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IDENTIFICATION OF SCALES OF FLUCTUATION IN THE CONDITION OF ROTATED ANISOTROPY OF THE SOIL BASED ON LIMITED CPTU SOUNDINGS

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In the reliability analysis of geotechnical structures, stationary anisotropic random fields are increasingly used to describe soil parameters. It is usually assumed that the principal anisotropy directions in such fields are vertical and horizontal. While rotated anisotropy is also possible), existing methods rarely focus on determining the rotation angle of the principal anisotropy directions. This paper proposes a method for identifying that angle, which is an extension of the method proposed for identifying the horizontal scale of fluctuation. It enables the simultaneous identification of the rotation angle of the anisotropy and the principal values of the fluctuation scale of the stationary random field, based on the results of a limited number of CPTu soundings. As shown, the proposed method is particularly effective and precise in identifying the value of the rotation angle; this value is very accurately identified for both artificial (generated) and real (measured) CPTu values. For the artificial CPTu where the exact values used for generation are known the difference between the original and recognized value was not larger than 0.2°.For real data the correctness of results is confirmed by good convergence and significant improvement in convergence of scales of fluctuation. The rotation angle value is important since in many cases, it can significantly influence the probabilistic analysis results for the given problem. Moreover, the assumption of wrongrotation angle can lead to identifying wrong values of fluctuation scales. In this view, the common assumption of the principal directions being vertical and horizontal can lead to an incorrect assessment of the failure probability.

Keywords: Rotated anisotropy, random field identification, Bayesian method.

1. Introduction

In geotechnical risk analysis, stationary random fields are frequently employed to describe the spatial variability of soils. These random functions are characterized by the probability distribution of the modeled property and their spatial variability, described through autocorrelation functions and, more precisely, by their parameters referred to as scales of fluctuation (SOFs). One of the most widely adopted probabilistic analysis methods utilizing such descriptions is the Random Finite Element Method (RFEM, Griffiths and Fenton, 2001; Fenton and Griffiths, 2008), which integrates the theory of random fields with numerical analysis. This method is highly versatile and facilitates the investigation of various geotechnical challenges, including the bearing capacity of foundations, slopes, and retaining structures (eg. Kawa et al. 2021).

Recent developments have increasingly considered random fields as anisotropic, characterized by distinct fluctuation scale values in different directions. Due to the predominantly horizontal stratification of soil layers and sublayers, the principal anisotropy directions are conventionally assumed to be horizontal and vertical. Determining fluctuation scale values in these directions has been the subject of extensive study, leading to the development of at least two distinct methodologies.

It is well established that some soils may exhibit rotated anisotropy, where the principal directions deviate from the standard horizontal and vertical orientations. Works addressing such cases have been published in some works over the last decade (Zhu and Zhang, 2013; Huang et al., 2019; Ghazavi et al., 2021; Luo and Luo, 2021; Das and Chakraborty, 2024). However, relatively limited attention has been given to the development of methods for detecting the principal directions of anisotropy. This study introduces a novel method designed to identify these directions. The proposed approach extends the method previously introduced by Ching et al. (2018) for determining vertical and horizontal fluctuation scales. By extending the vector of unknowns, the method enables the determination of the rotation angle of the random field parameter anisotropy, thereby providing a comprehensive framework for analyzingsuch soil conditions.

2. Identification of fluctuation scale values using the maximum likelihood method with TMCMC

The method developed by Ching (2018) is based on maximum likelihood estimation proposed by DeGroot and Baecher(1993). By employing Bayesian analysis and the likelihood function, it enables the determination of a credible distribution of the most probable values of the fluctuation scale for given field values or their fluctuations. The likelihood function for a field following a Gaussian distribution can be expressed as:

$$f(\mathbf{S}\,|\,\mathbf{R}) = \frac{1}{(2\pi)^{\frac{1}{2}}\sqrt{\det(\mathbf{R})}}\exp(-\frac{1}{2}\mathbf{S}^T \times \mathbf{R} \times \mathbf{S}) \qquad (1)$$

where **S** denotes the data vector in the random field (random field realization), and **R** represents the covariance matrix obtained for the assumed fluctuation scale values. The individual components of the covariance matrix can be expressed using the values of the autocorrelation function, determined based on the distances between points associated with selected random variables, and substituted into the covariance function. This function can be defined for arbitrarily chosen principal directions of the fluctuation scale. For example, the Gaussian function for two perpendicular, arbitrarily chosen directions ξ andηtakes the form:

$$\rho(\tau_\xi, \tau_\eta) = \exp\left[-2\left(\frac{|\tau_\xi|}{\theta_\xi} + \frac{|\tau_\eta|}{\theta_\eta}\right)\right] \qquad (2)$$

Since in the case of function (2), defining the vector of unknowns as **X**=(θ_ξ, θ_η) enables the determination of the matrix**R**, for a given arrangement of points, equation (1) can be rewritten as*f*(**S**|**R**) = *f*(**S**|**X**). Consequently, this allows for the evaluation of the likelihood of the data SS obtained from soil investigations (e.g., CPTu) relative to arbitrary values of the fluctuation scale (θ_ξ, θ_η).

The method proposed by Ching et al. (2018) involves utilizing the likelihood function to assess the most credible pairs of fluctuation scales among many possible generated values. Specifically, based on likelihood values, the transitional Monte Carlo Markov chain (TMCMC) algorithm, previously developed by Ching and Chen (2007), facilitates an iterative transition from an assumed prior distribution of potential fluctuation scale values to a distribution of the most probable values. This final distribution is obtained in the last step of the TMCMC algorithm.

The prior distribution is typically assumed to be uniform. Given the strictly positive nature of the fluctuation scale, it is convenient to adopt a uniform distribution of the logarithms of the scale. As demonstrated in the work by Ching et al. (2018) using artificially generated CPTu data, and assuming ξandηas the horizontal and vertical directions, respectively, the procedure yields unambiguous results: starting from a uniform bivariate distribution of the logarithms of the fluctuation scale, TMCMC converges to a solution where the most likely scales are clustered around the values initially used for data generation. Similarly, convergent solutions are frequently (though not always) achieved for data obtained from real soil investigations.

3. Identification of principal fluctuation scale values and anisotropy rotation angle using TMCMC

As mentioned above, the soil parameter field can be characterized by rotated parameter anisotropy which can quite significantly affect the identification of the fluctuation scale. In addition, failure to detect such rotation may result in practice in modeling a different soil and perhaps even a larger error in the probabilistic model than that resulting from the adoption of incorrect scales. To identify not only the fluctuation scale values but also the rotation angle of the principal fluctuation scale values α, the vector of unknowns X can be extended, for example, to the form **X=**(θ_ξ, θ_η, α). To check the validity of the results of such identification, it was carried out on artificially generated data. It was assumed that the simulation concerns parameters in four vertical quasi-continuous profiles, similar to those obtained in CPTu studies. Single-plane profiles with mutual distances of 1m, 2m, and 4m were simulated. The vertical interval between points along the profile was 0.05m. The values were assumed to benormalized random field fluctuations following a Gaussian standard distribution (μ= 0 and σ= 1). For data generation, the scales in the ξ and η directions were taken as 10m and 1m, respectively, and the angle of rotation of these directions relative to the horizontal and vertical directions α was taken as 15º. The data thus generated by the Cholesky decomposition method (single realization) using the correlation matrix R obtained for the appropriately oriented function (2) is shown in Fig 1.

For these artificially generated data, an attempt was made to identify the magnitude of the fluctuations and the rotation angle of the principal anisotropy directions as if they were unknown. A function of the form (2) was again used to determine the **R** matrix. The results obtained in the final step of the TMCMC procedure for this case are presented in Fig. 2 and Fig. 3. In Fig. 2, the most reliable fluctuation scale values determined by the procedure are represented by blue points. As seen, these values are clustered around the red point, which corresponds to the data used for generation. Fig. 3 displays the histogram of the most reliable rotation angle values. The red line marks the 15º value used for data generation. It is visible that the procedure demonstrated high convergence for both types of information. The rotation angle of the principal directions relative to the vertical and horizontal axes was identified with particularly high precision, with deviations not exceeding 0.5º.

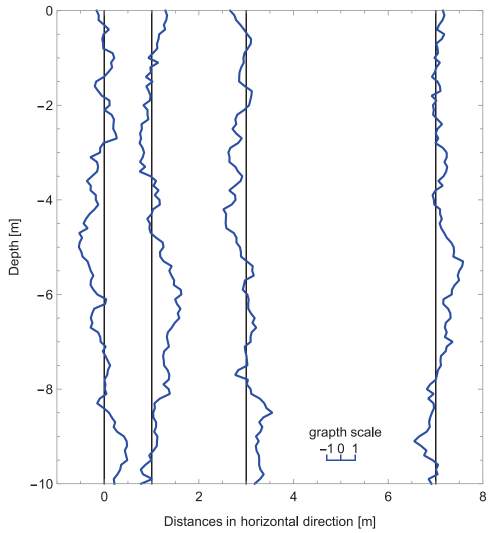

Fig. 1. Artificially generated data assuming an exponential autocorrelation function (2), SOF values of $\theta_\xi = 10$ m and $\theta_\eta = 1$ m and anisotropy rotation angle of α = 15º

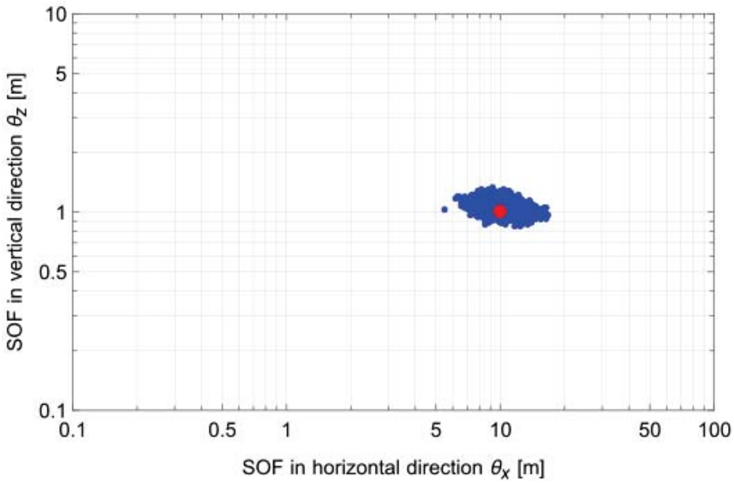

Fig. 2. Identified SOF values for the assumed exponential correlation model based on artificially generated data

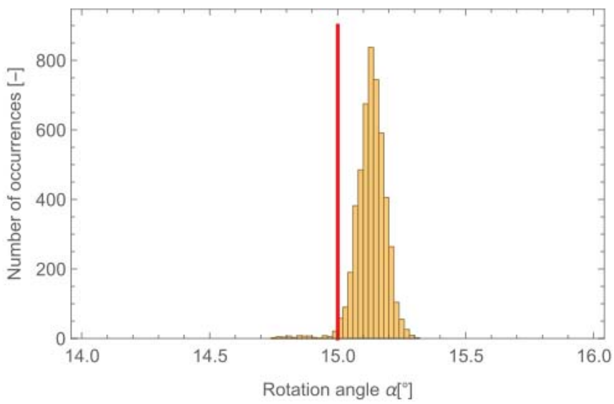

Fig. 3. Identified rotation angle α values for the assumed exponential correlation model based on artificially generated data

Similar identifications were made for actual soil tests, although the results are not presented here. Additionally, due to the unknown form of the correlation function, the Whittle-Matérn model was used for which the so-called "sample path smoothness" (Ching et al., 2019) was also identified. In many cases, a convergent solution was obtained allowing the identification of the angle of rotation of the principal directions.Moreover, the identification of the rotation angle was associated with a small scatter of values each time. Some of the identified values were significant (say 35° or even more). Furthermore, a convergent solution was obtained in cases including a possible rotation of the principal directions in situations where such a solution could not be obtained under the assumption of no anisotropy rotation (i.e. where it was assumed that the principal directions must coincide with the horizontal and vertical directions). Additional analysis on artificially generated data further showed that even if the angle of rotation of the anisotropy is small, it can significantly alter the identified fluctuation scale values. Thus, the consideration of the rotation of the anisotropy would have to be carried out in the identification of each random field: in the case of a significant value of the angle α, this will allow this value to be identified and the random field to be properly described. If, on the other hand, the value of this angle is small, then the procedure is worth introducing to identify the correct values of the fluctuation scale.Some 3D analysis was also performed.

4. Conclusions

The TMCMC can be used to efficiently search not only the principal values of the fluctuation scale, but also the angle of rotation of the principal anisotropy directions. As shown, the procedure for identifying such an angle for its known values (for artificially generated data) allows this value to be determined without error and with high precision. A high precision of the identification of the rotation angle is also obtained when the procedure is applied to data obtained in real soil tests. In some cases, the identified values of the anisotropy angle are significant.

Determining not only the value of the fluctuation scale is of great importance. In the case of significant angle values, it leads to a proper description of the random field which has a great impact on the results of probabilistic modelling. The identification of the angle of rotation of the anisotropy together with the principal values is also important for soils in which the rotation of the anisotropy axis is small, say in the range of a few degrees. For such a case, taking this small angle into account can lead to significant differences in the identified fluctuation scale values, which can consequently also significantly affect the probabilistic soil model.

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