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INCORPORATING EFFECTS OF UNCERTAINTY IN GEOTECHNICAL PARAMETERS VIA PARTIAL FACTORS DERIVED FROM PROBABILISTIC ANALYSIS

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Geotechnical engineering frequently encounters significant uncertainties in soil properties and environmental conditions. While design has traditionally relied more on deterministic approaches, the increasing complexity of projects now calls for a shift towards probabilistic frameworks and the second generation of the Eurocode has now included a detailed guideline for reliability-basedverification of limit states in design and assessment of geotechnical structures. Probabilistic methods offer substantial advantages for evaluating uncertainties in input parameters and their effects on analysis outcomes. However, since current codes are structured around partial factors, practicing engineers often prefer designs expressed in these terms. Translating probabilistic analysis results into partial factors can thus enhance the accessibility of the framework for decision-making under uncertainty. This paper presents approaches for interpretingresults from probabilistic analyses using partial factors, ensuring alignment with code requirements. Examples from slope stability and bearing capacity analyses are included to illustrate these approaches.

Keywords: uncertainty, probabilistic analysis, partial factor, reliability index, consequence class, variability class

1. Introduction

Deterministic methods offer engineers a straightforward approach for determining design parameters. However, these methods often fall short in systematically addressing uncertainties. In contrast, probabilistic methods provide a robust framework for quantifying and managing these uncertaintiesto account for a realistic range of outcomes.They enable data-driven decision-making, refine design parameters using empirical data, and enhance reliability for future projects. These methods also promote regulatory flexibility by accounting for local conditions, aiding the development of adaptable codes and standards.Additionally, sensitivity analyses within probabilistic frameworks pinpoint critical parameters influencing failure risks or serviceability limitations, which will enable designers to prioritize key project aspects and tailor site investigations accordingly.As geotechnical projects grow in complexity, the adoption of probabilistic approaches is becoming increasingly crucial.

While the full adoption of probabilistic methods remains limited in the state-of-the-practice of geotechnical engineering, some of their principles are integrated into guidelines such as Eurocode 7 and various national standards. Examples include:

- ~ Consequence and reliability classes(Eurocode 7, 2004)
- ~ Characteristic values(Eurocode 7, 2004)
- ~ Partial factors(Eurocode 7, 2004)
- ~ Safety classes (DIBK, 2017)

In this paper,the main focus is on the partial factor approach. In geotechnical design, partial factors are commonly employed toaccount for uncertainties in soil properties. These factors are applied to characteristic values to derive design values of geotechnical parameters. Interpreting these values in a probabilistic context provides a more nuanced understanding of the safety margins involved. In this paper, three different ways in which probabilistic methods can be applied through the partial factor technique are discussed.

2. Design values according toEurocode 7 (2004)

Eurocode 7, clause 2.4.6.2 (1P) provides guideline for calculating design values of geotechnical parameters stating:*Design values of geotechnical parameters (X_d) shall either be derived from characteristic values (X_k) using the following equation:*

$$X_d = X_k / \gamma_M \tag{1}$$

or *shall be assessed directly,* where γ_M is the partial factor.

It is obvious from Eq.1 that the selection (definition) of characteristic values is important for the determination of the design value. Eurocode(Eurocode 7, 2004), Clause 2.4.5.2 gives some suggestions for the selection of characteristic values. There are also other guidelines that define characteristic values based on statistics. For example, (DNVGL-RP-C207, 2017)suggests characteristic values based on the 2.5% quantile, the 5% quantileand the most probable value.

3. Interpretation of results from probabilistic analysis in relation to partial Factors

Eurocode 7 clause 2.4.6.2 (1)P opens for the assessment of design values directly. In the same section,clause 2.4.6.2 (3)Pfurther states that*if design values of geotechnical parameters are assessed directly, the values of the partial factors recommended in Annex A should be used as a guide to the required level of safety*.We maythus take advantage of probabilistic frameworks for considering uncertainties when directly assessing design values. By performing a reliability analysis,the design values of parameter i may be found, by evaluating the value of the parameter at the design point in general, according to:

$$x_{d,i} = F_{x,i}^{-1}\left[\Phi[(u_i^*)]\right] \tag{2}$$

where $u_i^* = \frac{x_i^* - \mu_i}{\sigma_i}$ is the transformed value of the parameter i at the design point (x_i^* being the value of the parameter i at the design point, $\mu_i$and $\sigma_i$being its mean and its standard deviation respectively), $\Phi()$ is the standard normal distribution function and $F_{x,i}^{-1}$ is the inverse of cumulative distribution function.

With the assumption of normally distributed variables, the design value of parameter i can be found as

$$x_{d,i} = \mu_i(1 - \alpha_i \beta V_i), \tag{3}$$

where β is the reliability index, α_i is sensitivity of the limit state to parameter i, μ is the mean value of the reliability index, V is the coefficient of variation (COV) which includes the inherent, measurement, transformation and statistical variability. We may also write the characteristic value, for normally distributed variables, as

$$x_{k,i} = \mu_i(1 - \kappa_i V_i) \tag{4}$$

whereκ_i is a constant determined by the definition of the characteristic value;$\kappa_i = 0$, when the mean value is used for defining the characteristic value.Combining Eq.3 and Eq.4 the partial factor may be calculated as:

$$\gamma_{m,i} = \frac{1 - \kappa_i V_i}{1 - \alpha_i \beta V_i}. \tag{5}$$

The influence of the various parameters on the resulting partial factor is illustrated in Figure 1.For a given coefficient of variation, the higher the reliability index the higher the partial factor, the higher k_i the lower the partial factor, the higher α_i the higher the partial factor (CUR-publicatie 190, 1997).

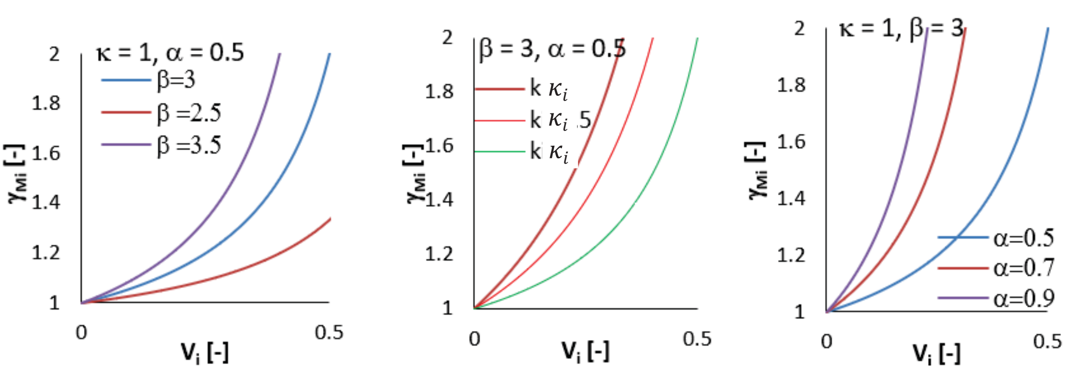


Figure 1: Illustration of the influence of coefficient of variation, κ coefficient, reliability index and parameter sensitivity in the determination of partial factors, normally distributed parameters

Reliability analyses can be considered through the partial factor method using following three approaches.

(i) **Passive approach**: The definition of partial factors in several codes including the Eurocode may be categorized in this approach. In this approach, probabilistic methods are primarily used to assess and prioritize key variables, define the required reliability index based on the assessed importance and perceived risk, and establish the statistical coefficient of variation for these variables which are further used for specifying patrial factors. Once established, these partial factors remain unchanged until substantial new data justifies a revision. The flow diagram in Figure 2 is established based on (CUR-publicatie 190, 1997). This method can be used to determine partial factors when national annexes are prepared.

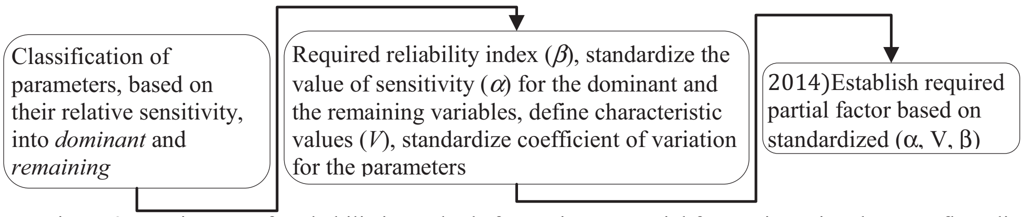


Figure 2: Passive use of probabilistic methods for setting up partial factors in national annex, flow diagram adapted from*(CUR-publicatie 190, 1997)*

(ii) **Intermediate approach**: This approach offers a balance between flexibility and structured guidance by discretizing both the coefficient of variation and the consequence (through the reliability index), as illustrated in Table 1. Practicing engineers can select both the consequence class and the variability class, which allows for design adaptability based on project risk levels and data availability. In this setup, a higher coefficient of variation correlates with a higher required partial factor, thereby addressing increased uncertainty with increased conservatism.The coefficient of variation can be determined either quantitatively, when sufficient data is available, or qualitatively based on professional judgment, experience, and site-specific conditions in the absence of detailed data.

Table 1: Defining partial factors according to the consequence and the variability class (intermediate approach)

Consequence class	Variability class (based on COV*)		
	1	2	3
	(0 – 0.2)	(0.2-0.5)	(>0.5)
CC1 Low consequence	?	?	?
CC2 Medium consequence	?	?	?
CC3 High consequence	?	?	?

*The ranges of COV given in each variability class are randomly chosen and should be investigated further.

(iii) **Active approach**: This approach aligns with Eurocode 7, clause 2.4.6.2 (3P). It entails the calculation of project-specific partial factors derived from probabilistic analysis, with the factors in Annex A of Eurocode 7 serving as the minimum benchmark. See the flow diagram in Figure 3.Other criteria such as the percentage improvement for natural slopes proposed by the Norwegian Water Resources and Energy Directorate (NVE Veileder 7/2014, 2014) may also be considered. The active approach enables the designer to leverage probabilistic methods,offering a flexible framework to address project-specific uncertainties effectively. The Bayesian updating technique can also be utilizedto incorporate the added value of new information in the design and decision-making process. A key limitation of this approach is its reliance on substantial project-specific data, which may not be readily available or feasible for all projects.

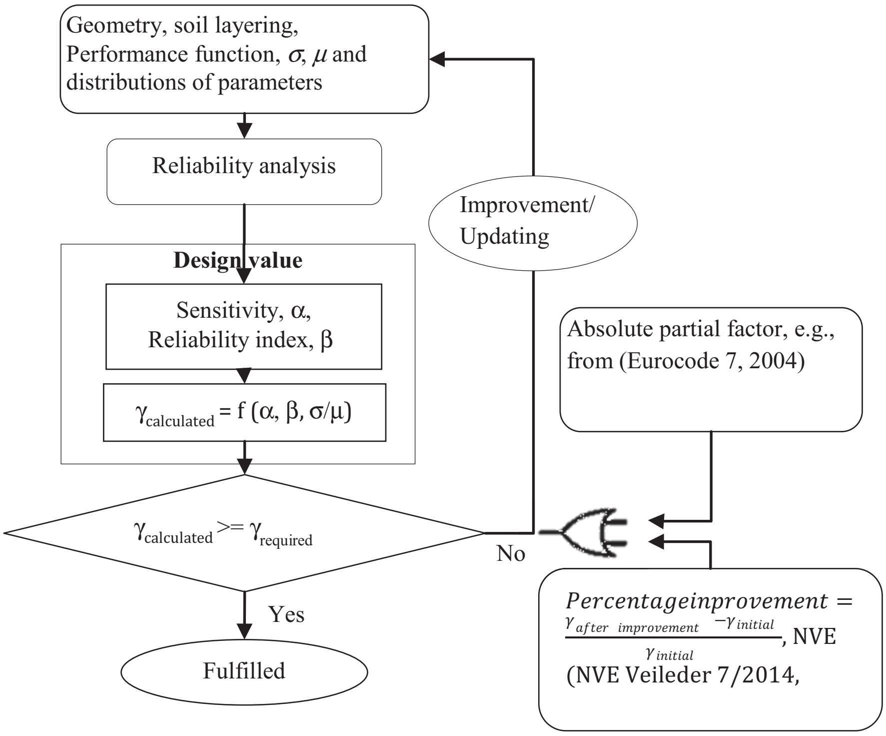


Figure 3: Flow diagramfor the active approach based on*(Tsegaye & Gylland, 2019)*)

2. Illustrative examples

Simple examples of the active approach applied to slope stability and bearing capacity problems are provided in the references(Tsegaye & Gylland, 2019; Tsegaye et al., 2023). Next, examples are briefly presented.

4.1 Short term slope stability

In this example, we will consider a simple probabilistic slope stability analysis using Jabu's stability charts(Tsegaye & Gylland, 2019). The slope under consideration has a height of 10.3 meters and an inclination of 26 degrees.The performance function (safety factor) is defined using Janbu's direct method(Janbu, 1954a) as:

$$F = \frac{N_0 c_u}{p_d}, p_d = \gamma H + q - \gamma_w H_w \tag{6}$$

where N_0 is the stability number, taken as deterministic in this example and N_0 =6.6 is obtained from Janbu's stability chart, and p_d is driving force. The variables includec_u, the undrained shear strength;γ, the unit weight of the soil;q, is surcharge loa;γ_w, the unit weight of water; H, the hight of the slope; and H_w, the tail water height from the slope toe (seeTable 2).

In the passive approach, the design values of the undrained shear strength and the load are determined by applying code-specified partial factors. The safety factor F is then calculated based on these design values, with the requirement that $F \geq 1$is met. If $F < 1$, additional measures are required to ensure adequate safety. This method does not explicitly account for the uncertainties associated with local conditions.

Now, let us consider the active approach where we can obtain calculated partial factor by performing probabilistic analysis where Eq. 6 is considered to define the performance function. Performing Monte Carlo simulation with 107278 realizations (Tsegaye & Gylland, 2019)yielded a reliability index of 1.49. Sensitivityanalysis identified the undrained shear strength as the dominant variablewith $\alpha_{cu} = 0.97$for this problem. This analysis results in a calculated partial factor for the undrained shear strength, $\gamma_{c_u - calculated} \approx 1.4$assuming the characteristic value is equal to the mean.

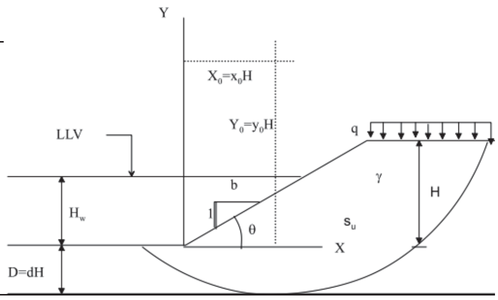
For normally distributed variables, and under the assumption of statistical independencein Eq. 6, the expression for partial factor of the undrained shear strength,γ_{c_u}, can be derived analyticallyas follows:

$$\gamma_{c_u - calculated} = \frac{1 - \kappa_R V_{cu}}{1 - \frac{N_0^2 \mu_{cu} \sigma_{cu} - N_0 \sigma_{cu} H \mu_\gamma - N_0 \sigma_{cu} \mu_q}{N_0^2 \sigma_{cu}^2 + H^2 \sigma_\gamma^2 + \sigma_q^2} V_{cu}}. \tag{7}$$

Here$\mu_i's$ are mean values of the respective parameters, σ_i's are their standard deviations, V_{cu} is the coefficient of variation of the undrained shear strength. Using Eq. 7, the calculated partial factor is $\gamma_{c_{u-calculated}} \approx 1.33$ wherethe mean value of the shear strength is taken as its characteristic value (*i.e.*, $k_R = 0$). The value is lower than the result from the Monte Carlo simulation, primarily due to the conservative nature of the analytical solution which assumes a normal distribution for all variables.In the active approach, the calculated partial factor is compared with the code-specified partial factor to determine if additional measures are required.

Table 2: Parameters used in the slope stability example

Variable	Dist.	mean	Stdv.
Unit weight of soil, γ_s[kN/m^3]	LN	20	1
Undrained shear strength, c_u[kPa]	LN	44	8.8
Surcharge load, q [kPa]	LN	10	5



4.2Undrained bearing capacity

In this example, a 6 m wide strip foundation on a clay bedding andloaded with a centric inclined load presented in(Tsegaye, Nøst, & Gjelsvik, 2023) is considered. The NTNU bearing capacity equation (Grande & Emdal, 2010)is used for defining the performance function as follows:

$$\sigma_v = p + N_c c_u, N_c = \pi - 2\tan^{-1} f_\omega + \frac{2}{1 + f_\omega^2}$$
$$f_\omega = \frac{1}{r}\left(1 - \sqrt{1 - r^2}\right) \tag{8}$$

where σ_v is the bearing capacity, p is the surcharge load, N_c is the bearing capacity number and r is the mobilized roughness ratio which depends on the mobilized horizontal shear stress and thus on the horizontal load. The bearing capacity envelope is constructed by plotting the vertical bearing capacity across various values of r.

A MonteCarlo simulation with one million realizations is performed, where the performance function is defined by the minimum distance to the bearing capacity envelopeconsidering the input parameters given in Table 3.The design value of the shear strength is found to be 23.2 kPa resulting in a partial factor of $\gamma_{c_{u-calculated}} \approx 1.29$ when the characteristic value is set equal to mean value. This is lower than specified in Eurocode 7 and therefore measuresmust be implemented.

Table 3: Parameters used in the bearing capacity example

Variable	Dist.	Mean	Stdv.
Undrained shear strength, c_u [kPa]	LN	30	6
Vertical load, q_v [kPa]	LN	100	10
Vertical load, q_h [kPa]	LN	10	5
Surcharge load, p [kPa]	LN	10	2

3. Conclusion

In this paper, we present approaches for incorporating probabilistic analysis results into geotechnical design using the partial factor principle. Selected illustrative examples from previous publications are included. Three methods for addressing uncertainties in design parameters through the partial factor approach are explored: the passive, intermediate, and active methods. The passive method, currently employed in Eurocode and similar standards, applies fixed partial factors universally. In contrast, the intermediate approach and active approach offer greater flexibility, enabling project-specific evaluation of partial factors based on local conditions. The active approach is suitable for projects with extensive data, allowing for tailored probabilistic analyses. Insights gained from implementing the active approach can contribute to refining future practices and standards.We wish to follow up this article with different applications and further elucidation of the intermediate and the active approaches.

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