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TOWARDS BAYESIAN CONSTITUTIVE MODEL PARAMETER CALIBRATION FOR STRAIN-SOFTENING SOILS

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Conventional element tests are limited in their ability to observe certain behaviors relevant to offshore engineering (e.g. strain softening). In this paper, an alternative approach to parameter calibration, incorporating full-field measurements, was expanded using Bayesian methods. A constitutive model's strain softening parameters were calibrated using numerical biaxial compression test datasets, altered with noise.Requirements regarding the quality and quantity of data required are recognized with a view to ultimately performing experimental trials of the proposed calibration approach.

Keywords: constitutive modeling; parameter calibration; full-field measurements; strain-softening; Bayesian inference

1. Introduction

Numerous constitutive geotechnical models attempt to characterize strain-softening behavior, which is prevalent in a variety of offshore geotechnical problems. Standard element tests, used for conventional calibration, implicitly assume homogeneous deformation, contrary to the true nature of soils (Muir Wood, 2012), and cannot apply sufficient deformation to observe complete remoulding. The subsequent selection of parameters, as well as the model itself, is often qualitative and therefore subjective. These uncertainties in model fit and parameter calibration could potentially be addressed by adopting a radically alternative approach based on the Virtual Fields Method (VFM) (Grédiac, 1989). Crucially, the technique enables full-field deformation measurements - better-suited to capturing remoulding - to be incorporated into the calibration process. Thus far, the approach has proved successful (Avril et al., 2008; Charles, 2018) but has only been numerically validated for realistic constitutive models as a “proof-of-concept” prior to experimental trials (Singh et al. 2023). By varying the additive noise and extent of compression in a numerical undrained biaxial compression simulation performed by Singh et al. (pers. comm.), this paper statistically justifies such an approach by considering on the quantity and quality of experimental data required.

2. Methodology

2.1. *Numerical Simulation*

Undrained biaxial compression simulations were selected as the focus of this study. In Abaqus, a 0.2×0.2 m square sample was modeled as a quarter space domain, leveraging symmetric boundary conditions. The external top boundary was restrained from horizontal displacements and subject to 50 kPa vertical pressure, the external horizontal boundary was unconstrained and subject to 100 kPa horizontal pressure, and the internal boundaries restrained the respective normal displacements. The domain was discretized into a 10×10 (i.e. 0.01×0.01 m) element mesh of eight node quadratic plane strain quadrilateral elements (CPE8RP in the Abaqus Standard Library). The sample was subjected to 0.01 m vertical compression in displacement-controlled loading. To prevent mesh dependent strain localization, a non-local regularization technique was implemented, as detailed in Singh et al. (2021) following Chow et al. (2011).

2.2. *Constitutive Model*

The Structured Modified Cam Clay (SMCC) model (Mašín, 2009; Singh et al., 2023) was adopted in this paper as it facilitates modeling of strain-softening behavior in a simple manner. A sensitivity state parameter, s^{ep}, which acts as a scalar multiplier on the yield surface, enables strength degradation induced by plastic strain in proportion to a strain softening rate parameter, k. Thus, s_i^{ep} (the initial sensitivity) and k, which are difficult to determine via conventional element tests, were the target parameters for calibration. In the numerical simulation, the constitutive model parameters were given values typical of a soft, normally consolidated marine clay, with $s_i^{ep} = 6.8$ and $k = 0.8$. A full constitutive model description may be found in Smith (2020).

2.3. *Calibration*

Full-field deformation measurements were analyzed by evaluating the work increments following the VFM (Grediac, 1989). External work increments applied at the boundary, ΔW_E, were computed through the product of the forces and displacements. The internal work increments dissipated by the sample, ΔW_I, were evaluated through the measured strain field and inferred stress field defined by the constitutive model. Therefore, the internal work increments were inherently dependent upon the material parameters, allowing manipulation of the target strain-softening parameters in order to minimize the objective function:

$$\delta_W = \sqrt{\sum_{n=0}^{T} (\Delta W_{I,n} - \Delta W_{E,n})^2} \tag{1}$$

where δ_W is the residual difference, a function of the different in work increments across all time steps, n, where T is the final time step.

2.4. *Bayesian Methods*

To investigate the effect of measurement noise and uncertainty, it was assumed that the associated error could be represented by spatio-temporally independent additive Gaussian noise, $\epsilon \sim \mathcal{N}(0, v)$ (where v is the variance), applied to the internal work increments directly. The target strain softening material parameters were characterized as uncertainty distributions through Bayesian inference. Specifically, Markov

Chain Monte Carlo (MCMC) sampling within the Metropolis-Hastings algorithm (Hastings, 1970) was used to generate kernel density estimates (KDEs) focused on high-likelihood parameter regions. In this paper, a uniform prior over the region $1 < s_i^{ep} < 10$ and $0 < k < 1$ was applied with a diagonal covariance matrix in which the variances of s_i^{ep} and k were $v_s = 0.5$ and $v_k = 0.1$, respectively.

3. Results

The results for three levels of noise variance (corresponding to the errors of 0.01%, 0.1% and 1.0% in the increments of internal work) and two compression levels are shown in Figure 1 and Table 1. A distinctive high-likelihood region is evident in Figure 1(a), of the form $y = 1/x^n$ (where $n \geq 1$) and centered on $(k = 0.81,\ s_i^{ep} = 7.05)$. Along this ridge, which correctly encompasses the true parameter set, the parameter sets produce a compelling fit between the internal and external work increments. The shape of the high-likelihood region suggests significantly greater model uncertainty in s_i^{ep} than in k which could be associated to the full extent of strain softening remaining unobserved.

As anticipated, decreasing the extent of compression, and thereby the degree of strain-softening observed, and increasing the additive noise acted to diminish the distinctiveness of the high-likelihood ridge, degrading towards the prior (i.e. a uniform plateau across the parameter space). Relative to the distribution of Figure 1(a), Figure 1(f) observed substantial increases in spread ($\Delta\sigma_k = 50.7\%$ and $\Delta\sigma_{s_i^{ep}} = 29.5\%$). However, the relative similarity between Figure 1(a) and Figure 1(b) suggests some resilience to noise within the input data, which in this case was from numerical simulations, but would be from Particle Image Velocimetry/Direct Image Correlation (PIV/DIC) experiments in reality.

4. Conclusion

This paper confirms the conclusions of Singh et al. (2023), that an image-based geotechnical parameter calibration technique is worth pursuing in physical testing,

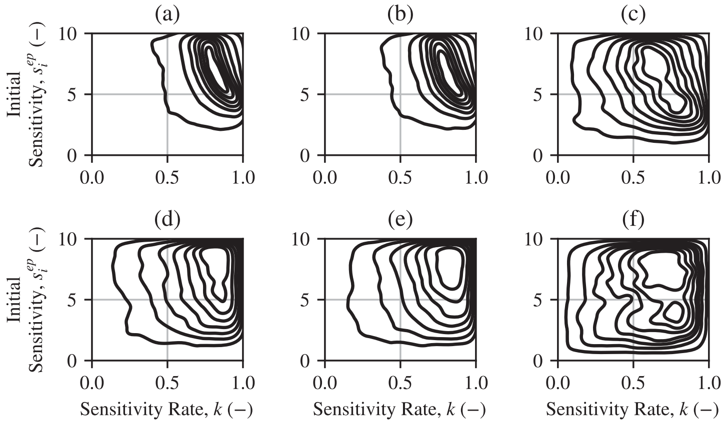

Fig. 1. Kernel density estimates (KDEs) demonstrating the effect of increasing the magnitude of additive noise (left to right) and decreasing the proportion of strain softening observed (top to bottom).

Table 1. Posterior likelihood distribution metrics associated with Figure 1.

Reference	Vertical Compression, δ_v (m)	Additive Noise, v $(-)$	Mean, $(\mu_k,\ \mu_{s_i^{ep}})$	Standard Deviations, $(\sigma_k,\ \sigma_{s_i^{ep}})$
(a)	0.10	1×10^{-7}	(0.76, 6.54)	(0.17, 2.03)
(b)	0.10	1×10^{-6}	(0.75, 6.50)	(0.18, 2.00)
(c)	0.10	1×10^{-5}	(0.59, 5.71)	(0.24, 2.43)
(d)	0.05	1×10^{-7}	(0.65, 6.00)	(0.24, 2.40)
(e)	0.05	1×10^{-6}	(0.65, 6.04)	(0.24, 2.40)
(f)	0.05	1×10^{-5}	(0.54, 5.32)	(0.26, 2.63)

whilst highlighting the potentially restrictive data quality requirements upon imaging. The apparent threshold in the degradation of calibration quality with additive noise emphasized the dependence of the method upon PIV/DIC performance limitations not considered here. Furthermore, a "proof-of-concept" physical test should maximize the proportion of strain-softening observed, as evidenced in the reduction of calibration quality when withheld. With respect to risk, this paper has demonstrated the seamless integration of Bayesian methods within the proposed calibration technique. As well as providing for better informed design and modeling, the characterization of uncertainty provided a metric for further refining the calibration approach.

References

1. Avril, S., Pierron, F., Pannier, Y., and Rotinat, R. (2008). Stress reconstruction and constitutive parameter identification in plane-stress elasto-plastic problems using surface measurements of deformation fields. *Experimental Mechanics*, **48**:403–419.
2. Charles, J., Smith, C., and Black, J. (2018). Identification of soil stress-strain response from full field displacement in plane strain model tests. In *9th International Conference on Physical Modelling in Geotechnics*, London, UK.
3. Grédiac, M. (1989), Principe des travaux virtuels et identification/principle of virtual work and identification, *Comptes Rendus de l'Académie des Sciences*, **302**:1–5.
4. Hastings, W. K. (1970). Monte carlo sampling methods using markov chains and their applications. *Biometrika*, **57**:97–109.
5. Mašín, D. (2009). Comparison of predictive capabilities of selected elasto-plastic and hypoplastic models for structured clays. *Soils and Foundations*, **49**:381–390.
6. Singh, V., Stanier, S., Bienen, B., and Randolph, M. F. (2021). Modelling the behaviour of sensitive clays experiencing large deformations using non-local regularisation techniques.*Computers and Geotechnics*. **133**.
7. Singh, V. Stanier, S. Bienen, B. and Randolph, M.F. (2023) Calibration of strain-softening constitutive model parameters from full-field deformation measurements. *Canadian Geotechnical Journal*. **60(6)**:817-833.
8. Smith, J.D.K. (2020) Large Deformation Testing for Strain-Softening Parameter Calibration. MEng Thesis, University of Cambridge.
9. Wood, D. M. (2012). Heterogeneity and soil element testing. *Géotechnique Letters*, **2**:101–106.