

## PROBABILISTIC PULLOUT CAPACITY ANALYSIS OF STRIP ANCHORS

Pengpeng He

*School of Science and Engineering, University of Dundee, Dundee, UK. E-mail: [phe001@dundee.ac.uk](mailto:phe001@dundee.ac.uk)*

Gordon A. Fenton

*Department of Engineering Mathematics and Internetworking, Dalhousie University, Halifax, Canada. E-mail: [gordon.fenton@dal.ca](mailto:gordon.fenton@dal.ca)*

D. V. Griffiths

*Department of Civil and Environmental Engineering, Colorado School of Mines, Golden, USA. E-mail: [d.v.griffiths@mines.edu](mailto:d.v.griffiths@mines.edu)*

Plate anchors have been recognized as a cost-effective and efficient solution for offshore applications. Conventional methods for predicting anchor pullout capacity often assume the soil is homogeneous with uniform properties. However, natural soils, particularly in offshore environments, typically exhibit significant spatial variability due to their geological history and formation process. To account for the inherent spatial variability, this paper conducts a probabilistic analysis of the pullout capacity of strip plate anchors embedded in both clay and sand using the Random Finite Element Method (RFEM) over a wide range of soil and anchor parameters. The soil undrained shear strength and friction angle are represented as random fields, and the mean and standard deviation of the anchor pullout capacity factor are estimated. The results show that while the soil correlation length has a minimal impact on the mean pullout capacity, it strongly influences the standard deviation. This indicates that understanding the spatial variability of soil properties is essential in order to achieve accurate anchor pullout failure probability predictions. Overall, the findings of this study can aid in the probabilistic analysis of anchor pullout capacity and guide the design of reliable plate anchors in spatially variable soils.

*Keywords:* Plate anchor; pullout capacity; spatial variability; probabilistic analysis, random finite element method, clay, sand.

### 1. Introduction

Floating offshore wind turbines (FOWTs) have been an attractive option for harnessing the more substantial wind resources over deep water. To anchor these systems, plate anchors are recognized as a cost-effective and efficient option, particularly for taut and semi-taut FOWTs, due to their affordability and performance (Randolph and Gourvenec 2017; Cerfontaine et al. 2023). The pullout capacity of these anchors is traditionally estimated as a function of parameters such as soil strength and anchor embedment depth ratio. However, existing studies often assume a homogeneous soil with uniform properties over the soil mass, ignoring the spatial variability in the soil caused by the geological history of soil formation (Phoon and Kulhawy 1999). This spatial variability needs to be considered in the reliability-based and thus economic design of plate anchors.

A probabilistic approach can explicitly address various sources of uncertainties and is able to produce more cost-effective and consistently reliable designs across varying site conditions. A number of previous studies have shown that soil spatial variability significantly influences the failure probability of ultimate capacity and deformations for a wide range of geotechnical problems, such as the bearing capacity of shallow (Soubra and Massih 2010; He et al. 2022) and deep (He et al. 2023) foundations, slope stability (Griffiths and Fenton 2004), etc. However, probabilistic studies on plate anchors in spatially variable soils are still sparse. To address this, this study aims to estimate the statistical moments of the pullout capacity of plate anchors in soil with spatially variable considering both undrained and drained soil conditions.

### 2. Random Soil Models

The pullout capacity of plate anchors is primarily influenced by the soil strength parameters, i.e., the undrained shear strength,  $s_u$ , for undrained soil conditions and the soil friction angle,  $\phi$ , for drained soil conditions. These parameters are represented by spatially variable random fields, characterized by their means,  $\mu_{s_u}$  and  $\mu_{\phi}$ , standard deviations,  $\sigma_{s_u}$  and  $\sigma_{\phi}$ , and correlation length,  $\theta$  (assumed the same for both parameters).

In this study,  $s_u$  is assumed to follow a lognormal distribution since it is non-negative and has a simple relationship with the normal distribution. Following Fenton and Griffiths (2008),  $\phi$  is assumed to be bounded between  $20^\circ$  and  $45^\circ$  with a bounded tanh distribution, which is a simple transformation from a standard normal random field. The correlation coefficient between the soil properties at one spatial location,  $x_i$ , and a second

location,  $x_j$ , is assumed to follow a simple isotropic Markovian correlation function (e.g., Vanmarcke 2010), i.e.,

$$\rho(\tau_{ij}) = \exp\left(\frac{-2\tau_{ij}}{\theta}\right) \quad (1)$$

where  $\tau_{ij} = |x_i - x_j|$  is the absolute distance between the two points in the soil mass; and  $\theta$  is the spatial correlation length. The correlation length is loosely defined as the distance beyond which the soil properties at two points become largely uncorrelated. Smaller  $\theta$  indicate rapid spatial variations in the field, whereas larger  $\theta$  values correspond to more gradual variations. Note also that the correlation length for the soil random field is applied directly to the values of  $\ln s_u$  (or the underlying normal distribution leading to  $\phi$ ). However, the log-space correlation length and its real-space counterpart have been demonstrated to be very similar for most geotechnical problems (e.g., Pullar and Griffiths 2021).

### 3. Random Finite Element Method (RFEM)

A RFEM programme, 'ranch2d', was developed to estimate the stochastic pullout capacity of horizontal strip anchors in spatially varying soils. The programme uses a two-dimensional plane strain model, with the soil property random fields generated using the Local Average Subdivision (LAS) method (Fenton and Vanmarcke 1990). The anchor was modelled to be rigid and rough, with no horizontal displacement allowed at the soil-anchor interface. The anchor width and thickness were taken as  $B = 6$  m and  $t = 0.6$  m, respectively. The finite element model adopted an elastic-perfectly-plastic soil constitutive model with a Mohr-Coulomb failure criterion, which degenerates to a Tresca failure criterion under undrained soil conditions. Boundary conditions included fixed horizontal displacements at the two vertical sides of the soil domain and fixed displacements at its base.

For undrained soil conditions,  $s_u$  was assumed to have a mean of  $\mu_{s_u} = 30$  kPa, with coefficients of variation of  $v_{s_u} = 0.2, 0.3,$  and  $0.5$  (i.e.,  $\sigma_{s_u} = 6, 9,$  and  $15$  kPa). For drained soil conditions, the lower and upper bounds of  $\phi$  were taken to be  $\phi_{\min} = 20^\circ$  and  $\phi_{\max} = 45^\circ$ , with a mean of  $\mu_\phi = 32.5^\circ$ . The coefficient of variation of  $\phi$  was taken as  $v_\phi = 0.05, 0.10,$  and  $0.15$ , which is consistent with the range of  $0.05 \sim 0.15$  suggested by Phoon and Kulhawy (1999). An associated flow rule (i.e., soil friction angle = dilation angle) was considered in this study. The soil submerged unit weight  $\gamma'$  was held constant at  $10 \text{ kN/m}^3$  for both undrained and drained soil conditions. A wide range of spatial correlation lengths was considered, i.e.,  $\theta = 0.1 \sim 1000$  m. The soil Poisson's ratio was taken as  $\nu = 0.49$  for undrained soil conditions and  $\nu = 0.3$  for drained soil conditions. The soil elastic modulus was held constant at  $E = 1000\mu_{s_u} = 30,000$  kPa for undrained soil conditions and  $E = 1000\gamma'B = 60,000$  kPa for drained soil conditions. Two anchor embedment ratios were considered, i.e.,  $H/B = 2$  and  $4$  ( $H$  is the anchor embedment depth). A total of  $n_{\text{sim}} = 2000$  realizations were used in this study, which has been demonstrated to provide reasonably accurate results in preliminary analyses. Refer to He et al. (2024a, 2024b) for more details.

### 4. Results

The stochastic pullout capacity factor is defined as

$$M_c = \frac{q_u}{\mu_{s_u}}, \text{ undrained soil conditions} \quad (2)$$

$$M_\gamma = \frac{q_u}{\gamma'H}, \text{ drained soil conditions} \quad (3)$$

where  $q_u$  is the random ultimate pullout pressure estimated using RFEM; and  $H$  is the anchor embedment depth from the ground surface to the anchor.

Fig. 1 presents the sample means and standard deviations of the pullout capacity factors as a function of  $\theta/B$  for the intermediate cases of  $v_{s_u} = 0.3$  and  $v_\phi = 0.10$ . Figs. 1(a) and (c) show that  $\theta/B$  has a marginal effect on the sample means, particularly for drained soil conditions (Fig. 1(c)). However, with an increase of  $\theta/B$ , both  $\hat{\sigma}_{M_c}$  and  $\hat{\sigma}_{M_\gamma}$  increase from nearly zero at small  $\theta/B$  ( $\theta/B < 0.02$ ) and eventually reach a plateau at large  $\theta/B$  values ( $\theta/B > 20$ ). This trend reflects the increasing variance reduction caused by local averaging over the

anchor failure zone as  $\theta/B$  increases. Specifically, when  $\theta/B \rightarrow \infty$ , the soil property random field become uniform, albeit remaining random across realizations, reducing to the single-random-variable (SRV) scenario. This indicates that using the SRV approach, without considering the spatial correlation length, leads to an overestimation of  $\hat{\sigma}_{M_c}$  and  $\hat{\sigma}_{M_\gamma}$ , which is unconservative.

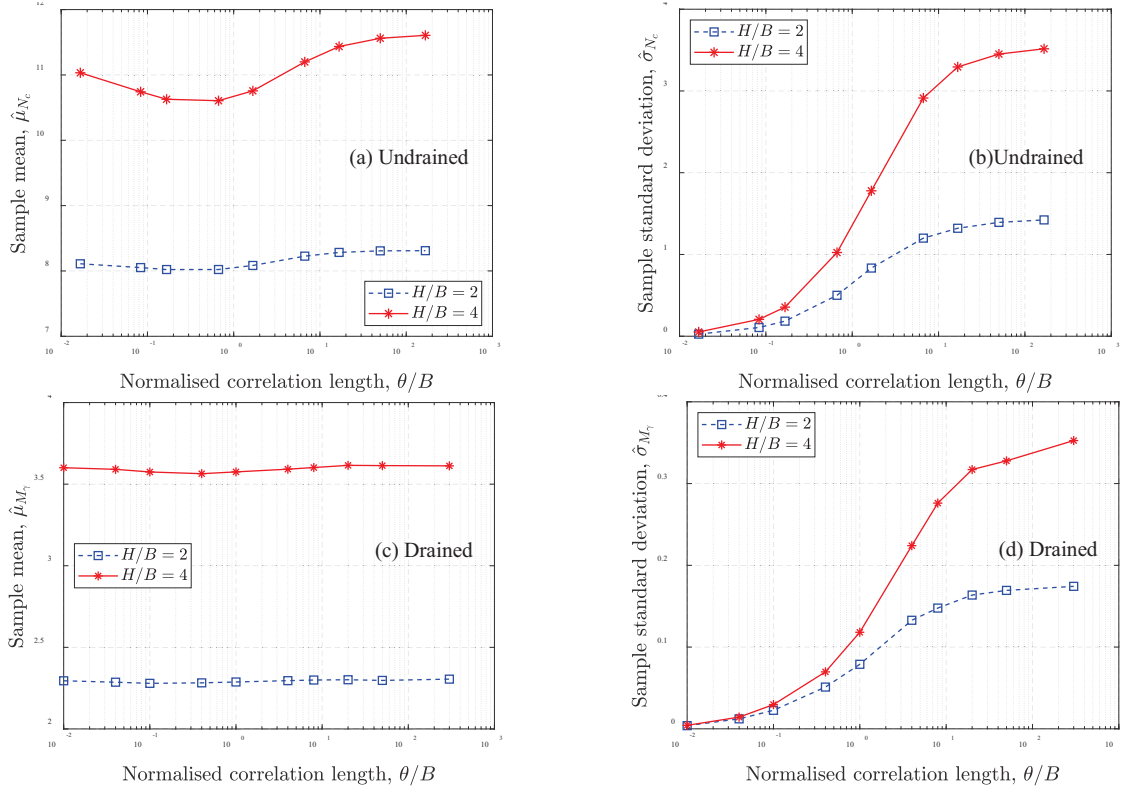


Fig. 1. Sample means and standard deviations of pullout capacity factors for  $\nu_{s_u} = 0.3$  (undrained) and  $\nu_\phi = 0.10$  (drained)

Fig. 2 illustrates the variation of the sample mean and sample standard deviation with  $\theta/B$  for  $H/B = 2$ . As shown in Figs. 2(b) and (d), the standard deviations increase with the coefficients of variation of the soil strength parameters, as expected. Figs. 2(a) and (c) exhibit a minimum value of the sample mean when  $\theta$  is of the same order as  $B$ . Similar minimums have also been observed in the undrained pullout capacity of deep rectangular anchors (Cai et al. 2022) and the undrained bearing capacity of shallow foundations (Fenton and Griffiths 2003). This behaviour can be explained as follows: at very small correlation lengths ( $\theta \rightarrow 0$ ), the soil properties in the random field become independent at every point. In this case, any local averaging results in a statistically uniform field (at the median for a lognormal distribution), so the weakest failure path (i.e., failure mechanism) through the soil is expected to be symmetric and identical to that in a homogeneous soil. At very large values of  $\theta$  ( $\theta \rightarrow \infty$ ), the random fields also become uniform, but random, with all local averaging tending to the mean. In this case, the failure path again becomes the same as those in a homogeneous clay. It is at intermediate correlation lengths that the failure path is expected to deviate from these two extreme cases, and thus has some freedom to choose a weaker failure path, leading to a lower mean pullout capacity than that for the homogeneous scenarios.

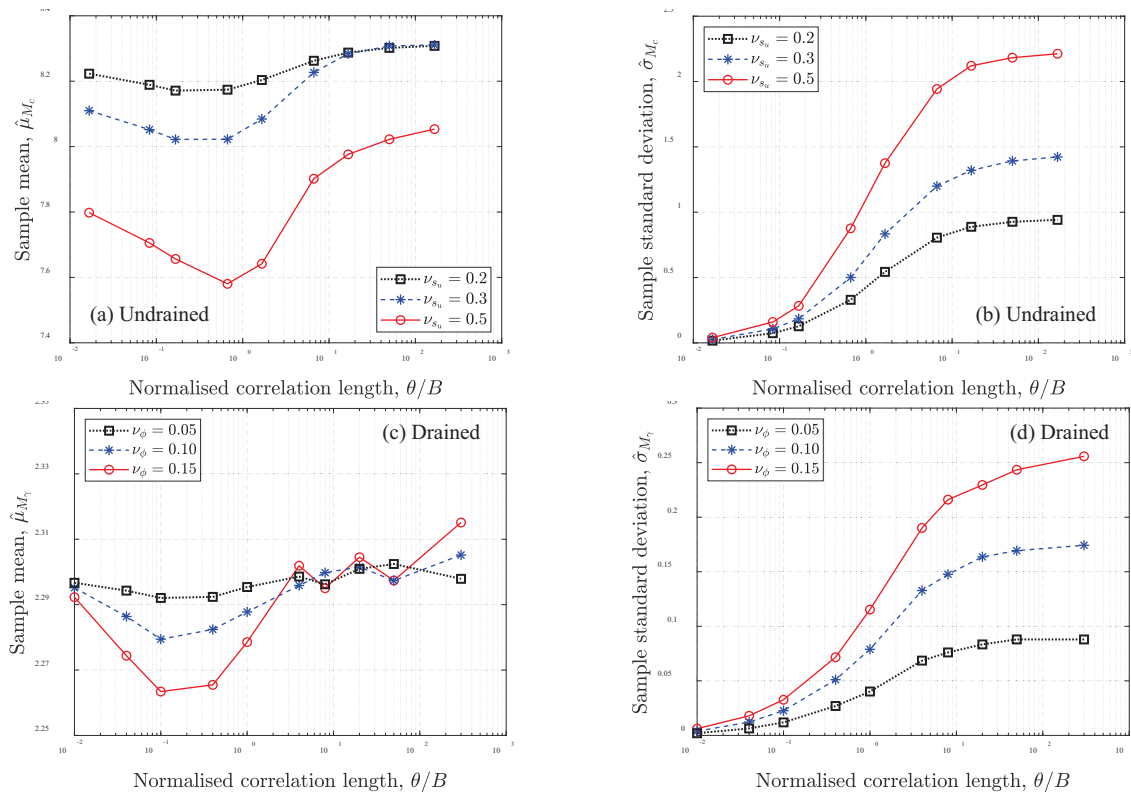


Fig. 2. Sample means and standard deviations of pullout capacity factors for  $H/B = 2$

## 5. Conclusions

This paper investigated the pullout capacity of plate anchors embedded in spatially variable soil under both undrained and drained soil conditions. The soil undrained shear strength and friction angle were considered to be spatially random fields. The Random Finite Element Method (RFEM) was used to estimate the mean and standard deviation of anchor pullout capacity factors for various input parameters. The results show that the soil correlation length  $\theta$  has a marginal influence on the means of the pullout capacity factors but exerts a significant effect on its standard deviations. Specifically, the standard deviations approach zero when  $\theta \rightarrow 0$ , and converge to constant values as a function of the soil coefficient of variation when  $\theta \rightarrow \infty$ .

The findings highlight the importance of sufficient site investigations in practical anchor design. However, when insufficient data are available to accurately estimate the correlation length,  $\theta \rightarrow \infty$  (i.e., the SRV method) can be adopted for a conservative design. This approach, however, may lead to increased anchor costs, which has been identified as a key challenge in the wide deployment of FOWTs (Cerfontaine et al. 2023). It should also be noted that the current study does not account for the effects of correlation length anisotropy. Typically, soils exhibit a smaller vertical correlation length compared to the horizontal correlation length. The results of the present study have shown that a larger correlation length results in a larger standard deviation of the pullout factor. Thus, the isotropic correlation structure used in this study can be seen to be conservative when compared to an anisotropic model with reduced vertical correlation lengths. The influence of the correlation length anisotropy is left for future research (while noting that estimating correlation length anisotropy is extremely costly). Overall, this study can aid in the probabilistic analysis of anchor pullout capacity and provide guidance to the design of reliable plate anchors in spatially variable soils.

## References

- Cai, Y., M. F. Bransby, C. Gaudin, and Y. Tian (2022). Accounting for soil spatial variability in plate anchor design. *Journal of Geotechnical and Geoenvironmental Engineering* 148(2), 04021178.
- Cerfontaine, B., D. White, K. Kwa, S. Gourvenec, J. Knappett, and M. Brown (2023) Anchor geotechnics for floating offshore wind: Current technologies and future innovations. *Ocean Engineering* 279, 114327.
- Fenton, G. A. and D. V. Griffiths (2008). *Risk assessment in geotechnical engineering*. John Wiley & Sons.
- Fenton, G. A. and E. H. Vanmarcke (1990). Simulation of random fields via local average subdivision. *Journal of Engineering Mechanics* 116(8), 1733-1749.
- Griffiths, D. V. and G. A. Fenton (2004). Probabilistic slope stability analysis by finite elements. *Journal of Geotechnical and Geoenvironmental Engineering* 130(5), 507-518.
- He, P., G. A. Fenton, and D. V. Griffiths (2022). Calibration of resistance factors for bearing resistance design of shallow foundations under seismic and wind loading. *Canadian Geotechnical Journal* 59(7), 1243-1253.

- He, P., G. A. Fenton, and D. V. Griffiths (2023). Calibration of resistance factors for design of deep foundations against lateral deflections. *Computers and Geotechnics* 156, 105250.
- He, P., G. A. Fenton, and D. V. Griffiths (2024a). Pullout capacity of strip anchors in spatially variable soil. I: Clay. *Journal of Geotechnical and Geoenvironmental Engineering* 150(11), 04024104.
- He, P., G. A. Fenton, and D. V. Griffiths (2024b). Pullout capacity of strip anchors in spatially variable soil. II: Sand. *Journal of Geotechnical and Geoenvironmental Engineering* 150(11), 04024105.
- Phoon, K. K. and F. H. Kulhawy(1999). Characterization of geotechnical variability. *Canadian Geotechnical Journal*36(4): 612-624.
- Pu~~B~~ W. and D. V. Griffiths (2021). Transformations of spatial correlation lengths in random fields. *Computers and Geotechnics*136, 104151.
- Randolph, M., and S. Gourvenec(2017). *Offshore geotechnical engineering*. CRC Press.
- Soubra, A. H. and D. Y. A. Massih (2010). Probabilistic analysis and design at the ultimate limit state of obliquely loaded strip footings. *Géotechnique* 60(4), 275-285.
- Vanmarcke, E. (2010). *Random fields: analysis and synthesis*. World Scientific.