

THE EFFECT OF THE MATERIAL SPATIAL VARIABILITY IN THE SLOPE STABILITY OF SAND TAILINGS DAMS

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Given Chile's status as one of the main copper producers in the world, it is essential to protect critical geo-infrastructure related to the mining industry, such as tailings dams. Tailings, the residual materials from mineral extraction, consist of ground rock, unrecovered metals, and chemical byproducts. The rising global demand for minerals has escalated large-scale mining activities, producing vast volumes of tailings, which are impounded by large embankments and sand tailings dams, where static and seismic slope stability analyses are crucial to prevent catastrophic failures, such as the Brumadinho dam collapse in Brazil, which caused 259 fatalities and severe environmental damage. The slope stability of tailings dams is typically evaluated using the factor of safety (FS), defined as the ratio of resisting to driving forces, and the critical seismic coefficient (k_y) from seismic analyses targeting an FS of 1.0. Such analyses are commonly conducted using deterministic approaches, which often overlook significant uncertainties. Probabilistic methods have gained interest in the industry to address uncertainties in slope stability analysis of tailing dams but generally disregard the spatial variability of geotechnical properties in large geostructures, assuming soil homogeneity. Nevertheless, factors such as material deposition, compaction, and construction processes in tailings dams significantly influence the spatial variability of geotechnical properties, thereby affecting slope stability analyses. This study evaluates the impact of spatial variability on the slope stability of tailings dams by incorporating inherent uncertainties using random fields derived from in-situ cone penetration tests (CPTu) data. The findings highlight the influence of spatial variability on FS and k_y , providing a foundation for ongoing research that will further investigate its impact on seismic performance of tailings dams.

Keywords: Geotechnical uncertainty, Spatial variability, Tailings dams, Slope stability, Random fields, Probabilistic analysis.

1. Introduction

As one of the leading copper producers globally, Chile must safeguard crucial geo-infrastructure linked to the mining sector, particularly tailings dams. These structures, responsible for storing mining waste, are especially susceptible to seismic threats prevalent in Chile and other seismically active regions. The collapse of such facilities can result in substantial human, economic, and environmental devastation. A dramatic example is the Brumadinho dam failure in Brazil, which resulted in 259 deaths and significant environmental degradation, as reported by the (Duncan et al., 2014; Global Tailings Review, 2020). Engineering practices and regulatory standards, both in Chile and globally, underscore the need for specialized geotechnical evaluations to ascertain the seismic stability of such geostructures. These evaluations entail conducting both static and seismic slope stability analyses, which are crucial elements of the design and operational controls. These analyses focus primarily on FS , defined as the ratio of resisting to driving forces, and k_y from seismic analyses targeting an FS of 1.0, as detailed by (Bray & Travasarou, 2009; Duncan et al., 2014). However, the literature and practical experience acknowledge that these assessments are loaded with uncertainties, both inherent and epistemic (Jones et al., 2002; K. K. Phoon & Kulhawy, 1999; K.-K. Phoon et al., 2022; Tang & Phoon, 2021; Xiao et al., 2016). The construction of tailings dams, which can extend over 25 years or more, involves a variety of machinery and equipment, as well as different sources of geomaterials and construction methodologies, as noted by (Guo et al., 2019). This highlights that geotechnical uncertainty, particularly relating to the spatial variability of geotechnical parameters, plays a significant role in the design and operational analysis of these structures. Additionally, the variability in these engineered soils often differs markedly from that observed in naturally formed soils. For instance, recent studies by (Macedo et al., 2024), which focus in characterizing the spatial variability of deposited tailings, underscores the need to incorporate spatial variability to improve the reliability of stability analyses. Accordingly, it is imperative to investigate how the spatial variability of soil geotechnical parameters, which arises from the construction of tailings dam structures, influences the slope stability analysis

in tailing dams. This research aims to address this gap through stochastic, static and pseudo-static, slope stability analysis for an archtype sand tailings dam, considering soil spatial variability of geotechnical parameters. The spatial variability of geomaterials is incorporated into the analysis through random field theory using in-situ CPTu data. This approach enables a comparison between traditional deterministic and probabilistic methods, and analyses that integrate spatial variability. The results show that accounting for spatial heterogeneity identifies critical strong and weak zones influencing failure surfaces, reducing uncertainty in FS and k_y computation. These findings highlight the importance of incorporating spatial variability to enhance the accuracy of slope stability probabilistic analyses in tailings dams. Ongoing work extends this analysis to evaluate the impact of spatial variability and geotechnical uncertainty on seismic performance of tailings dams through time-history analysis using Finite Element Analysis and the Material Point Method.

2. Case Study

This study focuses on a standard tailing dam located in the north-central region of Chile as representative case. Due to confidentiality agreements, the specific location and name of this geostructure remain undisclosed. Fig. 1 shows the main characteristic of the tailing storage facility, which includes a starter dike constructed of natural soil borrow material, topped with the main dam or tailing dam built using the downstream hydraulic construction method of cycloned sand derived from the coarse fraction of the tailings, the foundation formed from in-situ meteorized rock, and tailing composed by the mine waste. The downstream slope has a ratio of 1:4 (V:H), and the upstream slope 1:2. The dam height is 64 m, the crest width is approximately 20 m, and the freeboard is 5 m high. To characterize the spatial variability of the material as detailed below, CPTu were conducted at three different locations along the embankment cross-section, as shown in Fig. 1. These tests offer initial critical insights into the material properties at various elevations within the dam. Furthermore, additional in-situ CPTu data is being utilized for an in-depth characterization of the spatial variability of soil properties on tailing storage facility, aiming to establish benchmark spatial properties for this geomaterial to support future research and practical applications.

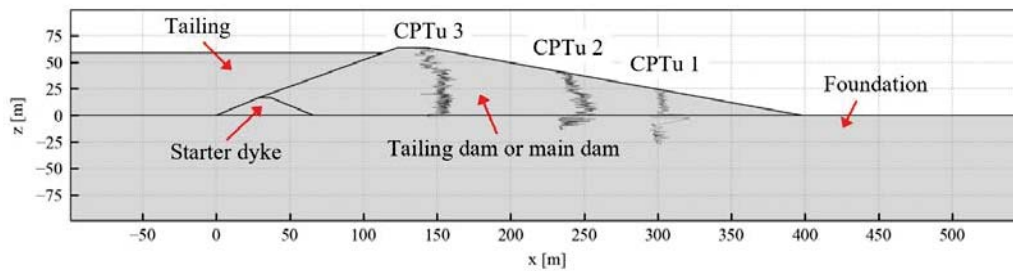


Fig. 1. Diagram of tailings dam and location of CPTu.

3. Geotechnical Uncertainty and Spatial Soil Variability

Spatial variability is a fundamental characteristic of geotechnical materials, affecting both natural and engineered soils. Natural soils, shaped under varying geological conditions, demonstrate considerable variations in properties even within what appears to be uniform units (Griffiths & Fenton, 2004; Jones et al., 2002). Similarly, in engineered geostructures such as sand tailings dams, variability can be influenced by construction factors such as compaction and layering techniques (Macedo et al., 2024). For instance, during tailings dam construction, the hydraulic deposition method creates cycloned sand layers approximately 30 cm thick, subsequently compacted using heavy machinery. This process, influenced by operational and climatic conditions, can lead to significant disparities in the geotechnical properties of the sand, both vertically and horizontally, diverging from those typically observed in natural soils (Macedo et al., 2024). This variability is crucial for predicting geostructural responses, especially in large geostructures like tailings storage facilities (K. K. Phoon & Kulhawy, 1999; K.-K. Phoon & Kulhawy, 1999; Zerva, 1991, 1992). This section outlines methodologies to describe and integrate soil spatial variability into static and pseudo-static analyses of tailing dam during slope stability analysis.

3.1. Geotechnical Properties and Random Fields

The design geotechnical properties for the geomaterials in the tailing's dams are listed in Table 1. Additionally, Table 1 outlines the types of material models used to represent the strength behavior of the geomaterials, as required by limit equilibrium method analysis. The properties in Table 1, defined by mean values from in-situ data, are utilized for the design and operational analysis of the tailing storage facility. These properties serve as a starting point for the deterministic analysis and the subsequent probabilistic analysis, as detailed later.

Table 1. Geomaterial models and design properties.

Material / Soil Classification	Unit weight (kN/m^3)	Strength model	Cohesion (kPa)	ϕ ($^\circ$)	Vertical Stress Ratio	Minimum shear strength (kPa)
Tailings / ML	16	Stress History and Normalized Soil Engineering properties) SHANSEP	-	-	0.1	10
Foundation / Rock	20	Infinite Strength	-	-		
Starter dyke / GW	19	Mohr-Coulomb	10	37		
Main dam or tailing dam / SM	19.9	Mohr-Coulomb	0.1	36.8		

In-situ soil data from CPTu soundings are utilized to model the spatial variability of tailing dam geomaterial using random fields, as suggested by (Griffiths et al., 2009; Jones et al., 2002). This spatial variability is characterized by a two-dimensional correlation that includes vertical (δ_v) and horizontal (δ_h) scales of fluctuation, coefficient of variation (COV), and trends and means derived from in-situ soil data and literature. The spatial correlations demonstrate that geotechnical properties of the soil at proximate locations are interdependent up to a certain distance, beyond which their relationships diminish, resulting in seemingly random behaviors. The CPTu data are normalized to represent the sum of spatial trends and residuals, adopting a modeling approach that accounts for non-stationarity in soil properties caused by varying confining stresses and construction processes in tailing dams. The analysis incorporates log-normally distributed variables, conforming to standard geotechnical practice and ensuring the natural variability of soil parameters. To evaluate the impact of COV in probabilistic slope stability analyses for tailings dams, two scenarios are defined based on the strength properties of the adopted model. Case A (CA) utilizes COV values derived from data reported in the literature, while Case B (CB) employs COV values obtained from in-situ CPTu measurements. This comparison highlights the variability between natural soils and engineered materials in tailings storage facilities, emphasizing the importance of site-specific data. Table 2 summarizes the COV values used in the CA and CB scenarios.

Table 2. COV for Case A (CA) and Case B (CB).

Property	Case A (CA)	Case B (CB)
COV _{γ} (%)	9	4.5
COV _{ϕ} (%)	10	11.4

Spatial correlation is modeled using the autocorrelation function defined by (Vanmarcke, 2010), Eq. (1), where ρ_{ij} represents the correlation coefficient between two points i and j , located at coordinates x and z , respectively. This modeling forms the foundation for generating two-dimensional random fields, integrated into the limit equilibrium analysis of the tailings dam. These random fields allow to identify zones of varying resistance that significantly influence potential failure surfaces, as shown in Fig. 2, which show one of the random fields used in the analysis.

$$\rho_{ij} = \exp \left\{ -\pi \cdot \left[\left(\frac{|x_i - x_j|}{\delta_h} \right)^2 + \left(\frac{|z_i - z_j|}{\delta_v} \right)^2 \right] \right\} \quad (1)$$

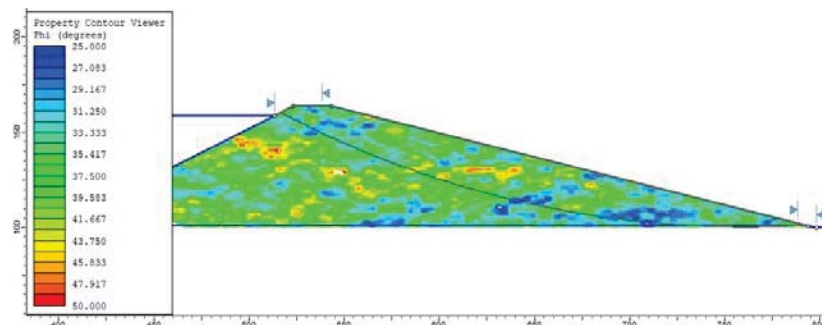


Fig. 2. Random field of internal friction angle, with statistical data from the CPTu.

It is important to note that δ_v for cone resistance (q_c) and friction ratio (R_f) are estimated using the methodology described by (Jones et al., 2002). Furthermore, the transformation equations from (Robertson &

Cabal, 2010, 2022; Robertson & Campanella, 2011) are employed to derive the unit weight (γ) and peak friction angle (ϕ') of the sand tailings from in-situ CPTu data. Subsequently, δ_v for these geotechnical parameters are also computed to assess the transfer of fluctuation scales from in-situ CPTu measurements to properties commonly utilized in slope stability modeling via the limit equilibrium method. The computed δ_v ranges from 3.32 m to 7.37 m, exceeding the average δ_v' values typically reported for natural soils for the same geotechnical properties (K. K. Phoon & Kulhawy, 1999; K.-K. Phoon & Kulhawy, 1999).

4. Slope Stability Analyses and Tailings Dam Model

The Morgenstern-Price method, widely used in limit equilibrium analysis, is selected for its precision and adaptability to diverse slope geometries and soil profiles. This method rigorously ensures both force and moment equilibrium in all force driving the slope stability and accommodates circular and non-circular failure surfaces. It is particularly suitable for deterministic and probabilistic analyses that exclude spatial variability, as well as stochastic analyses that incorporate spatial geotechnical property variability through random field models. For consistency across analyses, slope stability limits are constrained, with entry points at the dam crest and exit points near the downstream toe. A minimum failure depth of 25 m is adopted to focus on significant structural failures of tailings dams, adhering to standard geotechnical practices in Chile. Additionally, the mesh size for generating random fields is determined to ensure adequate density for the spatial distribution of parameters. This is set in accordance with the slice width specified for slope stability analysis, aligning with the modeling of spatial variability of geotechnical soil properties in the tailing dam soil mass. Horizontal seismic coefficients for the pseudo-static analysis are derived from data on maximum credible earthquakes (MCE) specific to the Chilean region (Olivares Allendes, 2023). The analysis identified the lognormal probability density distribution as the most suitable probability distribution function for representing the MCE data, with a mean horizontal seismic coefficient (k_h) of 0.21 and a standard deviation of 0.049. This methodology was implemented using Slide2 from the RocScience geotechnical software suite (Rocscience Inc., 2023).

Two types of analyses incorporating different levels of uncertainty are developed, adhering to established standards in geotechnical analysis in Chilean mining industry. The first is a probabilistic analysis that does not account for soil spatial variability, in which the geotechnical properties ϕ and γ of the main dam are modeled as lognormal random variables to ensure positivity. The third involves spatial variability, utilizing procedures detailed in Section 3 to depict the expected spatial variability of ϕ and γ in the tailings dam considering circular and non-circular failure surfaces. The probabilistic analyses incorporate 5,000 Monte Carlo simulations. This means that 5,000 realizations of the geotechnical properties are used for analyses without spatial variability, and 5,000 random fields are employed for analyses that considered the spatial variability of these properties.

5. Discussion

Fig. 3a–Fig. 3f present histograms derived from the probabilistic analysis of static scenarios. Fig. 3a and Fig. 3b show results from analyses that exclude soil spatial variability, while Fig. 3c and Fig. 3d incorporate this variability with circular failure surfaces, and Fig. 3e and Fig. 3f with non-circular failure surfaces. Each figure includes the mean (FS_μ) and COV(FS_{COV}) for FS, and reliability index (RI) calculated following (Duncan et al., 2014). A comparison between scenarios CA (Fig. 3a, 3c, 3e) and CB (Fig. 3b, 3d, 3f) shows slight differences in FS_μ , FS_{COV} and RI when using laboratory data versus in-situ CPTu data. This suggests that literature-derived laboratory data can effectively serve as prior information for slope stability analyses, particularly in tailings dam design. Analyses neglecting spatial variability (Fig. 3a and Fig. 3b) reveal noticeable differences in FS_μ and FS_{COV} compared to those incorporating spatial variability (Fig. 3c–Fig. 3f). Specifically, the mean FS_μ increases from 3.20 (CA) and 3.29 (CB) to 3.31 in scenarios with spatial variability, regardless of failure surface type—an increase of less than 3%. In contrast, the FS_{COV} decreases significantly, from 11.6% (CA) and 13.2% (CB) to approximately 4.3%, representing a reduction of over 63%. This reduction indicates lower dispersion and tighter clustering around the mean. Furthermore, the RI increases substantially, rising from below 8.77 in scenarios without spatial variability to above 25.17 in those incorporating spatial variability—a difference of 187%. This marked increase underscores the enhanced reliability and reduced failure probability in slope stability assessments when accounting for soil spatial variability, irrespective of failure surface type. For pseudo-static scenarios (Fig. 3g–Fig. 3l), the FS_μ remains consistent across all cases, with only marginal differences. However, the RI and FS_{COV} exhibit notable trends. Scenarios that account for soil spatial variability show an increase in RI compared to those without it, with increases ranging from 10.36% to 20.61%. Similarly, the FS_{COV} decreases significantly, by approximately 27% (CA) and 32% (CB). Although the seismic load reduces the differences between probabilistic analyses of scenarios with and without spatial variability, the results demonstrate the advantage of incorporating spatial variability. This approach enhances the reliability of geotechnical assessments for tailings dams under both static and seismic conditions.

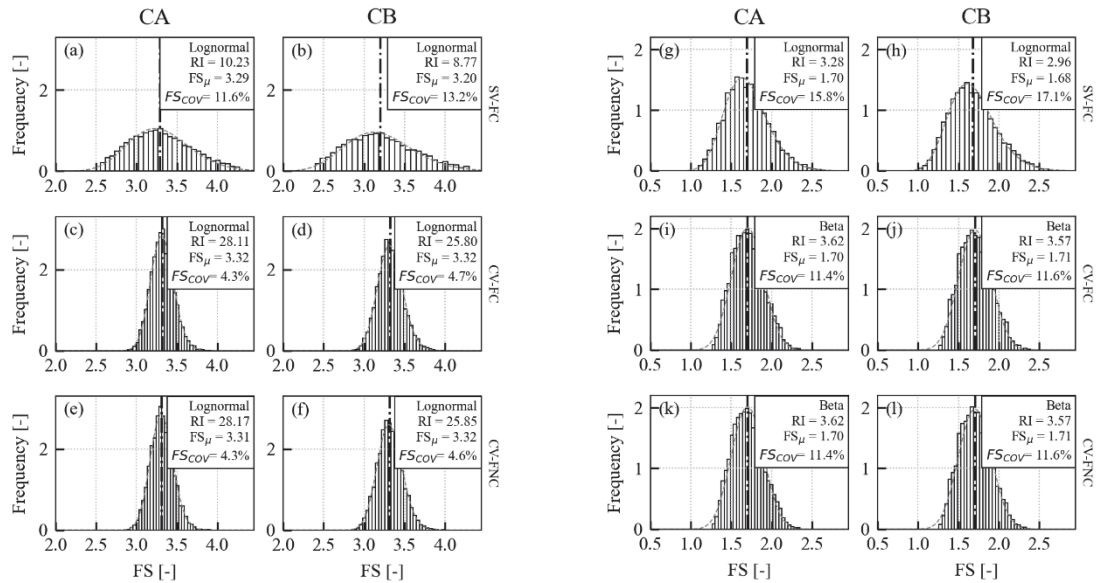


Fig. 3. Histograms of FS for static slope stability analyses (a–f) and pseudo-static slope stability analyses (g–l) of the tailing dam. (a, b) and (g, h) correspond to cases without soil spatial variability, while (c–f) and (i–l) incorporate soil spatial variability, considering both circular and non-circular failure surfaces. Scenarios CA and CB are labeled as defined in Section 3.1.

6. Conclusions

The results demonstrate that incorporating spatial variability significantly reduces the dispersion of FS histograms, with decreases ranging from 27% to 64%. This reduction reflects a higher concentration of values around the mean, which is particularly important as a decrease in FS_{COV} indicates more narrowly bounded distributions. Consequently, the likelihood of obtaining low critical values is reduced, leading to a lower probability of failure. By limiting scenarios where the FS is insufficient to ensure stability, the incorporation of spatial variability enhances the accuracy and reliability of probabilistic analyses, thereby reducing uncertainty and improving the reliability analysis of tailing dams. The reduction in FS dispersion resulting from soil spatial variability arises from its capacity to represent the inherent randomness of the soil, including localized weak and strong zones. Conversely, probabilistic and deterministic models that assume uniform soil conditions overlook these variations, leading to underestimate strength in certain areas, thereby increasing dispersion in slope stability analyses. Furthermore, a thorough investigation of autocorrelation lengths in engineered soils, such as those in tailings dams, is crucial, as they differ significantly from values typically reported for natural soils.

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