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# PROBABILISTIC CALIBRATION OF RESISTANCE FACTORS FOR PILE GROUP CONSIDERING THE SPATIAL VARIABILITY OF SOILS

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Resistance factors for pile groups are typically derived using empirical methods that do not directly account for system redundancy and overlook the correlation between individual piles, which are inherently influenced by the spatial variability of soils. While rigorous three-dimensional random finite difference or random finite element analyses could potentially address these issues, they are constrained by significant computational demands. To address these issues, this study introduces two novel probabilistic methods for calibrating resistance factors for pile groups based on individual load tests. These methods utilize Bayesian and machine learning techniques, respectively.Comparative analyses indicate that the results derived from both approaches exhibit good agreement.

*Keywords*: Pile group, Resistance factor, Bayesian approach, Machine learning.

## 1. Introduction

Resistance factors for pile groups are conventionally derived using empirical methods(Zhang et al., 2024b). Typically, to account for system redundancy, it is assumed that the pile group achieves a target reliability index of 3, while individual piles attain a lower reliability index, ranging from 2 to 2.5(Kwak et al., 2010; Ng & Sritharan, 2014; Paikowsky, 2004; Zhang et al., 2023). However, this approach is quite empirical as it does not directly consider the system redundancy and the correlation between individual piles, which arepredominantlyinfluenced by the inherent spatial variability of soils(Naghibi & Fenton, 2017; Zhang et al., 2024a).

For a robust calibration of resistance factors that encompasses the spatial variability of soils and results from site-specific load tests, a rigorous three-dimensional (3D) numerical analysis is required, such as the random finite difference method (RFDM).However, the computational demands are substantial, as the 3D numerical analysis must be repeated numerous times to update the reliability of pile groups(Huang et al., 2016).To address this challenge, a Bayesian approach integrated with RFDM is proposed by the authors(Zhang et al., 2024b), which combines the RFDM and Bayes' Theorem to calibrate resistance factors for pile groups. Alternatively, to reduce computational overhead, the development of surrogate models based on machine learning techniques, such as Convolutional Neural Networks (CNN), which have proven effective for reliability analysis of geotechnical systems considering spatial variability, is suggested(Wang & Goh, 2021; Wu et al., 2022; Xu et al., 2023).Employing a trained machine learning model facilitates numerous simulations efficiently, enabling the direct computation of failure probabilities for both individual piles and the pile group through counting. The calibration of resistance factors is straightforward by adjusting these factors to achieve a target reliability index (or probability of failure) of the pile groups.

In this paper, the procedural flowcharts of these two approachesarefirst presented. Then, an illustrative example is utilized to demonstrate these two approaches. Finally, comparison analyses are conducted, which revealgood agreementbetween the two approaches.

## 2. Methodology

In Load and Resistance Factor Design (LRFD), the design equation is expressed as(AASHTO, 2020):

$$\phi R_{gn} = \gamma_D Q_{Dn} + \gamma_L Q_{Ln} \quad (1)$$

where $\phi$, $\gamma_D$ and $\gamma_L$ represent the resistance factor, dead load factor and live load factor, respectively. $R_{gn}$, $Q_{Dn}$ and $Q_{Ln}$ are the nominal pile group resistance, nominal dead load and nominal live load, respectively.

For calibration purposes, a limit state function is formulated. In the Bayesian approach, the limit state function is represented by Eq. (2), whereas for the machine learning-based approach, it is expressed as Eq. (3) (Zhang et al., 2024b).

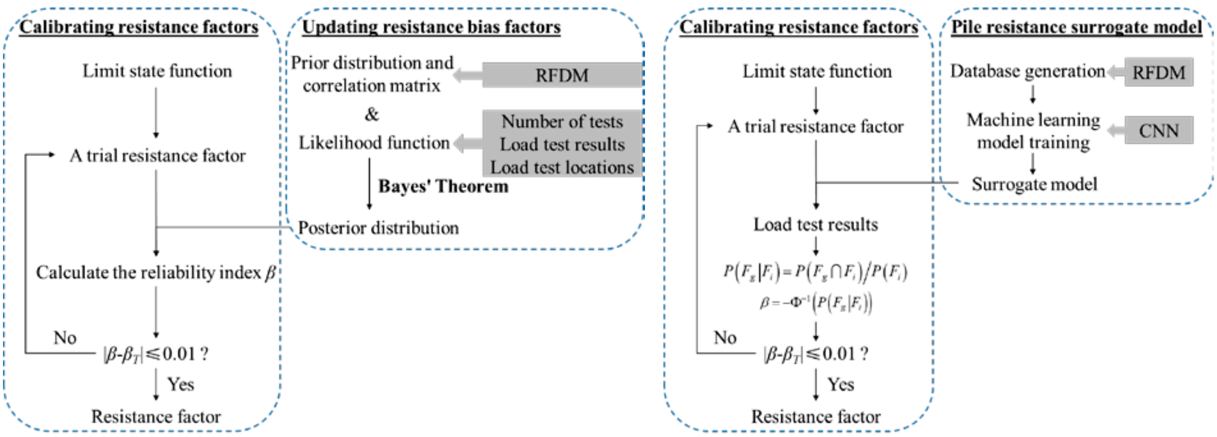
$$g = \frac{\lambda_\eta}{\phi N} \times \sum_{i=1}^{N} \lambda_{iR} \times (\gamma_D \kappa + \gamma_L) - (\lambda_D \kappa + \lambda_L) \tag{2}$$

$$g = \frac{R_g}{\phi N \eta_n R_n} \times (\gamma_D \kappa + \gamma_L) - (\lambda_D \kappa + \lambda_L) \tag{3}$$

In Eqs. (2) and (3), $\lambda_\eta$ , $\lambda_{iR}$, $\lambda_D$ , $\lambda_L$ , and $R_g$ are random variables. The group efficiency is a random variable when the spatial variability of soils is considered(Zhang et al., 2024a), thus, $\lambda_\eta$ is defined as the ratio of the group efficiency to its mean, $\eta_n$ . $\lambda_{iR}$ is the resistance bias factor of individual piles, defined as the ratio of the individual pile resistance to its mean, $R_n$ . Without proof load tests, $\lambda_{iR}$ is identical for all piles, as they are located within the same random soil field. However, when load tests are conducted, $\lambda_{iR}$ is updated based on load test results and test locations, leading to variations across individual piles. $\lambda_D$ and $\lambda_L$ are the dead and live load bias factors, respectively. These factors are generally assumed to follow a lognormal distribution, with statistical parameters obtained from design codes. $R_g$ is the pile group resistance, which is determined using the RFDM or a CNN surrogate model. $N$ is the number of piles within the group. $\kappa$ is the ratio of dead to live load.

The flowcharts illustrating the Bayesian and machine learning-based approaches are presented inFig. 1 (a) and (b), respectively. In Fig. 1(a), the Bayesian method assumes that the group efficiency and the resistance bias factors follow a lognormal distribution. Their statistical parameters (i.e., mean and standard deviation) are derived from a limited number of RFDM. Subsequently, using assumed load test results, the Bayesian approach is employed to update the resistance bias factors, which are then used to recalibrate resistance factors for pile groups to achieve a targeted reliability index.

Fig. 1(b)depicts the flowchart for the machine learning-based approach.In this approach, an appropriate number of RFDM simulations are conducted to train a CNN model, which then serves as a surrogate model. The surrogate modelis subsequently employed to generate a large pile resistance dataset,such as 100,000 pairs of resistances for individual piles and pile groups, based on the specified random field of soil properties. This enables the estimation of the failure probabilities of individual piles, $P(F_i)$, and the pile group, $P(F_g)$ through counting. The calibration of resistance factors is straightforward, involving adjustments to these factors to achieve a target reliability index (or probability of failure). Furthermore, when load tests are performed on individual piles, the failure probability of the pile group, conditional on the observed load test results, $P(F_g|F_i)$ is determined using conditional probability, facilitating the recalibration of conditional resistance factors to satisfy the desired reliability criteria.




(a) Bayesian approach (b)Machine learning-based approach

Fig. 1.Flowchart of the proposed approaches for calibration

## 3. Example

For demonstration, a 3×3 pile group subjected to vertical loading is analyzed, as depicted inFig. 2,with the pile and soil parameters detailed inTable 1.Additionally, the pile group is assumed to be free-standing with a rigid cap, undergoing only vertical deformation under vertical loading (i.e., no rotations). The proof test load is set to

$T = R_n$, with an associated test error of 0.1$T$.In the calibrationprocess, $\gamma_D = 1.25$ and $\gamma_L = 1.75$ are considered, while $\kappa$=2 and $\beta_T = 3$ are adopted.

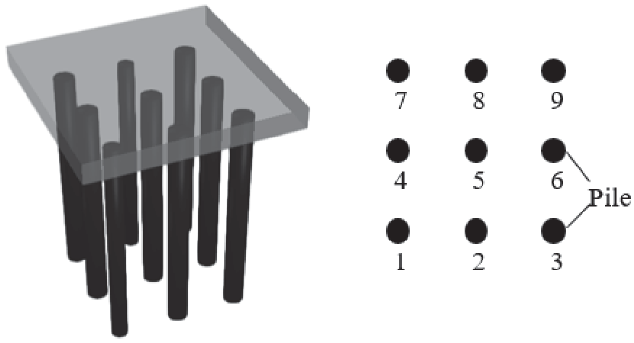



Fig. 2.Pile group configurations

Table 1. Details of pile and soil parameters for the illustrative example

| | Parameters | Value |
|---|---|---|
| Pile | Diameter, m | 1 |
| | Embedded length, m | 10 |
| | Spacing, m | 3 |
| | Elastic modulus, kPa | $2.2\times10^7$ |
| | Poisson ratio | 0.3 |
| Soil | Mean of undrained shear strength, kPa | 20 |
| | Coefficient of variation of undrained shearstrength | 50% |
| | Horizontalspatial correlation length, m | 40 |
| | Vertical spatial correlation length, m | 5 |
| | Shear modulus, kPa | $1.3\times10^3$ |
| | Bulk modulus, kPa | $6.0\times10^3$ |

### 3.1.*Effect of training sample size in machine learning-based approach*

This subsection evaluates the influence of training sample sizes on the calibrated resistance factorsin the machine learning-based approach. For illustrative purposes, three scenarios are considered: one proof load test conducted on pile 5 that passes, two proof load tests conducted on pile 1 and pile 5 that both pass, and three proof load tests conducted on pile 1, pile 5 and pile 9 that all pass. Fig. 3 presents the resistance factor for different training sample sizes, which shows that the resistance factors stabilize when 500 samples are used for training. Additionally, resistance factors increase as more piles pass the load tests, aligning with engineering practices. For example, when 500 samples are used for training, the resistance factor is 0.96 when pile 5 passes the load test. The resistance factor increases to 1.02 if an additional load test on pile 1 also passes.

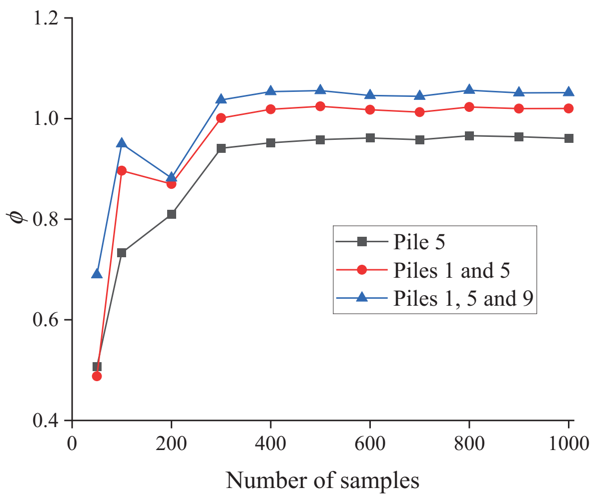



Fig. 3.Resistance factors as a function of the sample sizes and test results

### 3.2.*Comparison between Bayesian and machine learning-based approaches*

This subsection compares the resistance factors obtained from the Bayesian approach and the machine learning-based approach, considering various numbers of load tests and corresponding test outcomes summarized in Table 2. In the table, the notation 'S' denotes that the tested pile passes the load test, while 'F' indicates failure.The results from both approaches are observed to be very similar, with the absolute difference between the two methods being less than 0.03. It is also noted that a significant advantage of the machine learning-based approach is that it does not require assumptions concerning the distribution of resistance bias factors, group efficiency, or the correlations among these variables. Instead, these characteristics are inherently captured within the extensive dataset generated through thesurrogate model.

Table 2. Resistance factors obtained by Bayesian and machine learning-based approaches

| Load test results | Bayesian approach | CNN approach | Absolute difference |
|---|---|---|---|
| No tests | 0.63 | 0.66 | 0.03 |
| 1S | 0.97 | 0.94 | 0.03 |
| 1F | 0.61 | 0.63 | 0.02 |
| 2S8F | 0.92 | 0.89 | 0.03 |
| 2F8F | 0.62 | 0.63 | 0.01 |
| 1F5S9F | 0.89 | 0.87 | 0.02 |
| 1F5F9F | 0.61 | 0.59 | 0.02 |

## 4. Conclusion

This paper introduces two distinct approaches, the Bayesian approach and the machine learning-based approach, for calibrating resistance factors for pile groups using individual pile load tests while accounting for the spatial variability of soils. A synthetic example is provided to illustrate the application of these approaches. The results demonstrate that the resistance factors obtained using the machine learning-based approach align closely with those derived from the Bayesian approach, indicating strong agreement between the two methods.

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