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PHYSICS-INFORMED NEURAL NETWORKS EMBEDDED BAYESIAN FRAMEWORK FOR LONGITUDINAL TUNNEL PERFORMANCE ANALYSIS

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Physics-Informed Neural Networks (PINNs) have gained significant attention in geotechnical engineering in recent years. Their high efficiency in solving complex partial differential equations (PDEs) and estimating unknown parameters with limited observational data make PINNs particularly suitable and practical for geotechnical applications. However, when dealing with uncertain inputs, standard PINNs trained using nominal values of input parameters may become inaccurate. Moreover, PINNs for solving standard inverse problems lack built-in uncertainty quantification capabilities. These limitations make it challenging to account for uncertainties in many applications, leading to potentially inaccurate results, especially in scenarios with large measurement noise or strong parameter variability. In this paper, we present a physics-informed neural networks embedded Bayesian framework to address these limitations. We demonstrate its application to a practical problem in tunnel engineering: estimating the posterior distributions of the uncertain inputs and characterizing the longitudinal tunnel performance while considering the parameter variability, thereby quantifying the uncertainty arising from input parameter variations, noisy measurements and the physical modelling process. To this end, a parameterized physics-informed neural network is first trained by embedding the governing equations of the soil-tunnel interaction. This surrogate model is trained using an ensemble of tunnel performances generated from a set of realizations of the uncertain inputs. Then Bayesian updating is conducted to estimate the posterior distributions based on the surrogate model and on-site measurements. The effectiveness of the proposed framework is demonstrated through comparison with results from Bayesian updating using a standard numerical model. The results highlight the advantages of the framework in capturing uncertainties and improving predictive performance, while decreasing computational cost, showcasing its potential for practical applications in geotechnical engineering.

Keywords: Tunnel longitudinal performance; Parameterized Physics-informed neural networks; Bayesian updating; Uncertain quantification;

1. Introduction

The length of shield tunnels can extend up to several thousand meters, resulting in often significant longitudinal variations in structural stiffness, subgrade reaction of soil and loading conditions. These variations can have significant impact on tunnel performance measures. Therefore, a calculation model for longitudinal tunnel performance that incorporates these variations is required for ensuring the reliability of tunnel designs.

Modelling the tunnel as an elastic beam resting on an elastic foundation is the most practical approach. The advantage of this approach is the simplicity of the mathematical form of the underlying governing equation system, which enables its solution with various numerical methods. Determining the input parameters of the calculation model of longitudinal tunnel performance is crucial for obtaining accurate predictions. These input parameters can be identified through an inverse analysis in which model parameters are calibrated by utilizing on-site measurement data that are compared to mechanicalmodel predictions at the measurement locations. In this context,the Bayesian framework providesa systematic approach for learning model parameters through combininga prior probabilistic model with measurement, thus enabling quantification of uncertainties in parameter estimates.

Bayesian updating is usually performed numerically through sampling methods. While many efficient sampling techniques have been developed to accelerate the computation, they still require a large number of evaluations of the system's mechanical response, resulting in significant computational burden. To alleviate high computational demands, physics-informed neural networks (PINNs) are adopted in this paper for solvingthe governing equations of tunnel responsegiven their proven effectiveness in handling ordinary and partial differential equations (ODEs/PDEs) (Raissi et al. 2019).However, Standard PINNs trained using the nominal values of the input parameters may become inaccurate when dealing with variable inputs.

To address this, we adopt parameterized PINNs (Gasmi and Tchelepi2022) to infer the ensemble realizations of the tunnel performance with a set of samples from the input parameters. The trained network is then embedded within the Bayesian framework to estimate the posterior distribution of the input parameters and the corresponding tunnel longitudinal performance.

2. Methodology

2.1.*Soil-tunnel interaction model for shieldtunnels*

The tunnel is simplified as a continuous homogeneous Timoshenko beam,which accounts for rotation and shear deformation, resting on a Pasternak foundation model.Compared to the Winkler foundation model, the Pasternak foundation introduces a shear layer to consider soil continuity and interaction effects. The governing differential equationfor a tunnel settlement$w(x)$ subjected to a longitudinaldistributed loading can be expressed as Eq.(1):

$$\frac{d^4w(x)}{dx^4}+\frac{\left(-G_sD_t\xi(\kappa GA)_{eq}-(EI)_{eq}kD_t\right)}{\left((EI)_{eq}\xi(\kappa GA)_{eq}+(EI)_{eq}D_tG_s\right)}\frac{d^2w(x)}{dx^2}+\frac{\xi(\kappa GA)_{eq}kD_t}{\left((EI)_{eq}\xi(\kappa GA)_{eq}+(EI)_{eq}D_tG_s\right)}w(x)=-\frac{D_t}{\left(\xi(\kappa GA)_{eq}+D_tG_s\right)}\frac{d^2q(x)}{dx^2}+\frac{\xi(\kappa GA)_{eq}D_t}{\left((EI)_{eq}\xi(\kappa GA)_{eq}+(EI)_{eq}D_tG_s\right)}q(x) \quad (1)$$

where x denotes the longitudinal coordinate of the tunnel segment,$D_t$is the outer diameter of the tunnel, $(EI)_{eq}$is the equivalentlongitudinal bending stiffness, $(\kappa GA)_{eq}$is the equivalent shearstiffness, ξ is a modification factor introduced to consider the effect of the joints between the segments, $G_s$is the foundation shear modulus, k is the foundationcompressionmodulus, and $q(x)$ represents the loading acting on the tunnel along the longitudinal direction.

Among these parameters, ξ and $k$are subject to significant uncertainty. The former need to be determined through experiments or field monitoring, while for the latter, even within the same type of soil, guidelines provide a wide range of suggested values.Additionally,$q(x)$ is also uncertain,as it is challenging to precisely account for the environmental impacts on the tunnel.To account for these uncertainties, these parameters are treated as uncertain variables, whose distributions will be quantified through Bayesian updatingusing on-site measurements.

2.2.*Parameterized physics-informed neural networks*

Parameterized physics-informed neural networks (P-PINNs) extend standard PINNs to solve parameterized ODEs/PDEs, where the solution is approximated using a neural network $\tilde{u}(\boldsymbol{x},\boldsymbol{\theta};\boldsymbol{\phi})$. Here,$\boldsymbol{x}$represents the spatial coordinate,$\boldsymbol{\theta}$denotestheODE/PDE parameters, and $\boldsymbol{\phi}$ refers to the neural network parameters.The network is trained by minimizingtheODE/PDE residual loss at a set of collocation points, along withthe loss related to initial and boundary conditions. Through this approach, physical laws represented by ODEs/PDEs are embedded into the neural network.In the tunnel problem, the ODE is given by Eq. (1), where the uncertain parameters correspond to $\boldsymbol{\theta}$. With P-PINNs, the weights $\boldsymbol{\phi}$ are trained for a set of ensemble realizations of the uncertain parameters $\boldsymbol{\theta}$, thus ensuring fast and accurate response predictions for multiple parameter inputs.

2.3.*P-PINNs embedded Bayesian framework*

Fig. 1shows the schematic of the P-PINNs embedded Bayesian framework.The P-PINNs model is first trained using prior samples of the uncertain parameters. Following the Bayesian updating process(refer to Kamariotis et al. 2023), posterior samples of these parameters are obtained and then used as inputs to the trained P-PINNs model to predict the system's performance.

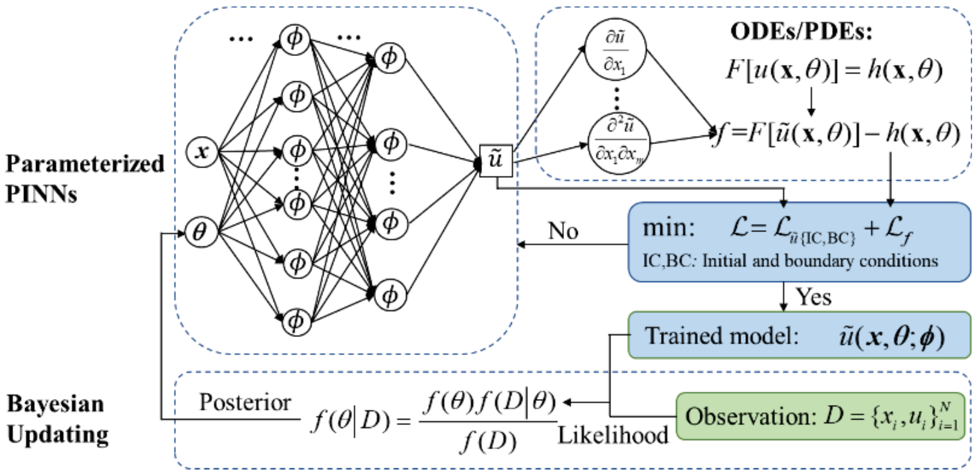

Fig. 1.Schematic of the P-PINNs embedded Bayesian framework

3. Applications and Results

To demonstrate the effectiveness of the proposed method, the probabilistic analysis of a tunnel section in Shanghai is conducted, and the results are compared to those from Bayesian updating using a numerical solution of Eq. (1) with thefinite difference method (FDM).

3.1.*General information onthe case-study*

A leakage accident occurred during the construction of the connecting passage of the twin tunnels in Shanghai Metro Line 18, causing significant longitudinal differential settlementof the twin tunnels in close proximity. The tunnels are situated insilty clay and sandy siltat an approximate depth of24.0m. The outer diameter $D_t$and the lining thickness $t$are 6.6 m and 0.35 m, respectively.Detailed description of the geological condition and the lining structure can be found in Liu et al. (2020).

The averageYoung's modulus of the soil is assumed to be 15 MPa andthe Poisson's ratio is 0.3. Usingthe empirical formula by Tanahashi(2014), $G_s$is calculated as 31730.8 kN/m.The estimated $(EI)_{eq}$ and $(\kappa GA)_{eq}$are 1.68×10^8kNm^2 and 2.51×10^6kN,respectively,based on the methods proposed by Shiba et al. (1988) and Wu et al. (2015).

3.2.*Calculation model*

The impact of water leakage is assumed as a loading acting on the tunnel, conforming to the Gaussian function (see Eq. (2)),which captures the localized pressure concentration and gradual dissipation,

$$q(x) = -ae^{-\frac{x^2}{2b^2}} \qquad (2)$$

in which$a$ and $b$represent the loading parameters, characterizing the amplitude and distribution range of the loading. Together with the structural parameterξ, they are treated as uncertain variables to be learned via Bayesian updating. These parameters are assigned weakly informative priors in the form of uniform distributions: a is uniformly distributed between 0 and 50,000, b between 10^{-5} and 25 and ξ between 10^{-5} and 20.This choice reflects an approach that minimizes prior bias while constraining the parameters to physically plausible ranges when prior knowledge is limited.The lognormal distribution is assumed for the soil parameter$k$. According to the Shanghai local code for geotechnical site investigation, the value of k in silty clay and sandy silt is expected to vary between 30000kN/m^{3}and 50000kN/m^3. These two values are taken as the 5% and 95% quantiles ofthe lognormal distribution for k, corresponding to a mean of 10.56 and a standard deviation of 0.16.

A total of 50,000 sets of $\boldsymbol{\theta}$= (k, ξ, a, b) are independentlydrawn from the prior marginal distributions. These parameter sets are then combined independently with 50,000 collocation points$x$, which are randomly sampled from the computation domain $x\in$ (-150, 150). Together, they form the input training data for the P-PINNs. The loss functions are minimized using the sequential optimization with Adam followed by L-BFGS (Shinet al. 2022).

The measuredsettlement of the lower line of the twin tunnels are used to update the prior distributions of the uncertain variables. It is assumed that the measurement issubject to a multiplicative error μ, which is described by a lognormal distribution. Specifically, the natural logarithm of the error, ln(μ), follows a normal distribution with a mean of 0 and a variance of σ^2, where σ is treated as an uncertain variable and follows a uniform distribution with lower bound of $10^{-5}$and upper bound of 10. The likelihood function is expressed as Eq. (3):

$$L_i(\boldsymbol{\theta}) = \frac{1}{\sqrt{2\boldsymbol{\pi}}\tilde{A}} e^{-\frac{(ln(y_i)-\ln(\tilde{u}(x_i,\boldsymbol{\theta})))^2}{2\tilde{A}^2}} \qquad (3)$$

in which y_i is the measurement at location x_i, $\tilde{u}(x_i,\boldsymbol{\theta})$ represents the corresponding settlement evaluated by the trained P-PINNs model. The multiple measurements are assumed to bestatisticallyindependent given the model parameter $\boldsymbol{\theta}$. Therefore, the combinedlikelihood of all measurements is expressed as the product ofthe likelihoods corresponding to each individual measurement.

Sequential Monte Carlo (SMC) method (Kamariotis et al. 2023) is employed for Bayesian updating.The posterior prediction of the settlement is obtained by usingsamples from the posterior distributions of the uncertain variables as inputs to the parameterized P-PINNs.

3.3.*Results and discussion*

Fig.2presents Bayesian updating results, with comparisons made between the P-PINNs and FDM approaches. The posterior distributions of the parameters and the tunnel response predicted by both methods exhibit high consistency, demonstrating the validity of the P-PINNs framework.However, there is a significant difference in computational efficiency: P-PINNs completes the analysis in about two minutes, while the FDM method requires several hours.Notably, the measurementfalls within the 95% confidence interval (CI) of the posterior distribution, further validating the rationality of the Bayesian updating results.

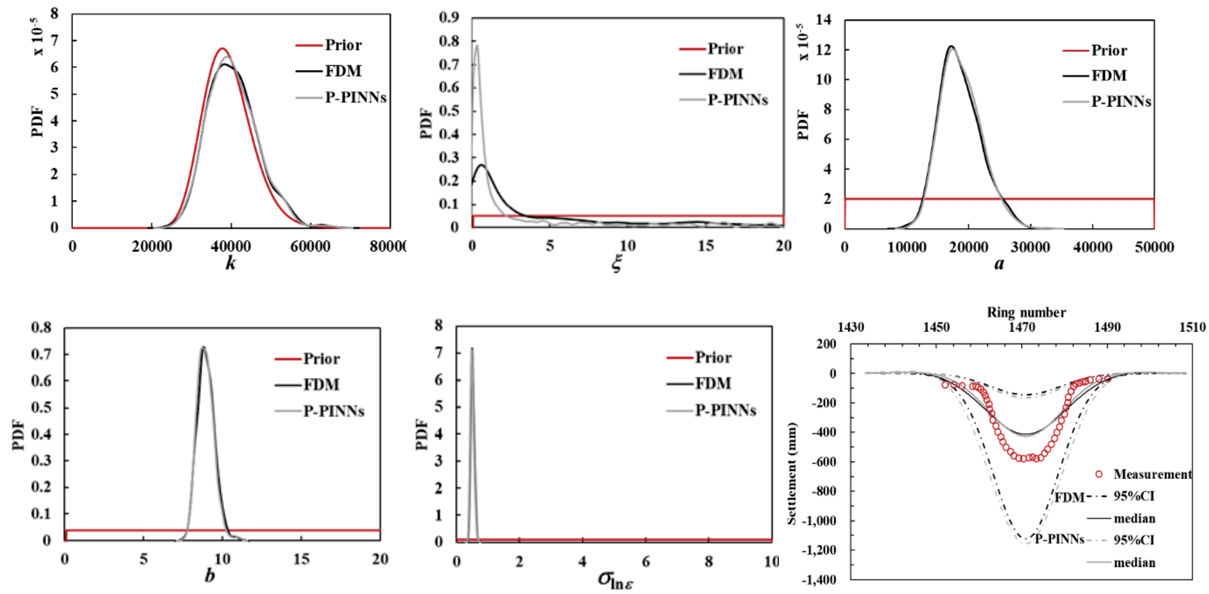

Fig. 2.Prior and posterior distributions of uncertain variables (k, ξ, a, b,$\sigma_{\ln\varepsilon}$) and posterior prediction of tunnel settlement (median and 95% CI) comparing P-PINNs and FDM results.

4. Conclusion

This study proposes a P-PINNs embedded Bayesian framework for updating the parameters of the calculation model for longitudinal tunnel performance. By integrating the P-PINNs with Bayesian updating, the method effectively quantifies uncertainties in input parameters and model predictions while leveraging the efficiency of P-PINNs in solving parameterized ODEs.

The framework is successfully applied to a real-life case study of longitudinal tunnel performance, showcasing its capability in addressing practical engineering challenges. The results highlight the high computational efficiency of the proposed approach, significantly reducing the time required for posterior estimation compared to conventional numerical methods, while maintaining accuracy. This demonstrates the potential of the P-PINNs embedded Bayesian framework for real-time and comprehensive assessments of tunnel performance under uncertainty.

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