

PROBABILISTIC ANALYSIS OF DEFLECTION OF AN ANCHORED DIAPHRAGM WALL FOR HARDENING SOIL MODEL AND NONLINEAR MODEL OF CONCRETE

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Modeling soil, structure, and soil-structure interaction are crucial components of the geotechnical structures' numerical analysis-based design. Advanced models for these components are becoming increasingly popular in deterministic numerical calculations. However, in probabilistic modeling based on Monte Carlo simulations, simplified models are often used for one or more of these components due to the significant computational power required for advanced models. The difference in structural response between precise and simplified models can be substantial, particularly in the small strain range. Recently, some papers have employed more advanced models of soil and soil-structure interaction in probabilistic modeling. However, the structure is still typically modeled using a linear elastic approach. This paper addresses more advanced probabilistic modeling for a diaphragm wall installed in normally consolidated sands. The analysis is conducted using the random finite element method and is based on $N = 1000$ Monte Carlo simulations. The soil is modeled using the Hardening Soil Brick model. Normalized CPTu parameters Q_m and F_r , with variability based on CPTu tests conducted in normally consolidated sands, are modeled using random fields. All the other model parameters are calculated using correlation formulas found recently by one of the authors. The analysis focuses on serviceability limit states, specifically deflection. Two different models are assumed for the structure: a classical linear model often assumed in engineering applications and a non-linear concrete model, which allows for a more precise simulation of the cracking process. The deflection obtained with the nonlinear concrete model is greater than that obtained with the linear model. This leads to a greater probability of failure which is discussed in the paper.

Keywords: Random fields, Hardening Soil model, diaphragm wall

1. Introduction

Over the last few decades, the random finite element method (RFEM, Griffiths and Fenton 1993, Fenton and Griffiths 2008) has been increasingly used for risk analysis in geotechnical engineering. This method, which uses random field theory to describe the spatial variation of soil parameters and Monte Carlo simulations to assess the associated risks, is very versatile. Its most significant disadvantage, which is probably the reason why it has still not penetrated engineering practice, is the long computation time resulting from the need to carry out a full numerical analysis for individual simulations of the problem. This time also determines the assumed computational models, which are simple whenever possible.

Of course, simple models are not always able to reflect complex soil mechanics problems. In particular, for phenomena related to the modeling of retaining walls or settlements, complex soil models have been increasingly used in the last two decades, taking into account in particular the significant stiffness of the medium in the range of small strains. One of the more popular models is the Hardening Soil small proposed by Benz (2007), reimplemented by Cudny and Truty (2020) under the name Hardening Soil brick. The popularity of both versions of this model is mainly due to the ease of determining all parameters using triaxial tests together with CPTu sounding. Despite the significant computational cost of using this model over the past decade, it has also been increasingly used as part of the random finite element method. The Hardening Soil brick model has been used by the authors in particular for probabilistic analysis of retaining walls (Kawa et al. 2021,2022).

In their previous work (Kawa et al. 2021,2022), the authors used a simplified linear elastic model for the diaphragm wall. The use of such a simplification will have a significant impact on the response of the structure. The present work extends the previous analyses. In the previous analysis, the linear concrete model was used. The novelty of the current work is employing a non-linear concrete model that takes into

account the phenomenon of concrete cracking. According to the best authors' knowledge until now this kind of approach with an adequately complex model for both spatially variable soil and diaphragm wall material is not present in the literature. The work focuses on comparing the displacements obtained using two different concrete models.

2. Numerical model of the problem and random fields.

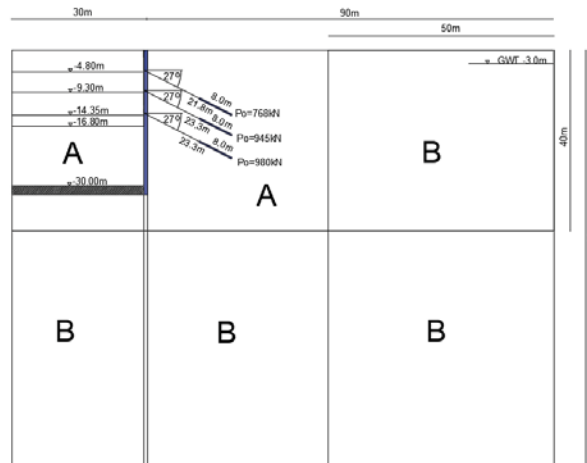


Fig 1. Engineering draft of the excavation(after Kawa et al. 2022)

A 32 m-long triple-anchored diaphragm wall was analyzed. The numerical model of the problem corresponded to that presented by Kawa et al (2022) and referred to the benchmark by Schweiger (2002). A scheme of the structure including anchor positions, anchor lengths, and prestressing forces is shown in Fig 1. Due to the need for spatial elements (solid shell in fact) for the non-linear concrete model, a 1m wide spatial model was analyzed for the entire sentence. The soil was modeled with a grid of different sizes in zones A and B, also shown in Fig. 1. In zone A, the size of the 8-node EAS brick elements was 0.43m x 0.245m x 1m; in zone B, it was 0.86m x 0.49m x 1m. A width of 0.8m was assumed for the wall and it was therefore modelled with the grid size 0.8m x 0.245m x 1m. In addition, a 2m thick anti-seepage diaphragm, made of jet-grouted soil, was modeled at the tip of the wall.

The Hardening Soil-brick model was used to model the soil; unlike Schweiger's (2002) benchmark, only one sand layer was modeled, but this was characterized by spatial variability. Based on CPTu tests performed in normally consolidated sands in central Poland, the probability distribution, cross-correlation coefficient, and scale of fluctuations in the vertical direction were characterized for the normalized CPTu parameters, i.e. Q_m and F_r (Kawa et al. 2022). These values were modeled as correlated random fields assuming a horizontal fluctuation scale (which was not determined) as infinity. $N=1000$ realizations of these two fields were generated using the relationships between these two parameters and the other parameters of the Hardening Soil model developed by Truty (2024), all other model values were determined from the field values for each realization. As a result, in the 1000 Monte-Carlo simulations of the problem performed, almost all parameters of the Hardening Soil model were characterized by spatial variability. The only parameters assumed to be deterministic were the Poisson ratio, set to $\nu=0.2$, the critical state friction angle $\phi_{cv}=32^\circ$, and the failure ratio $R_f=0.85$. Additionally, two physical parameters were also assumed to be deterministic: dry unit weight $\gamma_d=15.0 \text{ kN/m}^3$ and specific unit weight $\gamma_s=26.5 \text{ kN/m}^3$.

The diaphragm wall itself, as mentioned, was modeled using a modified version (Truty 2016) of Lee and Fenves (Lee et al. 1998) model for a concrete model that takes into account cracking. It was modeled as a layered shell in which concrete properties for class C30/37 are deterministic and equivalent to the mean values indicated by the EC2 standard. On both sides of the cross-section longitudinal reinforcement layers were added with different cross-section areas along the wall. The reinforcement set was designed in four segments based on the bending moments envelope resulting from the simulation in which the diaphragm wall is treated elastically while soil properties are derived from the mean values of the Q_m and F_r . The designed reinforcement sets (all per 1m in out of the plane direction) are summarized in Table 1.

In each simulation, the entire construction history of the structure was reproduced. As shown in Fig 1, the groundwater table was assumed to be 3m below ground level. Once the initial condition was resolved, the execution of the diaphragm wall and the diaphragm in the soil were modeled, followed by excavation deepening with dewatering and anchor installation in individual stages. Internal forces (stresses) and displacements were monitored throughout the calculation stages of each simulation. Only the wall displacements are presented in this paper, comparing them with the results obtained for the linear model in a previous paper (Kawa et al. 2025).

Table 1. Reinforcements in individual wall sectors

	sector 1	sector 2	sector 3	sector 4
depth [m]	0 ÷ 6	6 ÷ 11	11 ÷ 27	27 ÷ 32
A_{s1} [cm ²]	15.7	31.4	49.1	15.7
A_{s2} [cm ²]	15.7	31.4	15.7	49.1

3. Results for deflection

Of the results analyzed, only the results of the maximum horizontal deflections of the wall are presented in this paper. In each realization, the maximum deflection obtained throughout the excavation process was selected. The displacement itself was not analyzed and the deflection values were obtained from the analysis of the deformed axis of the wall, the lower end of which was assumed not to be displaced. In this way, wall displacement control was modeled based on inclinometric surveys without additional geodetic control.

A histogram of the final deflection values obtained is shown in Fig. 2. In the figure also the obtained values of mean (μ) and standard deviation (σ) are shown. Additionally probability distribution function of estimated continuous distribution. Of the known continuous probability models, the greatest likelihood that the data belongs to such a model was obtained for an estimated three-parameter Weibull distribution. The parameters of estimated distribution were $\alpha=2.80$, $\beta=43.30$, $\gamma=15.57$ as shown in the figure caption. Such distribution was also found to be most likely for obtained data in the author's previous work for the linear concrete model (Kawa et al. 2025). In the current work, however, Kolmogorov-Smirnov test results obtained were higher than in the previous work yielding 0.25 (it was only slightly higher than 0.05 in the previous work for the linear concrete model). For comparison, the displacement results for the linear model obtained in the previous work are shown in Fig 3. Here also the statistical moments are shown in the figure and parameters of the Weibull distribution estimated based on the data are reported in the figure caption.

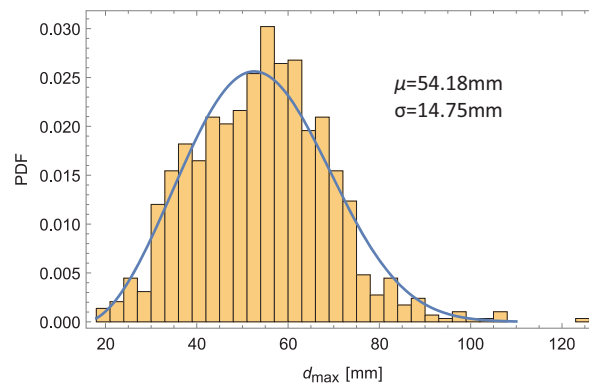


Fig 1. Histogram of wall deflection for a nonlinear model of concrete. Weibull distribution fit for $\alpha=2.80$, $\beta=43.30$, $\gamma=15.57$

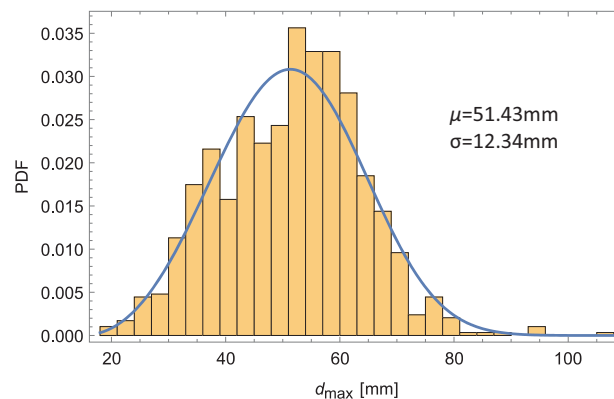


Fig 1. Histogram of wall deflection for linear model of concrete. Weibull distribution fit for $\alpha=3.29$, $\beta=41.37$, $\gamma=14.29$

Based on the estimated distributions, the failure probability values were calculated for models employing both models of concrete, namely nonlinear and linear. In each case, two criteria limiting the allowable displacement were assumed, namely 60mm and 70mm. The results obtained for the probability of exceeding these limits are summarized in Table 2. It is worth noting that according to Eurocode 0, the allowable probability of failure for

serviceability limit states, including wall deflection exceeding the limiting value, is 0.067. As can be seen, all the values collected in Table 2 are greater than this, which means that the structure under consideration is not designed correctly regarding serviceability. However, if 70mm is taken as the value limiting displacement for the linear concrete model, the probability criterion is almost satisfied. On the other hand, for a non-linear model that should better describe reality, the value of probability is exceeded two times. This shows the importance of using accurate models not only for the soil but also for the structure.

Table 2. Comparison of probability

Model of concrete	Probability of deflection failure p_f for limiting condition:		Allowable failure probability for SLS
	60mm	70mm	
Nonlinear	0.341	0.150	0.067
Linear	0.250	0.070	0.067

The results not presented in this paper included the ultimate limit state and the second service limit state, i.e. the crack width in the concrete. When two different conditions of serviceability limit state are considered, the so-called system probability is of great importance. Such system probability can be modeled using Nataf's transformation (e.g. Ditlevsen 1996). The great importance of such modeling will be shown in upcoming journal papers.

4. Conclusions

In the probabilistic modeling of engineering structures, it is not only important to have a sufficiently precise model of the spatially varying soil, but also of the structure itself. In the present study, the use of a non-linear concrete model for a diaphragm wall improved the precision of the failure probability estimation. The obtained increase of this probability from values almost satisfying the criterion to twice as large shows the importance of the problem. The paper focuses on only one type of result computed for the problem considered, i.e. deflection of the modeled diaphragm wall. However, an ultimate limit state and another serviceability limit state, namely crack width, were also investigated by the authors. When considering two conditions for the same limit state (e.g. deflection and crack width in the case of serviceability limit state), considering system probability is of great importance. More information on modeling this phenomenon using Nataf's transformation will be presented in an upcoming journal paper.

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