

PHYSICS-INFORMED MACHINE LEARNING OF SOIL-WATER CHARACTERISTICS CURVE FOR UNSATURATED FLOW

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A sound understanding of unsaturated soil properties is critical to the serviceability and resilience of coastal infrastructure (e.g., slopes). In tropical and subtropic areas, rainfall events often cause seasonal wetting and drying of surface unsaturated soils, leading to variations in the shear strength and deformation of slopes. The most fundamental unsaturated soil property that governs the hydro-mechanical behaviour of slopes is the soil-water characteristic curve (SWCC). In practice, high-quality soil samples are often collected from sites, and the SWCC curves are obtained from laboratory tests on those samples. The testing process can be time consuming. Alternatively, some studies are dedicated to inversely determining the SWCC based on extensive monitoring data. It is common practice to assume the SWCC follows a given parametric function form and to estimate associated hyperparameters using probabilistic methods. However, this is a nontrivial task, particularly when only limited site-specific measurements are available. To explicitly address this challenge, in this study, an ensemble learning framework is proposed to estimate the SWCC from limited site-specific data. A physics-informed neural network is adopted to solve the partial differential equation (PDE) governing unsaturated seepage. Instead of adhering to a single parametric function form, the SWCC is assumed to be a weighted summation of a series of SWCC bases. The adopted bases consist of representative SWCC curves derived from empirical relationships, and each base follows a different deterministic function. Therefore, the estimation of a suitable SWCC function evolves into the determination of appropriate weight coefficients. The performance of the proposed framework is demonstrated using a synthetic slope example. Results indicate that the proposed method can predict arbitrary SWCC curves, such as bimodal SWCC, with high accuracy.

Keywords: Soil slope, Ensemble learning, Data-driven, Parameter estimation, Water flow, Pore water pressure.

1. Introduction

Global warming has been identified as one of crucial factors that may adversely affect the resilience of existing infrastructure. For example, intensified rainfall events have led to more frequent incidents of slope failure due to a reduction in soil suction and shear strength. Many studies have been devoted to estimating the failure probability of slopes considering various uncertainties, such as soil spatial variability and stratigraphic variations (e.g., Qian and Shi 2024, Qian et al. 2024, Shi et al. 2024, Yang et al. 2025). The key challenge lies in the accurate estimation of governing soil unsaturated parameters, such as soil water characteristic curve (SWCC), a crucial soil parameter for solving unsaturated seepage. Although it may be possible to retrieve soil samples from the slope site of interest for direct measurements of SWCC, this process can be time-consuming and labor intensive. Alternatively, instruments (e.g., tensiometers and piezometers) can be meticulously and strategically placed at critical locations and inverse analyses can then be performed to estimate the SWCC from limited measurements using Bayesian approaches or statistical fitting methods. However, this practice can be problematic as it highly relies on predetermined parametric function forms and lacks the flexibility to account for variability in site data.

Recently, physics-informed neural networks (PINN) have gained significant interest within the scientific computing community and emerges as a new paradigm for solving forward and inverse analyses of geotechnical problems (e.g., Wang et al. 2024). PINN has the merit of bypassing the need for a discrete mesh and excels in estimating governing parameters for parametric functions. For instance, Wang et al. (2024) applied PINN to solve two-dimensional consolidation problems and revealed that PINN is good at out-of-sample predictions with the explicit consideration of estimation uncertainty. However, the current application of PINNs in civil engineering primarily focused on parameter estimation for equations with well-defined partial differential terms. To address the limitation, this study proposes an ensemble PINN method for estimating the SWCC from limited monitoring data. Rather than rely on predetermined partial differential terms, the SWCC is represented as a weighted summation of a series of basis functions, and each basis follows a certain parametric function form. Therefore, the estimation of SWCC evolves into the inference of weight coefficients associated with each basis function.

2. Machine learning of SWCC for unsaturated seepage

Figure 1 outlines the key components of the proposed ensemble learning method. The framework is adopted to solve the governing partial differential equation for a steady seepage problem as follows:

$$\nabla \cdot (k \nabla h) = 0 \quad (1)$$

where h is the hydraulic head; k is the unsaturated permeability and is a function of the normalized volumetric water content $\tilde{\omega}$. The solution to Eq. (1) can be concisely represented as $h = g(\mathbf{x}; \tilde{\omega})$, where \mathbf{x} denotes spatial coordinates and g is the mapping function between \mathbf{x} and h . In essence, the goal is to inversely estimate the SWCC function $\tilde{\omega}$ from limited hydraulic head measurements h . Rather than directly infer the governing hyperparameters associated with an assumed SWCC function $\tilde{\omega}$, this study proposes to represent $\tilde{\omega}$ at the site of interest as a weighted summation of a couple of representative SWCC basis functions (e.g., $\tilde{\omega}_1$ and $\tilde{\omega}_2$), characterized by unknown weight coefficients $\mathbf{w} = [w_1, \dots, w_i, \dots, w_n]^T$. Therefore, the estimation of the SWCC function $\tilde{\omega}$ evolves into the inference of weight coefficients \mathbf{w} , i.e., $h = g(\mathbf{x}; \tilde{\omega})$. The SWCC bases can be empirical SWCC functions obtained from case histories or SWCC samples in a regional database. In this study, the representative GD model (Gardner 1958), FX model (Fredlund and Xing 1994), and VG model (van Genuchten 1980) are taken as the three bases. Once the SWCC ensemble is established, the geometry and hydraulic boundaries of a slope domain for unsaturated seepage can be established, and a physics-informed neural network is built to approximate the mapping function g . More specifically, the physical laws are included as a loss term in the loss function \mathcal{L} for regulating the training of the neural network. More details about the architecture of PINNs can refer to Yang et al. (2025) and Wang et al. (2024).

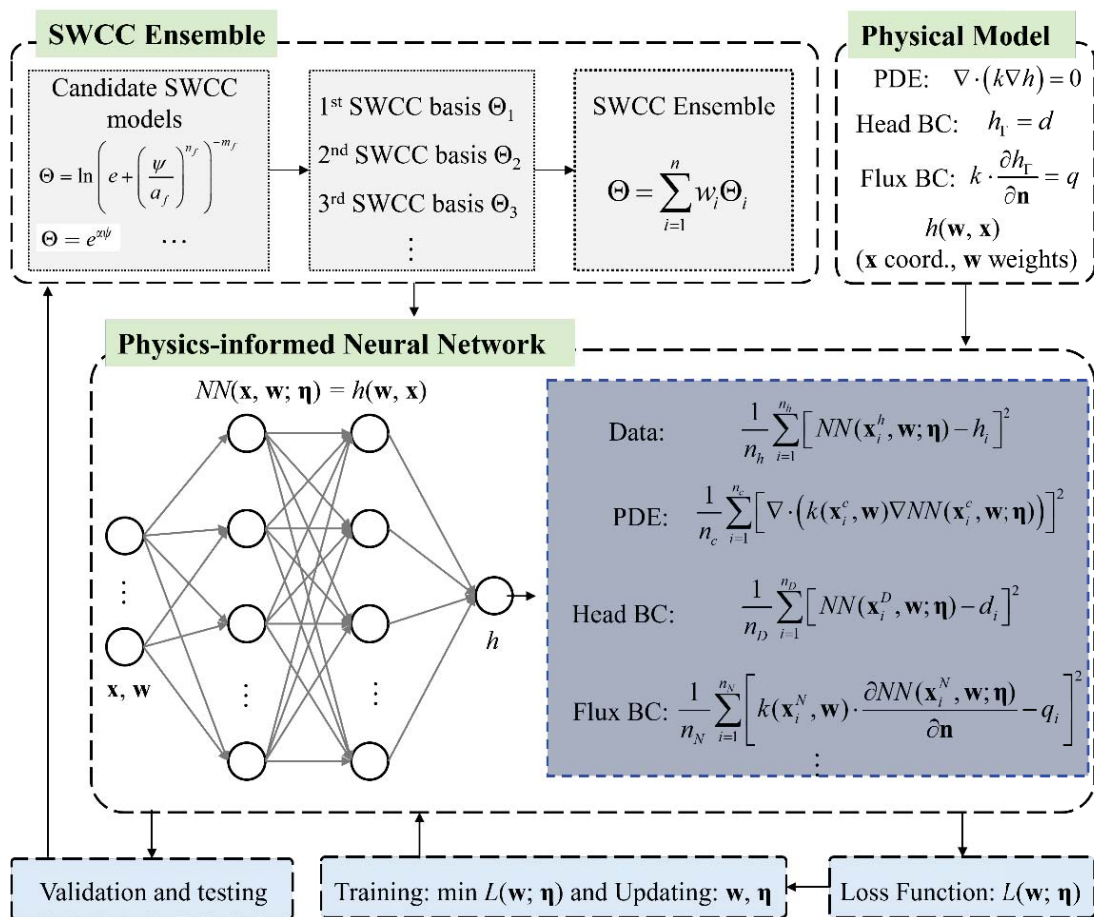


Figure 1 Illustration of the proposed ensemble learning method using PINN (modified from Yang et al. (2025))

3. Illustrative example

A hypothetical slope under steady seepage is adopted to demonstrate the performance of the proposed ensemble learning methods. Figure 2 shows the geometry of the slope, which has a height of 2m and a slope angle of 38 degrees with a constant rainfall flux of 1×10^{-7} m/s. A bimodal SWCC profile is assumed for the soil slope domain (see Figure 3), and the groundwater table aligns with the bottom of the slope. Three representative SWCCs are derived from the three empirical relationships (i.e., VG, GN, and FX model). Based on the imposed boundary conditions and an assumed permeability of 1×10^{-7} m/s, the spatial distribution of the hydraulic head is computed using the finite element method (FEM). The corresponding results are superimposed onto the slope domain. There is a general trend for the hydraulic head to decrease with the depth. For illustration purposes, computed results from seven selected points (i.e., piezometers) are taken as the site-specific measurements.

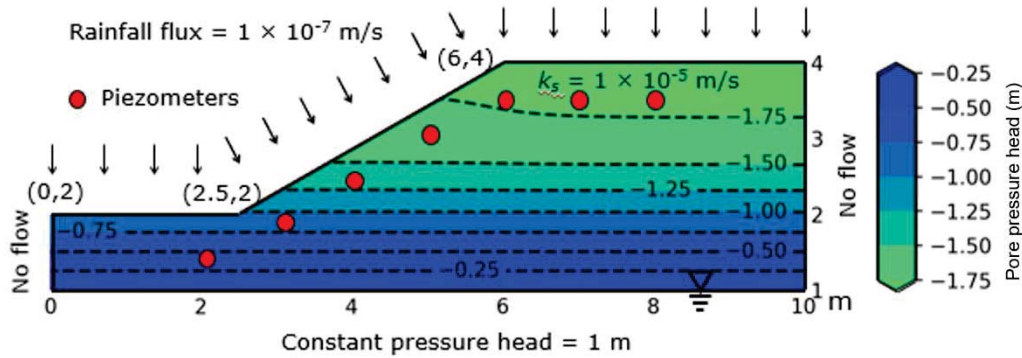
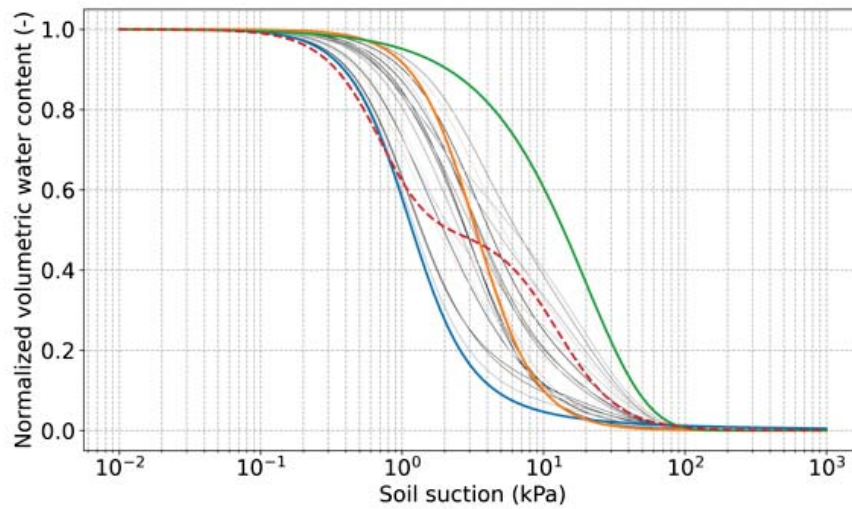


Figure 2 Geometry and model boundary conditions of a hypothetical slope



Benchmark:

- Bimodal SWCC: $\Theta_b = 0.5\Theta_{vg1} + 0.5\Theta_{vg2}$
- Macropores Θ_{vg2} : VG model with $a_g = 0.08 \text{ kPa}^{-1}$, $n_g = 2$, $m_g = 1$
- Micropores Θ_{vg1} : VG model with $a_g = 1 \text{ kPa}^{-1}$, $n_g = 2$, $m_g = 2$

SWCC Ensemble:

- Random SWCCs: $\Theta = w_1\Theta_1 + w_2\Theta_2 + w_3\Theta_3$
- 1st Basis SWCC Θ_1 : FX model with $a_f = 1 \text{ kPa}$, $n_f = 2$, $m_f = 2$
- 2nd Basis SWCC Θ_2 : VG model with $a_g = 0.3 \text{ kPa}^{-1}$, $n_g = 2$, $m_g = 1$
- 3rd Basis SWCC Θ_3 : GN model with $\alpha = 0.05 \text{ kPa}^{-1}$

Figure 3 Assumed bimodal SWCC and representative SWCC bases

4. Results from the proposed method

In this study, the open-access Python package DeepXDE (Lu et al. 2021) was modified to incorporate the ensemble learning procedures. The network shares a typical architecture similar to that of an Artificial Neural Network and comprises one input layer, four hidden layers, and one output layer. Each hidden layer consists of 50 neurons. The Adam optimizer is adopted in combination with the L-BFGS-B optimizer (Taylor et al. 2022).

Figure 4a shows a comparison between the inferred and benchmark bimodal SWCCs. To ensure robustness of the results, 10 random seeds are adopted to randomize the weights associated with network neurons. The estimated SWCCs can reasonably capture the general trend of the SWCC. The difference is relatively larger at low suction values. This is attributed to the fact that all the seven monitoring points have shallow embedment depths, resulting in relatively larger suction values. Therefore, the measurements can better restrain the SWCC for the portion with relatively larger suction values. This is also demonstrated by the spatial distribution of the computed hydraulic head in Figures 4b and 4c. The prediction errors are close to zero at the piezometer locations, and the errors increase as the distance from the measurement locations grows. In addition, the standard

deviation derived from 10 random results also confirms full alignment with the measurements at the seven piezometer locations. This implies that the proposed ensemble learning method is effective in inferring the governing SWCC from sparse site-specific measurements.

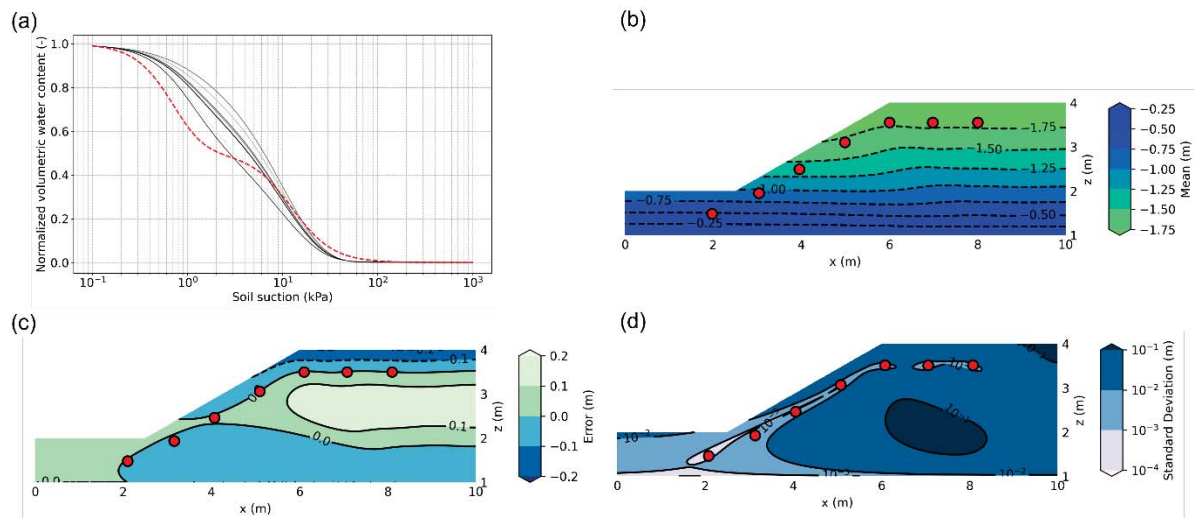


Figure 4 Computed results from the proposed method: (a) Estimated SWCC; (b) Contour of mean hydraulic head; (c) Contour of prediction error; (d) Contour of standard deviation

5. Summary and Conclusions

In this study, an ensemble learning framework that leverages physics-informed neural networks (PINN) is developed to estimate the soil–water characteristic curve (SWCC) for unsaturated seepage. Instead of directly solving parametric SWCC functions, the potential SWCC at a given site is taken as the linear combination of a series of SWCC bases. The coefficient associated with each SWCC basis is estimated based on limited field monitoring data under the framework of PINN. The performance of the proposed method is illustrated using a hypothetical example. Results indicate that the proposed method has the potential to discover new SWCCs that cannot be described by existing empirical parametric models. In addition, the performance of the proposed method can be further enhanced with the increase in the measurement data number.

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