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CHARACTERIZING THE VARIABILITY OF BEDROCK SURFACE USING AN EFFICIENT CONSTRAINT SEED METHOD

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Abstract: Bedrock surfaceis inherently uncertain due to spatial variability and the limited site-investigation data. Although random field theory is effective in evaluating spatial characteristics of soil and rock properties, it often overestimates site uncertainty when borehole data within the site are not incorporated. This paper introduces a novel Constraint Seed Method (CSM) for characterizing bedrock surface, effectively capturing spatial variability while integrating site investigation data. A construction site in Hong Kong, is used as an example to illustrate the effectiveness of proposed method, focusing on the determination of the Grade III rock surface depth. Site investigation data are utilized to continuously update and refine the bedrock surface. This method ensures that the updated Grade III rock surface depth can align with measured data at borehole locations and significantly reduces uncertainty in surrounding areas due to spatial correlation. By integrating data from these successive drilling stages, the method achieves a dynamic and increasingly accurate characterization of the bedrock surface. The proposed method offers a more precise bedrock surface, essential for geotechnical design and construction.

Keywords: Site investigation; Geology; Uncertainty; Analytical method; Updating characterization.

1. Introduction

The performance of geotechnical systems isoften influenced by the properties, depth, and thickness of soil or rock layers. Accurate characterization of a site's stratigraphic structure is therefore essential during the engineering design process (Ching et al., 2021; Yang and Ching, 2021). For instance, in Hong Kong, bored piles in foundation constructionmust be embedded in Grade III or higher-grade bedrock, with the embedment depth exceeds 5 m(GEO, 2006). However, the inherent complexity of geological processes leads to significant variability in geological conditions, making it a challenge to accurately characterize the bedrock surface. In practice, site investigations are typically conducted to gather information about the geological conditions. While due to time and budget constraints,usually only a limited number of boreholes can be drilled across the construction site. Consequently, the bedrock surface is difficult to deterministically define, even when extensive exploration work had been conducted.

Random field theory is a powerful tool for analyzing the spatial characteristics of soil and rock properties and had been widely applied to site characterization (Zhang and Dasaka, 2010). However, conventional random field methods, relying completely stochastic generation process, may yield the simulated valuesthat do not match the observed values at measured locations.This approach is thus called the unconditional random field (URF). When boreholes are drilled at specific locations, the random field must be conditioned to incorporate the measured values at these borehole sites. If these constraints are not considered, the variability of the random field is likely to be overestimated. To address this issue, the conditional random field (CRF) is more suitable for providing an accurate characterization of the bedrock surface(Li et al., 2016, 2018).

The purpose of this study is to propose a new Constraint Seed Method (CSM) for generating CRF using borehole data, andto apply this method to characterize the spatial variability of the Grade III rock surface depth at a construction site in Hong Kong. The structure of this paper is as follows. First, the CSM for generating CRF is introduced. Next, a case study is presented, demonstrating the application of the CSM in characterizing the bedrock surface. Finally, some conclusions are summarized.

2. Characterizing the Bedrock Surface Using the Constraint Seed Method

Consider $\boldsymbol{D}$is the depth of the bedrock surface at the study site. To characterize the bedrock surface using random field theory, one of the common methods, the midpoint method, is employed to discretize the bedrock depth into a correlated vector denoted as $\boldsymbol{D} = [D_1, D_2, \ldots, D_{ne}]^{\mathrm{T}}$, where *ne* represents the number of random field elements. When the statistical properties of these elements remain unchanged throughout the field, the random field is termed stationary. The simulation procedure for Gaussian random field can be expressed as (Liu et al., 2024a):

$$\boldsymbol{D} = [\xi_1 \quad \xi_2 \quad \cdots \quad \xi_{ne}] \begin{bmatrix} L_{1,1} & L_{1,2} & \cdots & L_{1,ne} \\ 0 & L_{2,2} & \cdots & L_{2,ne} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & L_{ne,ne} \end{bmatrix} = [D_1 \quad D_2 \quad \cdots \quad D_{ne}] \tag{1}$$

where ¾= $[\xi_1, \xi_2, ..., \xi_{ne}]^{\mathrm{T}}$ is the vector of independent Gaussian random variables, which is called the random seeds in this study;$\boldsymbol{L}$is the upper triangular matrixderived fromthe Cholesky decomposition of autocorrelation coefficient matrix$\boldsymbol{R}$, $\boldsymbol{R} = \boldsymbol{L}^{\mathrm{T}} \boldsymbol{L}$ .In this study, an exponential autocorrelation function is utilizedto characterize the spatial autocorrelation structure$\boldsymbol{R}$. Using Eq. (1), the stationary and unconditioned random field is generated.

When additional measured data are collected from the study site, the URF may not align withthe observed data at measuredlocations.The field should be conditioned to ensure consistency with the measured values. In this study, an innovative methodis proposed for generating the CRF that fully incorporates the site-specific measurements. At first, since $\boldsymbol{L}$ is an upper triangular matrix, Eq. (1) can be written as

$$\begin{cases} D_1 = \xi_1 L_{1,1} & \text{(a)} \\ D_2 = \xi_1 L_{1,2} + \xi_2 L_{2,2} & \text{(b)} \\ \quad \cdots & \\ D_{ne} = \xi_1 L_{1,ne} + \xi_2 L_{2,ne} + \cdots + \xi_{ne} L_{ne,ne} & \text{(c)} \end{cases} \tag{2}$$

From Eq. (2), it isevident that the$D$ values are determined by the random variables$\boldsymbol{\xi}$. Therefore, if we can constraint the random seed$\boldsymbol{\xi}$ using the site-specific measurements, we can ensure that the simulation values at measuredlocations match the observations,while the spatial autocorrelation structure at other locations is preserved.It is why this method called theconstraint seed method.

Suppose that$\boldsymbol{D^m}$ = $[D_1^m, D_2^m, \ldots, D_k^m]^{\mathrm{T}}$ are collected at $k$specific locations, $\boldsymbol{q} = (q_1, q_2, \ldots, q_k)^{\mathrm{T}}$, the CRF is required to match the observed site data at these$k$ locations. To account for potential measurement errors ε caused by imperfect measurement techniques, the measured values $\boldsymbol{D}^m$ can be expressed as:

$$D^m(q_i) = D(q_i) + \varepsilon_i \quad (i=1,2,...,k) \tag{3}$$

Since εisconsidered following the normal distribution witha mean of zero and a standard deviationof σ_ε. Therefore, the simulations $D(q_i)$ followsa normal distribution with the mean of $D^m(q_i)$ and the SD of σ_ε(Liu et al., 2024b).The key point of CSM is how to constraint$\boldsymbol{\xi}$ using the site data. At first, determine the D_1 values at the 1-th measured location, $D_1 = D_1^m$-ε_1. The seedξ_1 can be constrained as $\xi_1 = D_1 / L_{1,1}$. Then, the seed$\xi_2$should satisfy the measured data $D_2 = D_2^m$-ε_2, as well the spatial autocorrelation structure, $D_2 = \xi_1 L_{1,2} + \xi_2 L_{2,2}$, seed$\xi_2$can be constrained as:

$$\xi_2 = \left(D_2 - \xi_1 L_{1,2}\right)\left(L_{2,2}\right)^{-1} \tag{4}$$

Similarly, the seedξ_3 to ξ_k can be further derived as follows:

$$\xi_k = \left(D_k - \sum_{j=1}^{k-1} \xi_j L_{j,k}\right)\left(L_{k,k}\right)^{-1} \tag{5}$$

Finally, since the seeds$\boldsymbol{\xi}$are mutually independent, the remaining (ne-k) ξseeds can randomly sample from the independent Gaussian space.As a result, the constrained set of seeds $\boldsymbol{\xi}$ is obtained. Using these constrained seeds, the CFR of $\boldsymbol{D}$ can be simulated based on Eq. (1).

3. Illustrative Example

In this section, a case study from Hong Kong is presented to demonstrate the performance of the proposed CSM. Borehole data from the study site are used to condition and characterize the bedrock surface, i.e., determining the depth of the Grade III rock surface. Detailed information about the study site can be found in Zhang and Dasaka (2010), Dasaka and Zhang (2012), and Li et al. (2016). The exploration area covers 69 × 60 m, and the locations of 49 boreholes are shown in Fig. 1. The depthsof the Grade III rock surface for boreholesare shown in Fig. 2. For more comprehensive detailsof the boreholes data, readers can refer to Li et al. (2016).

To characterize the variability of bedrock surface, the research site is discretized into 460 random field elements, each with a size of 3 × 3 m. Given the availability of sufficient borehole data (e.g., 49 boreholes), the prior statistics of the field can be estimated through statistical analysis, with the scale of fluctuation estimated as 20.3 m(Dasaka and Zhang, 2012). Previous studies (Dasaka and Zhang, 2012; Li et al., 2016) also indicate that the depth of the Grade III rock surface shows a non-stationary trend across the site. Therefore, a linear trend is calculated using the least squares method based on the measured data. The trend function is:

$$t = 207.03 - 0.11x - 0.10y \tag{6}$$

where $t$denotes the trend depth ofGrade III rock surface;x and $y$represent the north coordinate and east coordinate, respectively.Fig. 2 presents the borehole data alongside the trend surface for the Grade III rock surface depth.

The depth of Grade III rock, D, is modelled as the sum of a trend component and a residual depth component, e, expressed as $D = t + e$. Where $e$is considered as a Gaussianfield, with a mean of 0 and a standard deviation of 1.85 m, estimated using the actual borehole data.Fig. 3 shows the prior mean values of residualfield.It is evident that the prior mean remains constant across the entire field, which does not match the actual residual depths at the borehole locations.Assuming that the measurement errorε follows a normal distribution with a mean of 0 and a standard deviation of 0.05 m (Li et al., 2016), the residual field can be constrained and updated using the proposed CSM.

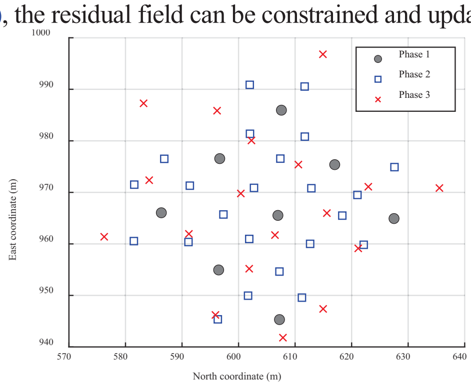

Fig. 1. Borehole locations at the study site.

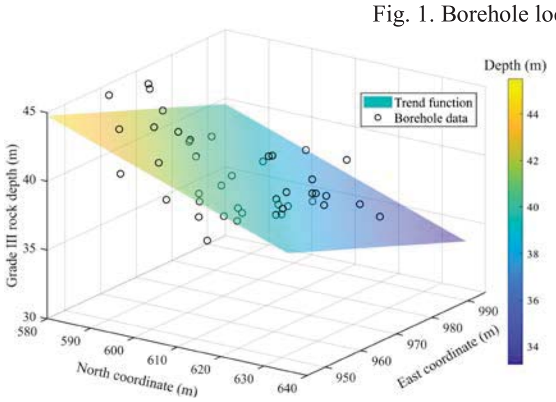

Fig. 2. The trend function and borehole data for depth of Grade III rock surface.

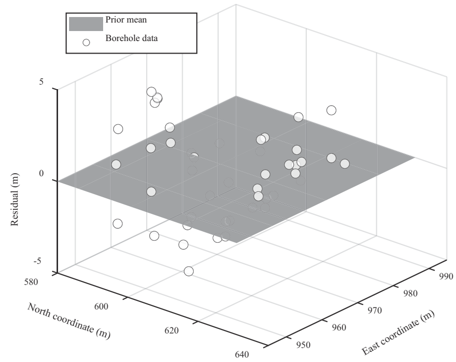

Fig. 3.Prior mean and borehole data of the residual depth field.

The CSM simulations are performed with 10,000 samples to generate the conditional random field. By averaging these samples, Fig. 4 shows the updated mean and borehole data for the residual depth field. The updated mean across the study site significantly differs from the prior mean, the updated mean values at borehole locations matching the borehole residuals (i.e., the actual measured value minus the trend value). Mean values at other locations are also updated due to the spatial correlation. The updated means at other locations increase with larger residuals at nearby boreholes and decrease with smaller residuals.By combining the trend function with the mean residual depth field, Fig. 5 shows the updated mean and borehole data for the depth of the Grade III rock surface. As expected, the updated mean aligns well with the borehole data, showing a non-stationary distribution across the study site. The depths are higher in the northwest and gradually decrease towards the southeast.

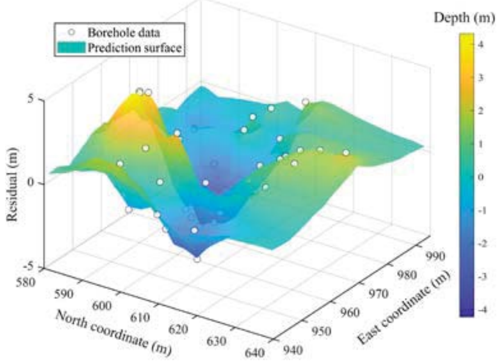

Fig. 4. Updated mean and borehole data of the residual depth field.

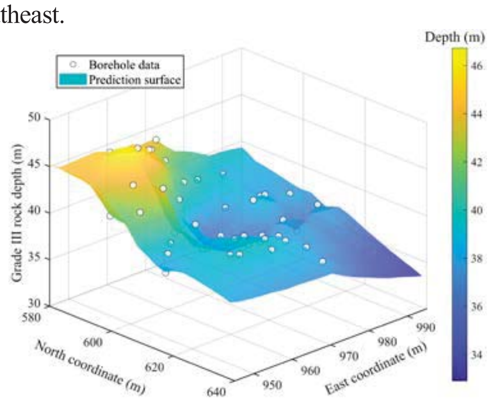

Fig. 5. Updated mean and borehole data for depth of Grade III rock surface.

To illustrate the reduction in uncertainty of the residual field after conditioning, Fig. 6 presents the updated standard deviations. The standard deviations far from the boreholes closely match the prior value of 1.8 m, while those near the boreholes decrease significantly, approaching 0.05 m (i.e., the standard deviation of measurement error). This demonstrates a substantial reduction in uncertainty in areas close to the boreholes.In the central region of residual field, a significant reduction in standard deviation is observed, primarily due to the high density of boreholes, which substantially reduces uncertainty. In contrast, at the corners of residual field, where boreholes are sparse, the reduction in standard deviation remains minimal, indicating a still higher level of uncertainty in these areas.

To quantify the global reduction in uncertainties, the percentage of global uncertainty reduction (η) is adopted here(Lloret-Cabot et al., 2012). The index $\eta$is defined as the ratio between the variance of the conditional field andthat of the prior field, thereby reflecting the extent ofglobal uncertainty reduction after conditioning.

$$\eta = \left(1 - \frac{u_{conditional}}{u_{prior}}\right) \times 100\% \quad (7)$$

where $u_{conditional}$ and u_{prior}denote the global uncertainty for the conditional and prior field, respectively; $u_{conditional} = \sum_{i=1}^{ne} \sigma^2 \left[x_{conditional}(i)\right]$ and $u_{prior} = \sum_{i=1}^{ne} \sigma^2 \left[x_{prior}(i)\right]$.Based on Eq. (7), the reduction percentageisestimatedto be45.05%, implying thatthe unconditioned random field willsignificantly overestimate the uncertainty.

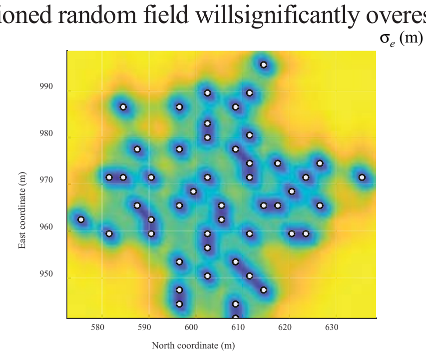

Fig. 6. Updatedstandard deviations of the residual depth field.

4. Conclusion

This paper introduces anovel constraint seed methodfor characterizing bedrock surface conditioned on borehole data. This method is applied to assess the spatial variability of the Grade III rock surface depth at a construction site in Hong Kong. The results demonstrate thatthe CSM effectively captures both the spatial variability and nonstationary distribution trend of the bedrock surface.The constrained field not only aligns with the borehole data at the borehole locations but also updates field values at nearby locations due to the inherent spatial autocorrelation structure.Furthermore, the standard deviations at borehole locationsclosely match theσ_ε of 0.05 m. In contrast, locations farther from the boreholes exhibit higher uncertainty, with standard deviations approaching the prior value of 1.8 m.

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