e}$ is the empty set.

Finally, the CIM for the $e$-th component in the $k$ initial disruption scenarios is proposed as
and the normalized CIM (NCIM) vector is proposed as

\[ NCIM^k = \frac{CIM^k}{\max(CIM^k)} \]  

where \( CIM^k = [CIM_1^k, CIM_2^k, \ldots, CIM_n^k]^T \) indicates the CIM vector for all \( n \) components in the system. (In the following numerical examples, the hazard index \( j \) is omitted assuming single disaster scenario.)

5. Numerical examples

The new causality-based CIM presented for \( \beta - \pi \) analysis is demonstrated and tested by two numerical examples: virtual lifeline network and electric substation network.

5.1 Virtual lifeline network

5.1.1 Proposed CIMs

The resilience constraint \( C_j \) in Eq. 24 is determined for the given per hazard de minimis risk of the earthquake \( P_{\text{dom}} = 10^{-5} \). For each initial failure scenario outside of \( C_j \), CIM\(^k\) of the related component is accumulated by Eq. 26 based on the \( \beta - \pi \) analysis of the virtual lifeline network in Fig. 1. Then, CIM\(^k\) is normalized and shown in Fig. 4.

In Fig. 4, NCIM\(^k\) for some components are zero, and the same ‘filtering’ effect is observed for other values of \( P_{\text{dom}} \). This is natural from the system reliability perspective in that the failures of components #1, 4, 5, 7, 10, 11, and 14 have no direct causal effect on the system failure, even if they are close enough (to the seismic source) to have a meaningful correlation between component failures.

Figure 4. Proposed normalized CIM in virtual lifeline network for the hazard occurrence rate \( \Delta H_1 = 10^{-5} \text{ f/yr} \)

5.1.2 Comparison with existing IMs

For comparison, the existing IMs in Eqs. 17-22 are computed for the same network example as shown in Fig. 5. Since CPIM and ICPIM, by their definition, cannot exclude spatial correlation caused by common source earthquake, the importance of component #1, 4, 5, 7, 10, 11, and 14 cannot be filtered out although they do not cause system performance degradation.

Figure 5. Existing IMs computed for components in virtual lifeline network: (a) CPIM, (b) ICPIM, (c) RAW, (d) RRW, (e) BP, and (f) FV

The other four IMs (RAW, RRW, BP, and FV) show appropriate filtering effects in Fig. 5. This is because the system failure probability terms \( P(F_{sys}^{(e)}) \) and \( P(F_{sys}^{(c)}) \) in the definition are the same as the causal terms \( P(F_{sys}^{(e)}|do(E_x)) \) and \( P(F_{sys}^{(c)}|do(E_x)) \) expressed with do-operation, which eliminate spurious correlations from component failure to system failure. However, since these IMs only reflect the probability of system failure when the single component failure is a conditional event and calculated as the ratio of the difference between two system failure probability terms, the initial failure of multiple components that has a significant impact on the system (outside of the resilience constraint in the \( \beta - \pi \) diagram) cannot be fully considered.

5.2 Electric substation network

Fig. 6 shows the geographic location of the components and disaster scenarios for the two-transmission-line substation network [15]. The system fails when it loses the connectivity between the two nodes S and T, representing input and output of the electricity respectively.

Figure 6. Topology of electric substation network and earthquake scenario
When the information of bedrock PGA $A$ is introduced, the failure probability of the $i$-th component by the $j$-th hazard in Eq. 9 is re-expressed as

$$P(E_i|H_j) = P(\ln a_{DS} - \ln A - \ln \text{PGA}_j \leq 0).$$  (28)

Based on component failure probability in Eq. 28, the reliability index of a single component failure in Eq. 10 and the correlation coefficient of component failure between two different components located in the $m$-th and $n$-th site in Eq. 11 are derived as

$$\beta_{ij}^1 = \frac{\ln a_{DS} - \ln A - gm(M_j, R_i)}{\sqrt{\left(\beta_{DS}^1\right)^2 + \sigma_\xi^2 + \sigma_\xi^2}}$$  (29)

$$\rho_{mn} = \frac{\sigma^2_{\xi} + \rho_{\xi\xi}(\Delta)\sigma^2_{\xi}}{\sqrt{\left(\beta_{DS}^m\right)^2 + \sigma_\xi^2 + \sigma_\xi^2}(\left(\beta_{DS}^n\right)^2 + \sigma_\xi^2 + \sigma_\xi^2)}.$$  (30)

where $\sigma^2_{\xi}$ denotes the variance of the bedrock PGA $A$.

In $\beta$-$\pi$ analysis, the identical component capacity properties, i.e., the same $\alpha_{DS}$ and $\beta_{DS}$ values as those in [15], and the GMPE and spatial correlation models in Section 5.1 are used. As a result, the reliability and redundancy indices of the initial disruption scenarios are visualized by different markers according to the number of disrupted components in Fig. 7. The resilience limit-state surfaces are also shown by contours for three return periods of the earthquake (2,475, 975, 475, and 225-year).

Fig. 7 demonstrates that the $\beta$-$\pi$ diagram facilitates identifying critical disruption scenarios in terms of disaster resilience of the system for a given earthquake hazard. In addition, the results provide further insights regarding disaster resilience of the network. First, in this particular example, for a group of scenarios with similar reliability index values, the variability of redundancy index is not significant enough to affect the resilience of the system. Next, overall, the reliability index is much larger than the redundancy index. This is because the connectivity of the given network, e.g., the average degree of component nodes, is relatively low. For this reason, as the number of initially disrupted components increases, the proportion of the scenarios leading to the system failure quickly increases.

By computing $\Delta P_c(F_{xyj})$ in Eq. 23 for each component, NCIMs with 2,745-year and 475-year return period earthquake are obtained as shown in Figs. 8(a) and (b) respectively. As shown in the $\beta$-$\pi$ diagram in Fig. 7, the number of critical initial failure scenarios outside of the resilience constraint decreases as a longer return period is considered. Accordingly, NCIM shows different trends depending on the return period of interest.
similar to that of the proposed NCIM. However, they have, by their mathematical definition, an essential limitation that initial disruption scenarios critical to system failure cannot be prioritized and the effect of hazard occurrence rate is not properly considered.

6. Conclusions

The system-reliability-based disaster resilience analysis framework recently proposed by Lim et al., [9] was applied to infrastructure networks in this study. In the framework, the generalized reliability index ($\beta$) and redundancy index ($\pi$) are computed for each initial disruption combination of the components in the system to show each pair in a scatter plot termed $\beta$-$\pi$ diagram. The reliability and redundancy indices were defined for the purpose of lifeline network analysis and calculated using the correlation coefficient of component failures, GMPE and the spatial correlation model. The diagram provided a compact insight on the disaster resilience of the network from system reliability perspective. Based on the new framework, a causality-based CIM is newly proposed to incorporate the causality aspect that existing correlation-based IMs cannot describe. Applications to a virtual lifeline network and an electric substation network demonstrated that the CIM proposed in this study can identify relative importance of components in infrastructure networks from disaster resilience point of view. Further research is underway to apply the analysis framework and CIM to flow-based analysis of lifeline networks.

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References


