

IMPROVING DRAINED SOIL STRENGTH PARAMETER PREDICTION IN GEOTECHNICAL ENGINEERING: LEVERAGING TRIAXIAL STRESS MEASUREMENTS WITH HIERARCHICAL BAYESIAN MODELS

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In offshore geotechnics, empirical correlations from the literature are often used to interpret CPT data supplementing measurements on shear parameters. However, it has been demonstrated that site-specific correlations should be employed whenever possible, as they better account for the unique soil behavior at a given project site. For sandy soils, the friction angle is a parameter with a significant impact on a design and is typically derived from triaxial tests conducted on at least three samples per soil type, followed by linear regression analysis. These friction angles are then used to establish site-specific correlations with CPT parameters. This process inherently introduces uncertainty into the used friction angles. In this study, a new model is proposed that integrates the linear regression of sample data with a regression linking the CPT parameter and the friction angle, utilizing various Bayesian modeling approaches. The findings indicate that the hierarchical Bayesian model applied to the triaxial test samples provides the highest predictive accuracy. However, site-specific correlations derived from this method can still be readily adopted by practitioners, enhancing the reliability of project-specific soil behavior predictions.

Keywords: Hierarchical Bayesian model (HBM), site-specific correlation, transformation model, triaxial test, friction angle prediction, offshore data, transformation model.

1 Introduction

The characterization of soil strength is crucial for safe and reliable designs. This is often achieved by statistical interpretation of *Site Investigation* (SI) data collected from the project site. In most cases, SI includes performing both in-situ and laboratory testing. While the latter provides a more accurate description of soil strength, the former is performed faster and has lower cost. Thus, the characterization of soil strength can benefit from the use of transformation models, that connect measurements of in-situ test data to soil strength.

This study focuses on establishing a transformation model between the tip resistance q_t , a measurement of *Cone Penetration Testing* (CPT), and the effective friction angle φ' as peak friction angle for sandy soils, which determines the drained shear strength for an offshore wind project in the North Sea. The derivation of φ' itself already depends on the statistical analysis of measurements between the mean effective stresses σ'_m, τ_m collected from triaxial testing (linear regression of minimal three tests). In this study, a new approach is presented, in which the original data from the triaxial tests σ_m, τ_m is used, while also a transformation model for the engineer between q_t and φ' is derived. The different modelling approaches regarding the grouping of observations, pooled, unpooled and partially-pooled / hierarchical modelling, are taken into consideration. Particularly the latter has gained more attention in geotechnical engineering in recent years due to its capacity to address differences between groups (sites, units, etc.) while accounting for the problem of limited observations (Bozorgzadeh et al. 2019).

2 Evaluation of triaxial test data and correlations to CPT-data

2.1 Established correlations

Several correlations have been established between CPT measurement data and the friction angle (e.g. Ching et al., 2017). To determine these correlations, data pairs of (q_{t1}, φ') were used, where $q_{t1} = (q_t/p_a)/(\sigma'_{v0}/p_a)^{0.5}$ is q_t standardised by vertical stress σ'_{v0} at the measurement depth and the atmospheric pressure p_a . One of the most

commonly used correlations (Eq. (1)) was derived by Kulhawy and Mayne (1990), which is used as comparison in this paper:

$$\varphi' = 17.6 + 11 \cdot \log_{10}(q_{t1}) \tag{1}$$

2.2 Data description

The analysed data was published by the German Federal Maritime and Hydrographic Agency (BSH) as public institution for maritime tasks containing preliminary investigation data for various wind farms in the German North Sea (BSH 2024). A subset was used for this study consisting of three areas (sites), named sites N-6, N-7 and N-9. To compile the data, all field and laboratory test results, the geological and geotechnical investigation reports and a classification of the soil layers based on geophysical measurements were included.

The data structure is displayed in Fig. 1. The site data is divided into five soil units, whereby in the following analysis, for illustration purpose, only the second unit is considered. For this soil unit, 78 samples of sandy soil were taken and on each of them three triaxial tests were conducted.



Fig. 1. Data structure

2.3 Proposed model with changed input variables

Since the derivation of the friction angle for every set of triaxial tests is based on the *Mohr-Coulomb* (MC) model its computation is already a linear regression by itself. Hence, a two-stage model is proposed (Fig. 2), wheretwo model parts are combined: the $q_{t1} - \varphi'$ - relationship and the MC-model for the analysis of the triaxial test results.

For the CPT data the variable q_{t1} is taken as input value following Eq. (1). For simplicity a linear relationship as input for $q_{t1} - \varphi'$ was chosen, whereas the original Kulhawy and Mayne correlation uses a log-scaled q_{t1} -value. For derivation of the effective shear parameters the maximum shear stress at failure was calculated using ISO 17892-9(2018), Annex B. As only sandy soils are considered, which are assumed to have no effective cohesion, the cohesion was set to zero in the Mohr-Coulomb model, a modelling choice which is widely accepted in practice. The input variables for the statistical model are σ'_m , τ_m and q_{t1} . In the model, the latent variables are the regression parameters b_0 and b_1 for the $q_{t1} - \varphi'$ -model as well as the variable φ' .

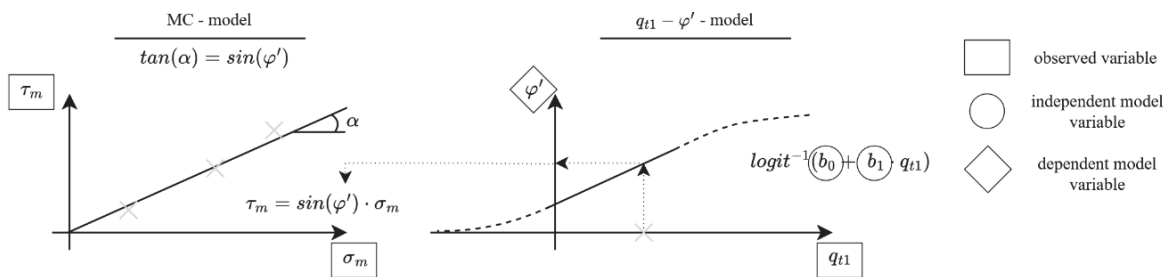


Fig. 2. Proposed model for prediction of τ_m

3 Bayesian Data Analysis

3.1 Examined models

The different possibilities for building the model – pooled (P), unpooled (U), hierarchical (H) – were combined (see Table 1). For explanations of the statistical approaches see Feng et al. (2021) or Gelman et al. (2014).

Table 1. Overview of compared models

model name	$q_{t1} - \varphi'$	MC-model
PP	pooled	pooled
UP	unpooled	pooled
PH	pooled	hierarchical (regarding tests)
UH	unpooled	hierarchical (regarding tests)

3.2 Mathematical form of model UH

Since model UH is the most complex model and the models are comparable in their structure, the mathematical formulation of the model UH is shown in Eq. (2)-(7). The logit transformation constrains the variable to the range $[0^\circ, 90^\circ]$. For $b_1 = 0$ the prior of b_0 results in a mean of 34° and standard deviation of around 20° . The prior of b_1 is weakly informative since b_1 takes very small values due to the logit transformation and values of q_{t1} .

$$\text{Priors for } q_{t1} - \varphi' \text{-model: } b_0^k \sim \mathcal{N}(-0.5, 1), \quad b_1^k \sim \mathcal{N}(0, 0.05), \quad \sigma_\varphi \sim \mathcal{HN}^+(5) \quad (2)$$

$$\text{Functional form } q_{t1} - \varphi' \text{-model: } \mu_\varphi^{j,k} = (b_0^k + b_1^k q_{t1}^{j,k}) \quad (3)$$

$$\text{(Hyper-)prior for MC-model: } \varphi^{j,k} \sim \text{logit}^{-1}(\mathcal{N}(\mu_\varphi^{j,k}, \sigma_\varphi)) \cdot \pi/2 \quad (4)$$

$$\text{Prior for MC-model: } \sigma_\tau \sim \mathcal{HN}^+(100) \quad (5)$$

$$\text{Functional form MC-model: } \mu_\tau^{i,j,k} = \sin(\mathbb{A}^{i,j,k}) \cdot \tilde{\mathbb{A}}_m^{i,j,k} \quad (6)$$

$$\text{Likelihood: } \tau_m^{i,j,k} \sim \mathcal{N}(\mu_\tau^{i,j,k}, \sigma_\tau) \quad (7)$$

3.3 Results and model comparison

Selection of the fittest model is based on the *expected log predictive density* (ELPD) metric (Gelman, et al. 2014). The study determines the model fitness using the *Leave-One-Out Cross Validation* (LOO-CV) criterion and the *Leave-One-Group-Out Cross Validation* (LOGO-CV) criterion, which offer different assessments of the ELPD metric, and provide insights regarding model fitness depending on the model's expected use. Specifically, the former indicates the fittest model in case predictions need to be performed for one of the groups existing in the dataset. On the other hand, the latter suggests the fittest model when prediction for a new group is relevant. Fig. 3 summarizes the LOO-CV on the left and LOGO-CV on the right of the examined models. The models which use the hierarchical set up for the MC-model have higher ELPD values for both metrics and indicate better performance in prediction. Nonetheless, the ELPD difference indicates that model PH and UH are quite similar. It is assumed that the hierarchical approach on the MC model overpowers the influence of the transformation model and therefore the predictive accuracy is nearly the same.

Since the shear stress τ_m and not the friction angle φ' is modelled, the comparison with measured values can only be done for the shear stress. For the winning model, PH, the predictions (black circles) are compared to the measurement values (gray crosses) in Fig. 4, including the credible interval (CI) of the posterior and the prediction interval (PI) as gray lines. For the small normal stresses shown, the measured values lie in the range of the credible intervals, which underline the predictive accuracy of the model. But aiming also for a transformation model for practitioners, the indirectly derived transformation models for $q_{t1} - \varphi'$ are important. Based on the used data, site-specific / region-specific correlations are derived. Fig. 5 displays model PH on the left side and on the right side additionally model UH, which contains a transformation model for every considered site. The derived models are in grey, whereas the literature model of Kulhawy and Mayne (1990) is shown in black. Due to the modelling approach and the data the functional form differs to the literature one. This form has the consequence that the model is only applicable for $q_{t1} > 0$. The Kulhawy-Mayne correlation would overestimate φ' in comparison to the site-specific models for this data. The derived transformations could be used for calculating the mean of φ' . Although the transformation models of UH differ significantly, this does not influence the predictive accuracy compared to model PH. Further investigation has to be done to evaluate this influence directly.

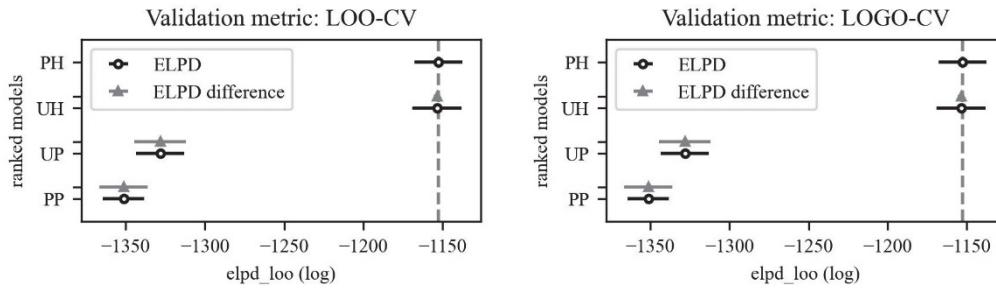


Fig. 3. ELPD metric for model comparison, left: Leave-one-sample-out (LOO-CV), right: Leave-one-group-out (LOGO-CV)

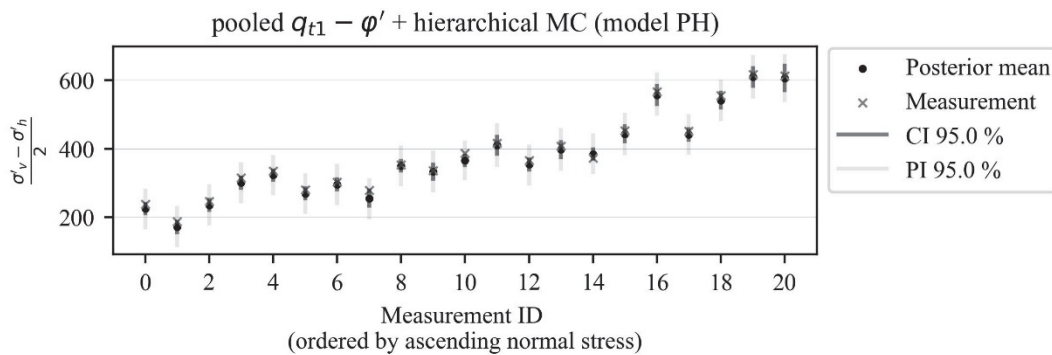


Fig. 4. Posterior means and 95 % credible and prediction intervals (CI and PI) for the model PH for the first 20 values of the first site

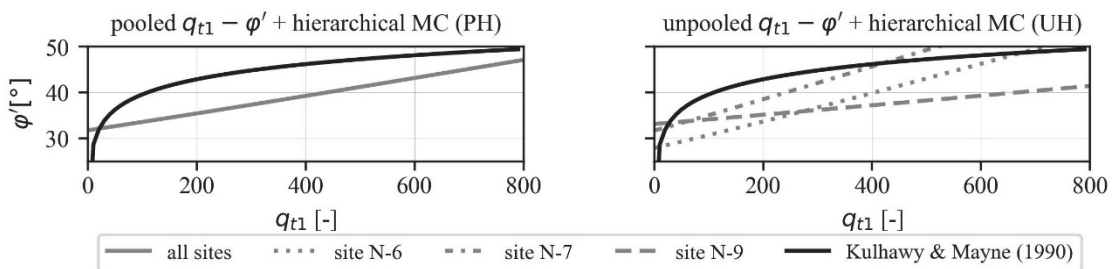


Fig. 5. Comparison of the results for the mean of the $q_{t1} - \phi'$ transformation models

4 Conclusions

Instead of limiting ourselves to the known regression procedures for deriving transformation models, this study has shown a new modelling approach for deriving transformation models between the standardised value q_{t1} of the CPT tip resistance and the friction angle ϕ' . Datasets of q_{t1} , σ_m and τ_m were used as input values to different Bayesian models. By comparing different combinations of pooled, unpooled and hierarchical models, it could be shown that the hierarchical modelling of the triaxial tests increases the predictive accuracy of the derived site-specific correlation models. The model PH (pooled on $q_{t1} - \phi'$, hierarchical on the MC model) shows the best performance based on ELPD assessment and shows that for this case, the established Kulhawy-Mayne correlation would overestimate the friction angle. The model, which is unpooled on the sites shows quite similar results for the prediction although different transformation models are derived. This indicates that the transformation models have much less influence in this model than the hierarchical approach on the samples. Further investigation has to be conducted on this topic, as well as using different functional forms or a different clustering approach.

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