Proc. of the 9th International Symposium on Geotechnical Safety and Risk (ISGSR)
Edited by Zhongqiang Liu, Jian Dai and Kate Robinson
Published by Research Publishing, Singapore.
doi: 10.3850/GRF-25280825_isgsr-188-P310-cd

ISGSR2025

HIERARCHICAL BAYESIAN MODELLING FOR UNCERTAINTY QUANTIFICATION IN SIMPLIFIED TUNNEL DEFORMATION MODELS

Yelu Zhou

Engineering Risk Analysis Group, Technische Universität München, Germany.
Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education and Department of Geotechnical Engineering, Tongji University, China. E-mail: yelu.zhou@tum.de

Xinyu Jia

Engineering Risk Analysis Group, Technische Universität München, Germany. E-mail: xy.jia@tum.de

Iason Papaioannou

Engineering Risk Analysis Group, Technische Universität München, Germany. E-mail: iason.papaioannou@tum.de

Dongming Zhang

Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education and Department of Geotechnical Engineering, Tongji University, China. E-mail: 09zhang@tongji.edu.cn

Hongwei Huang

Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education and Department of Geotechnical Engineering, Tongji University, China. E-mail: huanghw@tongji.edu.cn

Daniel Straub

Engineering Risk Analysis Group, Technische Universität München, Germany. E-mail: straub@tum.de

When analysing the longitudinal performance of shield tunnels, simplified model assumptions are typically employed. As a result, deformation predictions exhibit significant uncertainty. Routine maintenance for tunnels allows for deformation measurements at the same locations at different points in time. By analysing these deformation measurements, we aim to infer and quantify the parameter uncertainty in the simplified models, thereby enabling more accurate probabilistic estimations of future tunnel deformations. The classical Bayesian approach, while powerful in integrating prior knowledge with observed data to update beliefs about uncertain parameters, faces limitations due to the assumption that all uncertainties are encapsulated within a single level of probability distributions. This can lead to biased results and underestimation of variability when dealing with complex and heterogeneous datasets typical in geotechnical engineering. These limitations highlight the need for a more flexible and comprehensive modelling framework that can account for the intricate nature of the simplified model's parameters and their impacts on tunnel deformation. The hierarchical Bayesian modelling framework addresses these issues by allowing for multiple levels of uncertainty and variation to be modelled simultaneously. This method structures the problem into different layers, where parameters at one level are treated as random variables that depend on the hyperparameters at a higher level. By adopting a hierarchical Bayesian framework, one can better quantify the uncertainties associated with the parameters of the simplified tunnel deformation model and improve the predictive accuracy of tunnel performance. In this contribution, we adapt the hierarchical Bayesian framework to the prediction of tunnel deformation with aim at enhancing the forecasting ability in tunnel maintenance. The approach is tested on a real-life example: an interval tunnel of Shanghai metro line 1, located in the city center in Shanghai.

Keywords: hierarchical Bayesian modelling framework; uncertainty quantification; tunnel deformation; simplified model

1. Introduction

Since the pioneering framework developed by Beck and Katafygiotis (1998), Bayesian approaches have been widely used for model identification, selection, and response predictions in various engineering disciplines. However, challenges persist in accurately quantifying uncertainties, particularly as the classical Bayesian modelling (CBM) framework tends to underestimate parameter uncertainties when datasets grow larger. This limitation arises from its inability to capture parameter variability of probabilistic models of material properties, environmental conditions, measuring process, etc.

To address these challenges, the hierarchical Bayesian modelling (HBM) framework has emerged as a robust alternative (Behmanesh et al., 2015). HBM introduces an additional probabilistic layer, parameterizing the distribution of the model parameters with hyperparameters to be estimated based on multiple sets of measurements.

Rather than directly computing the posterior distribution of model parameters, HBM focuses on the posterior distribution of hyperparameters to capture the variability of model parameters.

Interest in HBM is increasing in geotechnical engineering, with most applications focusing on cross-site variability (Ching et al. 2021). In tunnel engineering, long-term settlement remains a persistent concern, largely influenced by external environmental conditions over time. These changing environmental states affect not only the representation of loads in simplified tunnel deformation models but also the values of spatially averaged soil properties and modified structural parameters within load-dependent influence zones. The HBM framework is well-suited to such scenarios, as it can capture the temporal variabilities in these parameters by leveraging monitoring data collected at different points in time. This study focuses specifically on environmental conditions that are relatively stable but exhibit minor fluctuations, excluding environmental changes with noticeable trends. This narrower focus allows for a more precise exploration of the variability within stable conditions.

In this work, the HBM framework is applied to long-term monitoring data from Shanghai Metro Line 1, Shanghai's first metro tunnel. The study integrates settlement measurements from a stable yet fluctuating period to quantify uncertainties of the time-varying parameters in a simplified tunnel deformation model. The results obtained using HBM are compared with those derived from the CBM, showcasing its advantages in capturing temporal variability and parameter uncertainties, enabling more accurate and reliable tunnel settlement predictions.

2. Brief Review of the HBM Framework

Consider the tunnel deformation predictive model $G(\boldsymbol{\theta})$ parameterized through a model parameter vector $\boldsymbol{\theta} \in \mathbb{R}^{N_\theta}$, where N_θ is the total number of uncertain parameters in the predictive model. Let $\boldsymbol{D}=\{\boldsymbol{D}_i; i = 1,2,\ldots,N_D\}$ represent N_D independent measured datasets collected from a shield tunnel during routine monitoring, where $\boldsymbol{D}_i$ defines a dataset consisting of settlement measurements taken at the same locations at a specific point in time. Classical Bayesian methods update the prior probability distribution of $\boldsymbol{\theta}_i$ with the i-th dataset $\boldsymbol{D}_i$. Due to the inherent variabilities introduced by modelling and measurement errors, fluctuating environmental and operational conditions, as well as uncertainties in soil properties and tunnel structure parameters, it is expected that the estimated uncertain parameters $\boldsymbol{\theta}_i$ will vary across datasets.

To capture such variations, HBM introduces hierarchy by assigning a prior probabilistic model for the model parameters $\boldsymbol{\theta}_i$. Specifically, $\boldsymbol{\theta}_i$ is assumed to follow a normal prior distribution parameterized by hyperparameters $\boldsymbol{\psi}$, including a hyper mean $\boldsymbol{\mu}_\theta \in \mathbb{R}^{N_\theta}$ and a hyper covariance matrix $\boldsymbol{\Sigma}_\theta \in \mathbb{R}^{N_\theta \times N_\theta}$, as expressed in Eq.(1):

$$f(\boldsymbol{\theta}_i|\boldsymbol{\psi})=N(\boldsymbol{\theta}_i|\boldsymbol{\mu}_\theta,\boldsymbol{\Sigma}_\theta) \tag{1}$$

Assuming that the multiplicative discrepancy ε between the measurement $\boldsymbol{D}$ and model predictions $G(\boldsymbol{\theta})$ follows the lognormal distribution with unit median, the natural logarithm of the error, $\ln(\varepsilon)$, is normally distribution with a zero-mean and a variance σ^2. For each dataset $\boldsymbol{D}_i$, the variance σ_i^2 may vary. Such variations are also modelled using a normal distribution with mean μ_{σ^2} and variance matrix Σ_{σ^2}. Note that the variance σ_i^2 is at the same level with the model parameter $\boldsymbol{\theta}_i$. In the following, we use $\boldsymbol{\theta}_i$ to represent all uncertain parameters, including the error term, and $\boldsymbol{\psi}$ to represent all hyperparameters.

To evaluate the posterior distribution of hyperparameters within the HBM framework, a two-step procedure is commonly employed (Wu et al., 2019). First, model inference is conducted separately for each dataset $\boldsymbol{D}_i$, using a uniform prior for the parameters $\boldsymbol{\theta}_i$, yielding samples of the uncertain parameters $\boldsymbol{\theta}_i$ from the posterior PDF $f(\boldsymbol{\theta}_i|\boldsymbol{D}_i)$,. Subsequently, the hyperparameters $\boldsymbol{\psi}$ are updated by leveraging the samples obtained from the first step. Specifically, the posterior samples of the hyperparameters can be obtained through Eq.(2) using an asymptotic-sampling method:

$$f(\boldsymbol{\psi}\,|\,\boldsymbol{D}) \propto f(\boldsymbol{\psi})\prod_{i=1}^{N_D}\sum_{k=1}^{N_s}\frac{f\left(\boldsymbol{\theta}_i^{(k)}\,|\,\boldsymbol{\psi}\right)}{f(\boldsymbol{\theta}_i^{(k)})} \propto f(\boldsymbol{\psi})\prod_{i=1}^{N_D}\sum_{k=1}^{N_s}\frac{N\left(\boldsymbol{\theta}_i^{(k)}\,|\,\boldsymbol{\mu}_\psi,\boldsymbol{\Sigma}_\psi\right)}{f(\boldsymbol{\theta}_i^{(k)})} \tag{2}$$

Here, $f(\boldsymbol{\psi})$ is the prior distribution of the hyperparameters, which is assumed to be a uniform distribution, and N_s is the number of samples for each $\boldsymbol{\theta}_i$. The detailed derivation of Eq.(2) can be found in Jia et al.(2020).

Upon obtaining the posterior distribution of the hyperparameters $f(\boldsymbol{\psi}|\boldsymbol{D})$, posterior samples of the uncertain parameters $\boldsymbol{\theta}$ at a future point in time can be generated based on Eq.(1). These samples can then be utilized for posterior predictions, enabling forecasting of the structure's deformation.

3. Application in tunnel deformation evaluation

3.1. *General information on the case-study*

Monitoring of the long-term behaviour of metro tunnels in Shanghai has been conducted semi-annually since construction was completed. This study analyzes data from Shanghai Metro Line No.1. Fig.1(a) presents the cumulative settlement measurements of a section of the line (South Huangpi Road Station to People's Square Station), plotted using data collected every three years between 1995 and 2012. Fig.1(b) displays the settlement-time curves corresponding to the maximum settlement and the intersections of different soil layers, providing a clearer view of deformation trends across different locations. As shown in the figures, the settlement curve began to stabilize around 2006, with only minor fluctuations observed thereafter. Therefore, the data from 2006 to 2012, resulting in a total of 13 datasets are selected for this study. Additionally, to focus on the most critical deformation, only the segment containing the maximum settlement trough, specifically from chainage 10,422m to 10,551m, is analyzed.

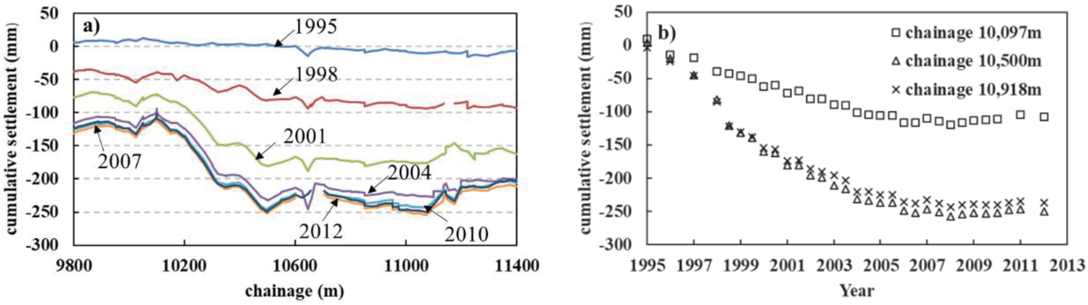

Fig. 1. The cumulative settlement measurements: a) from 1995 to 2012; b) over time at selected chainages

3.2. *Simplified tunnel deformation model*

The tunnel's longitudinal structure is modelled as a continuous Timoshenko beam, with the impact of longitudinal joints incorporated through the reduction factors ξ applied to the tunnel's longitudinal stiffness. The soil–structure interaction between the tunnel and the underlying ground is represented using a Pasternak elastic foundation model, where the foundation compression modulus k is taken as spatially averaged values over load-dependent influence zones. In this study, the distributing of the applied load along the tunnel segment is assumed to follow a Gaussian function (see Eq.(3)):

$$q(x) = ae^{-\frac{(x-c)^2}{2b^2}}$$

(3)

in which the parameter a represents the peak magnitude of the load, b controls the width of the load distribution and c is the location of the peak's center.

The load parameters are treated as uncertain variables, along with the structural parameter ξ and soil parameter k. The governing differential equation for the tunnel settlement $w(x)$ under longitudinally distributed load are given in Liang et al. (2017) along with the values of other model parameters. The finite difference method is employed to solve the equations and compute the tunnel deformation under the prescribed loads.

The prior distributions of the uncertain model parameters and error terms are listed in Table 1. Specifically, the second column shows the prior distributions used in the CBM, while the third and fourth columns present the hyper mean $\mu(\ln\theta)$ and hyper standard deviation $\sigma(\ln\theta)$ of the lognormal prior distribution assigned to θ_i to ensure the non-negative sampling for θ_i.

Table 1. Prior distributions of uncertain model parameters in the CBM and HBM

Parameter	CBM	HBM	
		$\mu(\ln\theta)$	$\sigma(\ln\theta)$
k	LN(9.9,0.42^2)	U(8,11.3)	U(0,1)
ξ	U(10^{-5},20)	U(-5,3)	U(0,1)
a	U(0,50000)	U(-5,10.8)	U(0,1)
b	U(60,300)	U(4,6)	U(0,1)
c	U(540,550)	U(6.29,6.31)	U(0,1)
$\sigma_{\ln(\varepsilon)}$	U(10^{-5},20)	U(-5,3)	U(0,1)

3.3.*Results and Discussion*

The first 12 datasets are used to perform the HBM analysis, and the 12th dataset is used to perform the CBM analysis. Figure 2 illustrates the posterior predictions from both HBM and CBM, with the 13th dataset, marked as the 2012 measurement in Fig. 1(a), used as the test dataset. The HBM-based predictions successfully include the 13th dataset within the 95% confidence interval (CI), whereas CBM fails to entirely encapsulate the data points. In other words, CBM tends to underestimate uncertainty, which may lead to an overestimation of reliability. In contrast, HBM provides a more comprehensive uncertainty quantification by capturing variability across datasets, resulting in more reliable predictive results.

It is noted that while CBM can be extended to handle multiple datasets over time, this requires explicitly incorporating time-dependent parameters into the model formulation, fundamentally altering its structure. This study focuses on comparing the standard CBM implementation, which analyzes each dataset separately, with HBM, which integrates multiple datasets through a hierarchical framework.

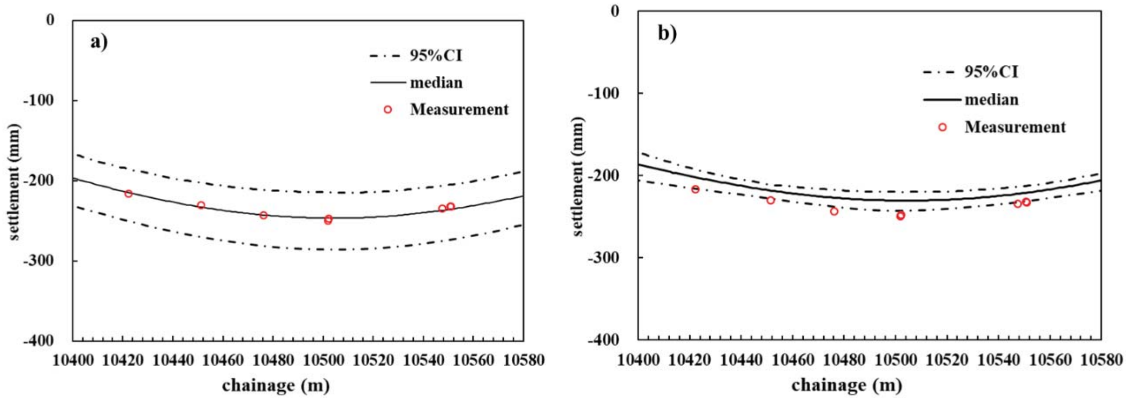

Fig. 2. Posterior prediction of tunnel settlement based on: a) HBM; b) CBM

4. Conclusion

This study presents a HBM framework for evaluating the long-term deformation of tunnels, using settlement data from a section of Shanghai Metro Line No.1 as a case study. The HBM approach is employed to quantify the uncertainties in model parameters as well as modelling and measurement error by leveraging multiple datasets collected over time.

Compared to the CBM, which relies on a single dataset for parameter inference, HBM demonstrates superior performance by integrating information across multiple datasets. The posterior predictions obtained from HBM are able to include unseen data within the 95% CI, whereas CBM fails to achieve the same level of reliability. This underlines the advantage of HBM in enhancing predictive accuracy for tunnel deformation and improving safety assessments by learning variability from diverse data sources.

The findings of this study demonstrate the potential of HBM as a powerful tool for assessing structural performance in complex geotechnical systems, providing a more comprehensive representation of uncertainties and enabling more reliable decision-making.

Acknowledgement

This work is supported by the National Key Research and Development Program of China (No.2021YFF0502200). In addition, the second author acknowledges the financial support from the China Scholarship Council (Grant No. CSC:202306260164) for her research at Technische Universität München.

References

Beck, J. L., and Katafygiotis, L. S. (1998). Updating models and their uncertainties. I: Bayesian statistical framework. *Journal of Engineering Mechanics, 124(4)*, 455-461.

Behmanesh, I., Moaveni, B., Lombaert, G., and Papadimitriou, C. (2015). Hierarchical Bayesian model updating for structural identification. *Mechanical Systems and Signal Processing*, *64*, 360-376.

Ching, J., Wu, S., and Phoon, K. K. (2021). Constructing quasi-site-specific multivariate probability distribution using hierarchical Bayesian model. *Journal of Engineering Mechanics*, *147*(10), 04021069.

Jia, X., Sedehi, O., Papadimitriou, C., Katafygiotis, L., and Moaveni, B. (2020). Two-Stage Hierarchical Bayesian Framework for Finite Element Model Updating. In *Model Validation and Uncertainty Quantification, Volume 3: Proceedings of the 38th IMAC, A Conference and Exposition on Structural Dynamics 2020* (pp. 383-387). Springer International Publishing.

Liang, R., Xia, T., Huang, M., and Lin, C. (2017). Simplified analytical method for evaluating the effects of adjacent excavation on shield tunnel considering the shearing effect. *Computers and Geotechnics*, *81*, 167-187.

Wu, S., Angelikopoulos, P., Beck, J. L., and Koumoutsakos, P. (2019). Hierarchical stochastic model in Bayesian inference for engineering applications: Theoretical implications and efficient approximation. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering*, *5*(1), 011006.