Dynamic response and reliability analysis of long-span high-pier rigid bridge subjected to multi-support and multi-component ground motion

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Abstract

In recent years, the probability density evolution method for structural random dynamic analysis can capture the refined probability density function and its evolution process with regard to both linear and nonlinear, single degree of freedom and multi-degree of freedom structural systems, which provides the satisfactory requirements of accurate structural design and analysis. In this study, firstly, the dimension-reduction simulation method of fully non-stationary \( mD-nV \) stochastic vector processes is applied to generate multi-component non-uniform seismic excitation for long-span bridge structures. Secondly, the finite element model of a long-span and high-pier rigid frame bridge with practical engineering background is established. And the representative time-histories of multi-component non-uniform and non-stationary ground motion realized by the dimension-reduction representation are used as random external excitation of the target bridge structure. Then, the probability density evolution method is employed to analyse the refined dynamic response of bridge structure subjected to different seismic excitations. Finally, combined with the equivalent extreme events, the seismic reliability of bridge structure under multi-dimensional non-uniform seismic excitation is investigated. This study verifies the effectiveness of the dimension-reduction representation for simulation realization of multi-support non-uniform ground motion. Meanwhile, the influence of multi-component non-uniform random ground motion on the seismic response and dynamic reliability of long-span and high-pier rigid frame bridge is thoroughly scrutinized, which can provide the basis for the seismic design and analysis of bridge structures.

Keywords: Earthquakes; Dimension-reduction; Bridge; Reliability analysis

1. Introduction

Studies show that, due to the significant spatial variability of ground motion, the spectrum, amplitude and phase of ground motion suffered by different supports of long-span structures are different under the action of earthquake, which is the main factor causing the seismic damage of long-span structures. At the same time, the seismicity is multi-dimensional, that is, three-dimensional translational components including two horizontal components and one vertical component can be observed in one ground motion event. The spatial variability and multi dimension of ground motion have great influence on the seismic response and damage mechanism of large spatial structures. Therefore, for the seismic analysis of large and complex spatial structures, the spatial correlation multi-support and multi-dimensional ground motion input is used, so as to make the structural design safe and reasonable [1].

In recent years, with the rapid development of China's social economy and transportation, the number of new bridges has increased significantly. Among them, the high-pier and long-span rigid frame bridges are widely used in Western China because of their reasonable internal force distribution, strong crossing capacity and beautiful appearance. At the same time, most of the above regions belong to the areas with frequent seismic activities, thus the seismic analysis and design of high-pier and long-span rigid frame bridges have gradually become a research hotspot in the engineering field. For the dynamic response analysis of high-pier and long-span rigid frame bridge under multi-support non-uniform seismic excitation, Li and Shi used a group of different ground motion time-histories as inputs of a four-span continuous rigid frame bridge considering the traveling wave effect. The analysis showed that the dynamic response of the bridge girder became larger under the influence of the traveling wave effect [2]. Pan et al. analyzed the influence of traveling wave effect on the difference between high and low piers of continuous rigid frame bridge under multi-support non-uniform excitation. The research showed that the greater the difference between high and low piers, the greater the influence of traveling wave effect on the structure [3]. Li et al. analyzed the vulnerability of a long-span continuous rigid frame bridge under multi-point excitation, and gave the failure probability of this kind of structure under earthquake action [4]. Based on the probability density evolution method, Liu et al. studied the random seismic response of the double track bridge of Shibanpo Yangtze River Bridge in Chongqing, and realized the evaluation of the seismic reliability of the bridge combined with the principle of equivalent extremum [5].

Based on the above research status, it can be concluded that the main challenges of seismic response analysis of high-pier and long-span rigid frame bridges are as follows: 1) At present, inconsistent seismic excitation is often used, and the influence of multi-dimensional effect of ground motion on structural dynamic response is seldom considered; 2) Generally, the measured records or a few synthetic seismic time-histories are usually selected for the seismic response analysis of the above-mentioned bridge structures as the input, which does not fully consider the randomness of the ground motion and the structural parameters, and fails to obtain a more refined seismic response and reliability assessment of the structure. Therefore, in this study, the idea of dimension-reduction of random function is introduced to establish the dimension reduction model of multi-support and multi-dimensional \( mD-nV \) fully non-stationary seismic random field, that is, just two elementary random variables can be used to accurately express the non-stationary \( mD-nV \) random field
on the probability density level. Furthermore, the finite element analysis model of high-pier and long-span continuous rigid frame bridge considering pile-soil effect is established based on the ANSYS general finite element analysis platform according to an actual project. Combined with the probability density evolution theory, detailed dynamic response analysis of the bridge structure under four multi-support and multi-dimensional seismic working conditions are implemented. This study can lay a foundation for the accurate dynamic response analysis and dynamic assessment of large and complex engineering structures under random earthquake excitation, and provide a theoretical basis for the seismic analysis and design of high-pier and long-span bridges.

2. The spectral decomposition expression of multi-support and multi-dimensional seismic random field

Suppose that \( X(t) = [X_1(t), X_2(t), L, X_n(t)] \) is a 3-D-nV random vector process, in which each component process is \( X_i(t) = [X_{i1}(t), X_{i2}(t), X_{i3}(t)]^T \). Generally, the probability information of the random vector process can be completely determined by the evolutionary power spectral density (EPSD) matrix:

\[
S(t, \omega) = \begin{bmatrix}
S_{11}(t, \omega) & S_{12}(t, \omega) & \cdots & S_{1n}(t, \omega) \\
S_{21}(t, \omega) & S_{22}(t, \omega) & \cdots & S_{2n}(t, \omega) \\
\vdots & \vdots & \ddots & \vdots \\
S_{n1}(t, \omega) & S_{n2}(t, \omega) & \cdots & S_{nn}(t, \omega)
\end{bmatrix}
\]  
(1)

where \( n \) is the number of support nodes at the seismic action; \( S_{ij}(t, \omega) \) is the EPSD matrix of three directional ground motion components at the same point; \( S_{ij}(t, \omega), (j \neq i) \) is the mutual EPSD matrix of three directional ground motion components at different points, respectively.

Further, \( S_{ii}(t, \omega) \) and \( S_{ij}(t, \omega) \) can be unified in the following form:

\[
S(t, \omega) = \begin{bmatrix}
S_{i1}(t, \omega) & S_{i1,j}(t, \omega) & \cdots & S_{i1,n}(t, \omega) \\
S_{i2,j}(t, \omega) & S_{i2,j}(t, \omega) & \cdots & S_{i2,n}(t, \omega) \\
\vdots & \vdots & \ddots & \vdots \\
S_{in,j}(t, \omega) & S_{in,j}(t, \omega) & \cdots & S_{in,n}(t, \omega)
\end{bmatrix}
\]  
(2)

with \( i, j = 1, 2, L, n \) where \( S_{i1}(t, \omega), S_{ij}(t, \omega) \) and \( S_{ij}(t, \omega) \) denote the EPSD of the three-directional seismic component of the \( i \)-th point; \( \gamma_{ij} \) is the coherence function of the \( i \)-th point and the \( j \)-th point (if \( i = j \), then \( \gamma_{ij} = 1 \)), respectively.

Furthermore, the EPSD matrix \( S(t, \omega) \) of multi-support and multi-dimensional fully non-stationary random seismic field can be decomposed into the following form [6]:

\[
\gamma(t, \omega) = D(t, \omega)\gamma(\omega)D^T(t, \omega)
\]  
(3)

where \( T \) denotes matrix transpose; \( D(t, \omega) = diag[D_1(t, \omega), D_2(t, \omega), L, D_n(t, \omega)] \) denotes diagonal matrix, in which \( D_i(t, \omega) = diag[\sqrt{S_{ii}(t, \omega)}, \sqrt{S_{ij}(t, \omega)}, \sqrt{S_{jj}(t, \omega)}] \).

In Eq. (3), \( \gamma(\omega) \) is the coherence function, which can be defined as:

\[
\gamma(t, \omega) = \begin{bmatrix}
\gamma_{11}(\omega) & \gamma_{12}(\omega) & \cdots & \gamma_{1n}(\omega) \\
\gamma_{21}(\omega) & \gamma_{22}(\omega) & \cdots & \gamma_{2n}(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_{n1}(\omega) & \gamma_{n2}(\omega) & \cdots & \gamma_{nn}(\omega)
\end{bmatrix}
\]  
(4)

where \( \gamma_{ij}(\omega) \) can be expressed as

\[
\gamma_{ij}(\omega) = \begin{bmatrix}
\gamma_{ix,iy}(\omega) & \gamma_{ix,iz}(\omega) & \gamma_{ix,jy}(\omega) \\
\gamma_{iy,iy}(\omega) & \gamma_{iy,iz}(\omega) & \gamma_{iy,jy}(\omega) \\
\gamma_{iz,iy}(\omega) & \gamma_{iz,iz}(\omega) & \gamma_{iz,jy}(\omega)
\end{bmatrix}
\]  
(5)

with \( i, j = 1, 2, L, n \).

Generally, the coherence function matrix \( \gamma(\omega) \) is a nonnegative definite Hermitian matrix, which can be decomposed by eigen orthogonal decomposition (POD). If it is a positive definite Hermitian matrix, it can be represented by Cholesky decomposition, i.e. spectral representation. From the above, the spectral decomposition expression of multi-support and multi-dimensional seismic random field can be obtained:

\[
X_{iv}(t) = 2\sum_{p=1}^{M} \sum_{k=1}^{N} \sqrt{S_{iv}(t, \omega_k)}\Delta\alpha G_p(\omega_k) \\
\times \left[ c_{mr}(\omega_k)(R_{rk} \cos \omega_k t - I_{rk} \sin \omega_k t) \\
- D_{mr}(\omega_k)(R_{rk} \sin \omega_k t + I_{rk} \cos \omega_k t) \right]
\]  
(6)

where \( X_{iv}(t) \) and \( S_{iv}(t, \omega_k) \) denote the acceleration component and the EPSD function in the \( v \)-th direction at the \( i \)-th point, respectively.

In Eq. (6), \( R_{rk} \) and \( I_{rk} \) are orthogonal random variables with zero mean, which satisfy the following basic conditions:

\[
E[R_{rk}] = E[I_{rk}] = 0, \quad E[R_{rk}I_{sk}] = 0, \quad E[R_{rk}R_{sl}] = E[I_{rk}I_{sk}] = \frac{1}{2} \delta_{rs}\delta_{kl}
\]  
(7)

3. Dimension-reduction simulation of multi-support and multi-dimensional seismic random field

In general, the simulation of multi-support and multi-dimensional fully non-stationary seismic random field, whether using spectral representation or POD, belongs to the traditional Monte Carlo simulation method, which requires a large number of random variables (up to millions), and the probability information of the generated sample set is incomplete, which brings great challenges to the refined dynamic response analysis of engineering structures. Therefore, this paper introduces the idea of dimension-reduction of random function [6], that is, the orthogonal random variable is defined as the following random function form:

\[
\begin{bmatrix}
\bar{R}_{sl} = \cos(s \times \Theta_1 + l \times \Theta_2) \\
\bar{I}_{sl} = \sqrt{2} \sin(s \times \Theta_1) \times \cos(l \times \Theta_2)
\end{bmatrix}
\]  
(8)

where the elementary random variables \( \Theta_1 \) and \( \Theta_2 \) are uniformly distributed over the interval \((0, 2\pi)\). It can be proved that the orthogonal random variables defined by Eq. (8) can completely satisfy the basic conditions defined in Eq. (7). In this way, the high-dimensional random variable is reduced to 2, and the dimension-reduction simulation of multi-support and multi-dimensional seismic random field is realized.
Four observation points (representing soil sites of I0, I1, II and III) with a distance of 50m between two adjacent points are selected on the ground along the X direction to simulate the acceleration process of multi-dimensional ground motion at the four selected points respectively. Then the multi-support and multi-dimensional non-stationary seismic random field can be regarded as a 1D-12V non-stationary seismic random vector process. Considering the hysteresis coherence effect and traveling wave effect of spatial variability of ground motion, the representative time-histories and the auto-correlation function of ground motion generated are shown in Fig. 1. For each soil site, it can be observed that the shape of the time-history curves of the ground motion in the two horizontal directions are relatively close, because the time-varying power spectrum model used in the modeling of the ground motion components in the two horizontal directions is exactly the same, but due to the different parameters of the intensity modulation function, it can be observed that the peak arrival time in the two horizontal directions is not exactly the same. In addition, there is a significant difference between the vertical acceleration time-history curve and the horizontal acceleration time-history curve, because the difference between the vertical component and the horizontal component is fully considered. Moreover, the simulated auto-correlation function fits well with the corresponding target value. Numerical examples show the effectiveness of the proposed method.

4. Dynamic response and reliability analysis of long-span and high-pier bridge

In this study, the finite element model of high pier long-span rigid frame bridge is based on the actual project. The bridge is a highway bridge, and its seismic fortification category is class B. According to the seismic parameter zoning map of China (GB 18306-2015), the soil site is classified as Type I (moderately weathered basalt), the characteristic period of the bridge site is 0.45s, and the seismic peak acceleration is 0.17g. The seismic fortification level of bridge is 8-degree. The superstructure of the main bridge is a continuous rigid frame with variable cross-section prestressed concrete, of which the total length is 281m. The height of No.2 and No.3 main piers are 123m, and the height of transition piers are 69.5m (No.1) and 62.2m (No.4) respectively. The finite element model of the bridge structure is shown in Fig. 2.

Figure 2. The finite element model of the bridge structure

In order to study the influence of multi-dimensional ground motion on the seismic response of long-span and high-pier rigid frame bridge, four loading conditions are set in this study, and the random seismic response of long-span rigid frame bridge with high piers is investigated in detail under the condition of the combination of different directions of ground motion components along the bridge direction (x-direction), transverse bridge direction (y-direction) and vertical bridge direction (z-direction), that is x-direction excitation (Condition 1), x+y-direction excitation (Condition 2), x+z-direction excitation (Condition 3) and x+y+z-direction excitation (Condition 4), respectively.

Fig. 3 shows the mean and standard deviation (Std. D) of the displacement time-histories along the bridge direction at the top of No.2 main pier under the action of multi-support and multi-dimensional random earthquake of the four different conditions. The average displacement of the pier along bridge direction is larger under condition 1 and 4, and the peak values of both are close. The average displacement of pier top along bridge direction is the smallest under condition 3. At the same time, the Std. D time-history of pier top displacement along the bridge direction has little difference. In fact, in addition to the displacement along the bridge direction, the seismic excitation in other directions will cause the displacement response along the corresponding direction. Therefore, in order to ensure the safety of the high-pier and long-span bridge structure, the influence of multi-support and multi-dimensional seismic input on the structural dynamic

![Figure 1. Simulation results of multi-support and multi-dimensional seismic random field](image-url)
Table 1. Dynamic reliability of the high-pier and long-span rigid frame bridge

<table>
<thead>
<tr>
<th>Inter-story drift</th>
<th>Dynamic reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.005</td>
</tr>
<tr>
<td>No. 1 pier</td>
<td>2.65%</td>
</tr>
<tr>
<td>No. 2 pier</td>
<td>16.72%</td>
</tr>
<tr>
<td>No. 3 pier</td>
<td>20.27%</td>
</tr>
<tr>
<td>No. 4 pier</td>
<td>1.87%</td>
</tr>
<tr>
<td>Global reliability</td>
<td>0.95%</td>
</tr>
</tbody>
</table>

response should be fully considered in the structural seismic analysis.

The proposed dimension-reduction representation of the multi-support and multi-dimensional stochastic ground motion filed is consistent with the probability density evolutionary method in essence, thus the accurate dynamic response analysis of the earthquake-excited bridge structure can be performed. Furthermore, the dynamic reliability evaluation of the structural system can be calculated combing with the equivalent extreme events. In this study, the inter-story drift is introduced to assessment the dynamic reliability of the rigid frame bridge structure. Table 1 shows the dynamic reliability of each pier and the overall dynamic reliability of the bridge structure with the inter-story drift of the pier as the failure criterion. From the table, it can be seen that the dynamic reliabilities of the two transition piers are close, correspondingly, the dynamic reliabilities of the two main piers are close. At the same time, the dynamic reliabilities of the main piers are higher than those of the transition piers, which is due to the strengthening of the main piers in the design to improve the seismic capacity of the main piers and the whole bridge structure.

5. Conclusions

By introducing the idea of random function, the dimension-reduction simulation of multi-support and multi-dimensional nonstationary seismic random field is realized. Furthermore, combined with probability density evolution method, the refined dynamic response and reliability analysis of high-pier and long-span continuous rigid frame bridge under multi-support and multi-dimensional random earthquake is conducted. Numerical analysis verifies the effectiveness of the dimension-reduction simulation method.

REFERENCES