

VARIABILITY CHARACTERIZATION OF MODEL PARAMETERS OF SWCC FOR SANDY SOILS

Ammavajjala Sesha Sai Raghuram

Assistant Professor, Department of Civil Engineering, Birla Institute of Technology and Science (BITS) Pilani Hyderabad Campus, Shamirpet, Hyderabad, Telangana, India – 500078

B. Munwar Basha

*Professor and Head, Department of Civil Engineering, Indian Institute of Technology Hyderabad, Kandi, Sangareddy, Telangana, India - 502285
E-mail: basha@ce.iith.ac.in*

A new statistical framework was employed to quantify the variability in the model parameters of the soil-water characteristic curve (SWCC) for sandy soils. It is essential to have precise estimates of the mean, standard deviation, and probability density function (PDF) of these model parameters in order to ensure a reliability based design in unsaturated soil mechanics. The conventional assumptions of Gaussian or lognormal distributions may not sufficiently capture the significant variability of these model parameters. The objective of this work is to determine the most appropriate continuous probability density functions (PDFs) by optimizing the mean and standard deviation in order to minimize errors in estimating quantiles, percentiles, and cumulative distribution functions (CDFs). Data from 228 sample points, drawn from comprehensive experimental studies on SWCC model parameters for sandy soils, were analyzed. The most appropriate distributions for model parameters of the SWCC vary significantly depending on the type of soil. In Fredlund and Xing's SWCC model, it was determined that the Gamma, Weibull, and Inverse Gaussian distributions are the most suitable for representing the model parameters a_f (related to air entry value), n_f (related to slope of SWCC), and m_f (related to residual suction), respectively, in the case of sandy soils. Additionally, the proposed PDFs for sandy soils were considered and applied for an infinite slope. Analysis of a shallow sandy slope is presented with the appropriate PDFs proposed in this study. The outcomes revealed that the selection of an inappropriate PDF resulted in a significant overestimation of the probability of failure of unsaturated infinite slopes composed of sandy soils. The suggested mean, coefficient of variation (COV), and probability density functions provided by the research are useful for designing and evaluating the performance of unsaturated soil slopes based on reliability.

Keywords: Variability, probability density function (PDF), unsaturated soils, soil water characteristic curve (SWCC), infinite slopes, reliability analysis.

1. Introduction

Most natural and compacted soils encountered in the field are typically unsaturated. In this state, soil properties are significantly influenced by changes in saturation and soil suction. The soil water characteristic curve (SWCC) describes the relationship between soil suction and water content. Fredlund and Rahardjo (1993) highlighted that properties such as shear strength, volume change, and permeability are linked to the SWCC. Unsaturated shear strength (USS) and hydraulic properties are critical for geotechnical design, but estimating SWCC using regression equations can be challenging due to precision concerns. Although SWCC is key to unsaturated soil behavior, knowledge of its fitting parameter variability is limited. Several methods are used to measure soil suction, such as filter paper, pressure plate, Tempe cell, and dew point potentiometer. These methods vary in their ability to measure suction types - matric, total, or osmotic - and each has limitations except for the filter paper method, which cover the full suction range (0 to 10^6 kPa). Zapata (1999) noted variability between methods. Thus, experimental method variability must be carefully considered when predicting SWCC. Leong and Rahardjo (1997) recommend using the Fredlund and Xing (1994) (FX) model for more accuracy. The FX model is adopted in the present study for statistical analysis. While the application of statistical methods to unsaturated soils has gained traction, limited studies have quantified the variability of SWCC fitting parameters. This variability is critical for assessing the probability of failure in unsaturated infinite and finite slopes.

2. Literature

Variability in geotechnical properties arises from measurement errors, transformation uncertainties, and inherent variability, as noted by Phoon and Ching (2014). Several studies (Fenton and Griffiths, 2003) focused on uncertainties in soil shear strength parameters. Statistical tools like coefficient of variation (COV) and probability density functions (PDFs) are commonly used to quantify these variabilities (El-Ramly et al., 2002). Gaussian and lognormal PDFs are often applied to model soil cohesion and friction angle (Kulhawey, 1992). Several studies

Gaussian and lognormal PDFs are often applied to model soil cohesion and friction angle (Kulhawy, 1992). Several studies addressed SWCC variability and its implications. Raghuram et al. (2020) highlighted the impact of fine content on SWCC shape. Zapata (1999) emphasized that high variability in SWCC fitting parameters can compromise design conservatism, with up to 54% variability reported in volumetric water content for Fountain Hills clay. Overall, the statistical modeling of SWCC fitting parameters using mean, standard deviation, and COV values are critical for understanding soil behavior. However, distributions and assumptions must be carefully selected, as some, like normal distributions, may not adequately represent shear strength or factor of safety (Phoon and Ching, 2014).

3. Data Compilation and Statistical Optimization

The study compiles fitting parameters of the SWCC (FX model) for sandy soils from experimental data spanning 1970 to 2016, sourced from diverse regions worldwide. The parameters were analyzed to assess their variability, with detailed experimental data. It is noticed that a_f ranges from 0.1 kPa to 6 kPa; n_f ranges from 1.19 to 14.55; m_f ranges from 0.2 to 3.279. The wide ranges of these parameters underscore the substantial variability inherent to the fitting parameters of the SWCC. This variability, as observed across sandy soil, emphasizes the importance of statistical tools to analyze and interpret the data effectively for geotechnical and environmental applications. Statistical optimization developed by Raghuram et al. (2021) was performed to calculate the mean, standard deviation, and coefficient of variation (COV) of the probability density function (PDF) for the fitting parameters of the SWCC. The process involved analyzing the experimental data using histograms and PDFs, along with evaluating errors from quantile-quantile (Q-Q), percentile-percentile (P-P), and cumulative distribution function (CDF) plots. Errors were computed by comparing the experimental data's fit to Q-Q, P-P, and CDF plots, which provide insights into how well the data align with the assumed statistical distributions. This process ensures the PDF represents the underlying variability of the data more accurately.

4. Results and Discussion

4.1. Fitting parameter a_f

Figure 1(a) reveals a significant right-skewness in the distribution of a_f values for sand. As shown in Figure 1(b), the Probability Density Function (PDF), Cumulative Distribution Function (CDF), P-P plots, and Q-Q plots for the Gamma distribution demonstrate its suitability for modeling the histogram of a_f values. The CDF, P-P, and Q-Q plots in Figure 1(b) show a strong alignment between the data points and their reference lines. Fig. 1(b) further supports this, showing that the errors for Q-Q and P-P plots are 7.69 and 7.81, respectively, the lowest among all considered distributions. However, the CDF error for the Gamma distribution is 15.53, which is not the minimum. This discrepancy arises from the sharp accumulation of data on the left side of the PDF, leading to a noticeable deviation in the CDF plot. Overall, the findings suggest a good agreement between the experimental data and the Gamma distribution, making it the most appropriate choice for modeling a_f for sand.

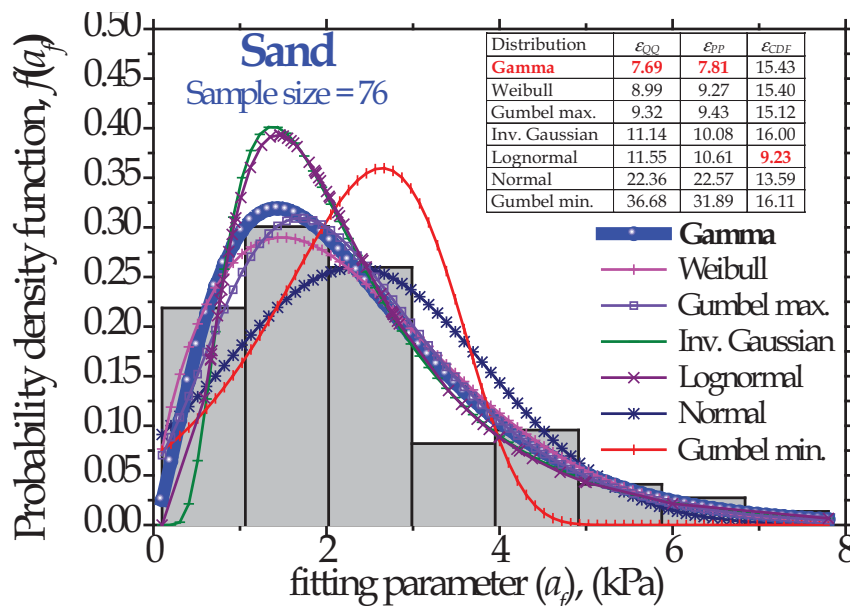


Fig. 1(a). Histogram superimposed with different PDFs for parameter a_f .

The optimal mean, standard deviation, and coefficient of variation (COV) for a_f are 2.4 kPa, 1.53 kPa, and 63.75%, respectively. Similar analysis was carried out for parameters n_f and m_f and the results are summarized in Table 1. Due to space constraints the plots are not presented.

4.2. Infinite Slope

An unsaturated infinite slope is analyzed in this study. The depth to the water table (Z_w) and the slip surface (Z_s) are measured from the ground level. The analysis employs the framework proposed by Bishop (1959), who extended Terzaghi’s effective stress principle to unsaturated soils. A recent study by Raghuram and Basha (2020) identified that the unsaturated model proposed by Vanapalli et al. (1996) provides the most conservative results. Consequently, this model is adopted in the reliability analysis to ensure a robust assessment of slope stability under unsaturated conditions. First order reliability analysis developed by Raghuram and Basha (2021) has been used in the present study to compute the reliability index and probability of failure of the sandy slope. For this purpose, a sandy slope with a slope angle of 30° is assumed. The depth of the slip surface and water table are assumed to be 1.5 m and 10 m below the ground level. The results in Fig. 2 indicate that the percentage difference (ϵ) between P_{f1} and P_{f2} increases significantly as the COV of m_f increases from 10% to 50%, with ϵ reaching up to 22.13%. These findings underscore the importance of using the most appropriate probability density functions (PDFs) for SWCC fitting parameters. Neglecting these can lead to significant overestimation of failure probabilities.

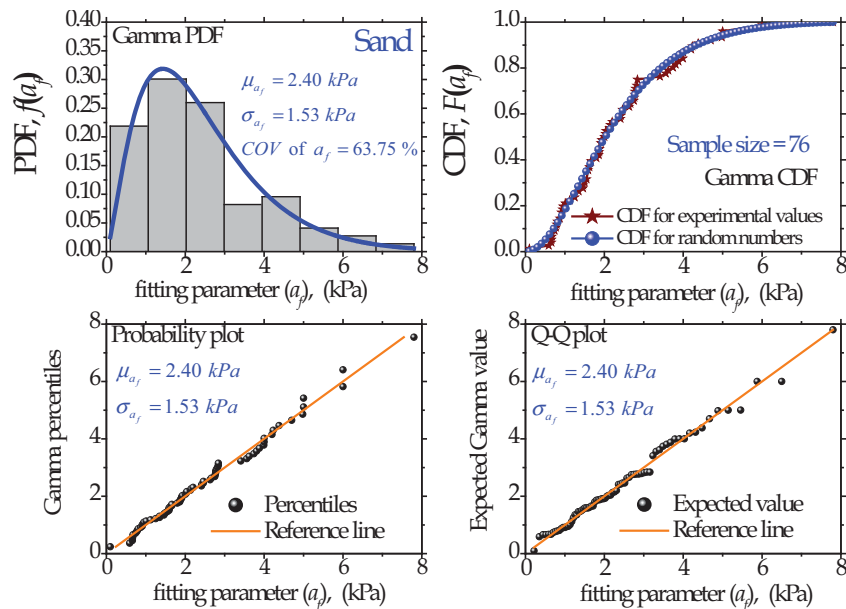


Fig. 1(b). Histogram with optimized PDF, CDF, P-P, and Q-Q plots for parameter a_f .

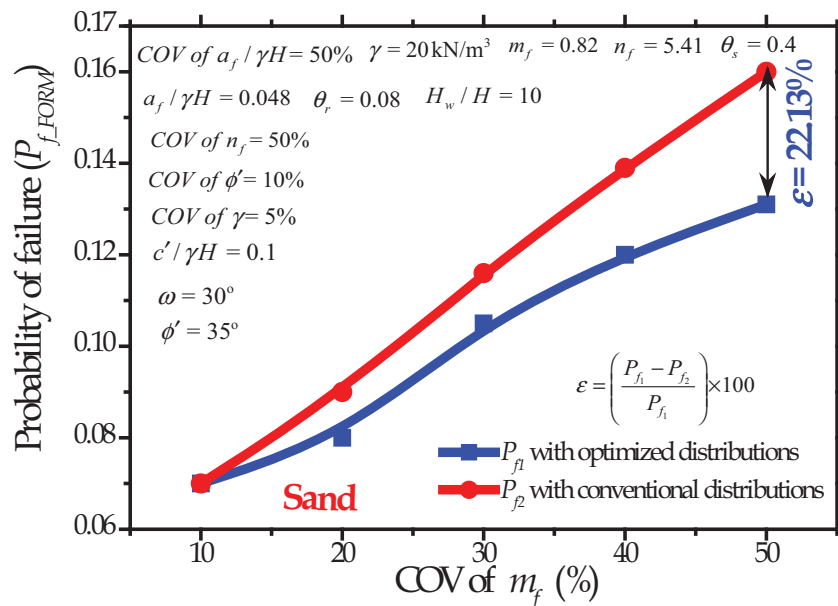


Fig. 2. Comparison between the P_f estimated using conventional and optimized distributions.

Table 3. PDFs, mean, standard deviation and COV of fitting parameters of SWCC for sands

SWCC fitting parameters	Optimized parameters			
	μ	σ	COV (%)	Distribution
a_f (kPa)	2.40	1.53	63.75	Gamma
n_f	5.41	3.68	68.02	Weibull
m_f	0.82	0.42	51.76	Inverse Gaussian

Conclusions

The variability of SWCC fitting parameters is inevitable. This study, based on a database of 76 samples collected from 1970–2016 for sandy soils, investigates the variability in parameters a_f , n_f , and m_f , related to air entry value, slope, and residual water content, respectively. Key findings include:

- Statistical Optimization:** Seven continuous PDFs (Gaussian, lognormal, Weibull, Gumbel minimum, Gumbel maximum, Gamma, and inverse Gaussian) were evaluated using histogram overlays and errors in quantiles (QQ), percentiles (PP), and cumulative distribution functions (CDFs).
- Sandy Soils:**
 - a_f : Gamma distribution models air entry value with a mean of 2.4 kPa, standard deviation of 1.53 kPa, and COV of 63.75%.
 - n_f : Inverse Gaussian distribution fits the slope parameter with a mean of 5.41, standard deviation of 3.68, and COV of 68.02%.
 - m_f : Weibull distribution models residual water content with a mean of 0.82, standard deviation of 0.42, and COV of 51.76%.
- Impact on Probability of Failure (Sandy Slopes):** Ignoring the most appropriate PDFs leads to a significant overestimation of the failure probability of sandy slopes by 22.13% when COVs of m_f increased to 50%.

Overall, the study highlights the importance of selecting the best-fit PDFs for SWCC fitting parameters to improve the accuracy of reliability-based design for unsaturated soil slopes.

References

- El-Ramly, H., N. Morgenstern, and D. Cruden. (2002). "Probabilistic slope stability analysis for practice." *Can. Geotech. J.* 39 (3): 665–683. <https://doi.org/10.1139/t02-034>.
- Fenton, G. A., and D. V. Griffiths. (2003). "Bearing capacity prediction of spatially random c–u soils." *Can. Geotech. J.* 40 (1): 54–65. <https://doi.org/10.1139/t02-086>.
- Fredlund, D. G., and H. Rahardjo. (1993). *Soil Mechanics for Unsaturated Soils*. New York: Wiley.
- Fredlund, D. G., and A. Xing. (1994). "Equations for the soil-water characteristic curve." *Can. Geotech. J.* 31 (3): 521–532. <https://doi.org/10.1139/t94-061>.
- Kulhawy, F. H. (1992). "On the evaluation of static soil properties." *Stability and performance of slopes and embankments II: ASCE geotechnical special publication no. 31*, 95–115. Reston, VA: ASCE.
- Leong, E. C., and H. Rahardjo. (1997). "Review of soil-water characteristic curve equations." *J. Geotech. Geoenviron. Eng.* 123 (12): 1106–1117. [https://doi.org/10.1061/\(ASCE\)1090-0241\(1997\)123:12\(1106\)](https://doi.org/10.1061/(ASCE)1090-0241(1997)123:12(1106)).
- Phoon, K. K., and J. Y. Ching. (2014). *Risk and reliability in geotechnical engineering*. Boca Raton, FL: CRC Press.
- Raghuram, A. S. S., B. M. Basha, and A. A. B. Moghal. (2020). "Effect of fines content on the hysteretic behavior of water-retention characteristic curves of reconstituted soils." *J. Mater. Civ. Eng.* 32 (4): 04020057. [https://doi.org/10.1061/\(ASCE\)MT.1943-5533.0003114](https://doi.org/10.1061/(ASCE)MT.1943-5533.0003114).
- Raghuram, A. S. S., Basha, B. M., and Raviteja, K. V. N. S. (2021). Variability characterization of SWCC for clay and silt and its application to infinite slope reliability. *Journal of Materials in Civil Engineering*, 33(8), 04021180.
- Vanapalli, S. K., D. G. Fredlund, D. E. Pufahl, and A. W. Clifton. (1996). "Model for the prediction of shear strength with respect to soil suction." *Can. Geotech. J.* 33 (3): 379–392. <https://doi.org/10.1139/t96-060>.
- Zapata, C. E. (1999). "Uncertainty in soil-water characteristic curve and impacts on unsaturated shear strength predictions." PhD thesis, Arizona State University, Tempe, USA.