

BAYESIAN MODEL CALIBRATION OF A TRIAXIAL SPECIMEN USING SURFACE DISPLACEMENT AND REACTION FORCE DATA

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Precise measurement of soil deformation across a wide range of strains is vital for geotechnical engineering applications. Traditional triaxial tests are limited in their ability to accurately capture small strain behaviour, end effects, and local non-uniformities, presenting a significant challenge. Digital Image Correlation (DIC) offers a non-contact and cost-effective solution for detailed surface displacement measurements, but the challenge lies in converting this data into actionable soil parameters. This study addresses this challenge by employing Bayesian model calibration in a synthetic case study of a triaxial specimen modelled using a Finite Element Model (FEM). A sequential Bayesian calibration approach is shown to be able to quantify uncertainties and refines parameter estimates dynamically, enhancing the prediction of soil behaviour under changing stress conditions. The results show significant reductions in parameter uncertainty and convergence towards ground truth values. This methodology represents a transformative step in soil mechanics, utilising advanced statistical techniques to improve the reliability and accuracy of soil behaviour models.

1. Introduction

Measuring soil deformation characteristics over a wide range of strains is essential for soil characterisation in most geotechnical applications. Despite its widespread usage, the conventional triaxial apparatus is not capable of reliably measuring small strains. Incorporation of local transducers within the triaxial apparatus has allowed for displacements with greater precision, however their implementation remains challenging (Nishimura 2022). Current research includes the utilisation of digital image correlation (DIC) capable of delivering cost-effective, non-contact strain measurements of triaxial specimens undergoing deformations across a range of strains. However, while DIC has the ability to provide detailed deformation data, the challenge remains in using this data to infer important soil characteristics.

In response to these challenges, this paper introduces sequential Bayesian calibration method as an approach for soil parameter estimation, integrating prior knowledge with observed surface deformation data to provide a probabilistic framework for identifying constitutive soil parameters. This Bayesian approach enables dynamic parameter updating as new information becomes available. Crucially, Bayesian model calibration quantifies the uncertainty associated with parameter estimates. This capability to assess confidence in parameter estimates is invaluable in geotechnical engineering, where it supports more informed decision-making under conditions of uncertainty (Wagner et al. 2024).

2. Implementation and results

A case study is presenting here on the use of Bayesian model calibration on a Finite Element Model (FEM) of a triaxial specimen modelled using ABAQUS (Smith 2009) assuming axisymmetry. A coupled hydro-mechanical FE simulation of a drained

triaxial compression test on an overconsolidated clay was carried out with a modified Cam-Clay model (Roscoe & Burland 1968). Unlike previous numerical optimisation techniques used for model calibration, this study emphasises the evaluation of uncertainties within the model parameters and their consequent effects on system responses dynamically, ultimately deriving a posterior probability distribution that accurately quantifies the probabilities associated with each parameter at all stages of testing. Bayesian model calibration was employed to refine the Cam-Clay model parameters based on the data obtained from experimental design. Four model parameters were calibrated as part of this study: the logarithmic hardening modulus, λ ; the logarithmic bulk modulus, κ ; the critical state ratio M and the preconsolidation pressure, σ'_{pc} .

For the experimental design, five hundred inputs were sampled from a 4 dimensional multivariate distribution. For the prior, each parameter is assumed to be defined by a uniform distribution with no correlation. Samples were generated using Latin-hypercube sampling and evaluated by Abaqus FEA. Based on the accuracy of a trained surrogate model, it was concluded that 500 samples was sufficiently large to confidently assess the posterior distributions of the different parameters. Data was collected along the surface of the model encompassing reaction forces and deformations. These metrics were chosen to reflect observations and measurements typical of real-life triaxial tests, ensuring the model's applicability and relevance in laboratory settings.

2.1. Observed data

2.1.1. Displacement

In Figure 1(a), shows the transformation of the surface displacement as generated by the FEM software to the vectorised representation used in the Bayesian calibration framework. Surface displacements for the different simulated models can be represented as a vector due to the assumption of axisymmetry. Total displacement was separated at each stage into radial and axial deformation. Figure 1(b) shows the typical profiles of displacement during loading across a range of strain levels.

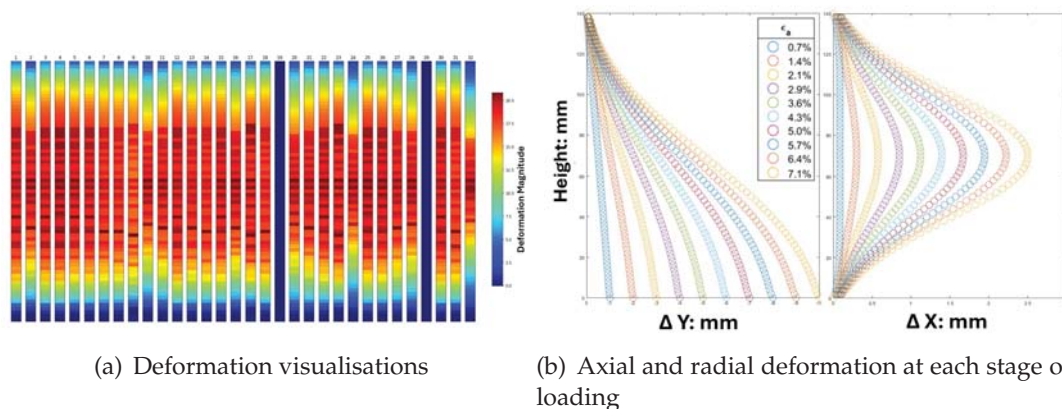


Fig. 1.: Comparison of deformation visualizations and material deformation stages

2.1.2. Deviatoric Stress

The deviatoric stress, measured during a triaxial test, is calculated as the axial load applied to the specimen divided by its cross-sectional area. Measuring deviatoric

stress allows for the determination of soil properties such as cohesion c' and the angle of internal friction, ϕ' . Together with deformation measurements and assuming a simple elasto-plastic relationship, it is possible to interpret the soil's stress-strain relationship in terms of stiffness moduli, Poisson's ratio, and the overconsolidation ratio.

2.2. Sensitivity Analysis

Sensitivity analysis is a useful tool in engineering that identifies the parameters or sets of parameters that have the most significant impact on the model output. This analysis provides a useful insight into which model inputs contribute most to the variability of the model outputs (Saltelli et al. 2000). Sobol indices (Sobol 1989) evaluates the contribution of each input parameter and their interactions to the total variance of the model output, providing a comprehensive understanding of the influence of individual inputs on model behaviour. Figure 2 illustrates the normalised Sobol' indices for three outputs collected from the experimental design. The influence of each parameter can be identified by the proportion of first-order indices. The change in relative importance of the four parameters through the analysis demonstrates the evolution of the sample from the overconsolidated to normally consolidated state during shearing.

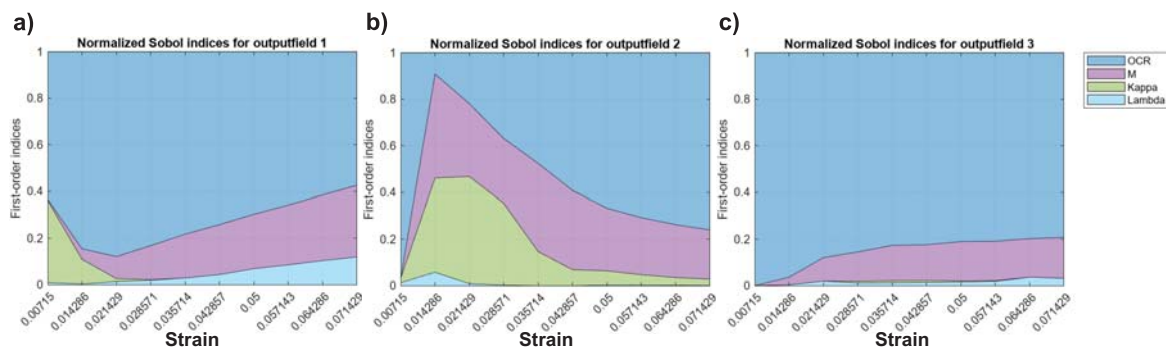


Fig. 2.: Sobol' indices for a) Reaction forces b) Axial displacement c) Radial displacement

2.3. Parameter Calibration

The reliability of the calibrated parameters is dependent on the data collected during the experimental design. Figure 3 details the posterior distributions of the four key parameters after inversion across the 10 different stages of loading modelled in Abaqus. Each subplot shows the evolution of the parameter estimates along with their associated uncertainties, with the shaded regions representing the different confidence intervals (σ , 2σ and 3σ), providing a visual of the variability at each stage. The red line represents the MAP estimate, and its evolution through the different stages which eventually tends towards the ground truth. This convergence indicates a strong agreement between the model predictions and the observed data, highlighting the model's confidence as more data is incorporated into the model. The progressive narrowing of the confidence intervals emphasises the reduction in uncertainty of the parameter space.

In order to assess the accuracy of the MAP estimates in Figure 3, the mean absolute percentage error (MAPE) is calculated at each stage. The MAPE provides a quantitative

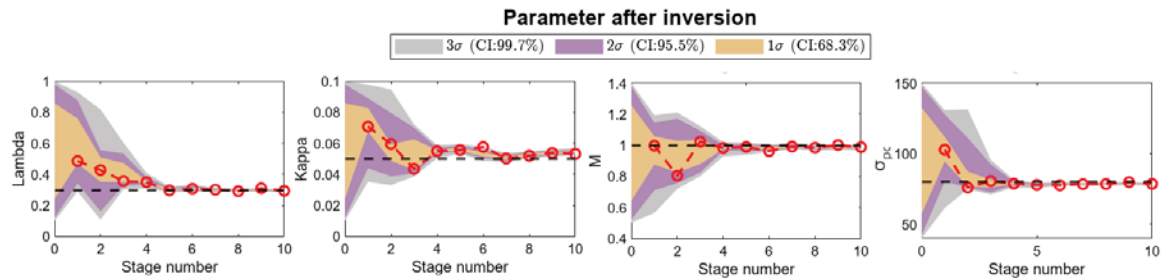


Fig. 3.: Parameter convergence for four Cam-Clay parameters

measure of the differences between the values predicted by the model and the actual observed values, thereby indicating the precision of the parameter estimates. A general trend of decreasing error can be observed with increasing stages of loading. This indicates an overall improvement in parameter estimation as more data is incorporated into the model, validating the accuracy. All four Cam-Clay parameters achieved convergence to errors below 10% by the end of 10 stages used for calibration. This indicates that, overall, the model was able to accurately capture the parameter values within acceptable error margins, ensuring reliable calibration outcomes.

3. Conclusions

The findings from this research hold significant potential for advancements within geotechnical engineering, particularly regarding triaxial tests and the understanding of soil responses and their non-uniformities. The results presented here demonstrated a significant reduction in uncertainty for the model parameters and MAP estimates showed convergence to ground truth values, evidencing the efficacy of the model. This research has highlighted the potential of Bayesian model calibration in improving the precision and reliability of soil behaviour predictions.

Building on these advancements, future research aims to explore the application of Bayesian model calibration to more complex geotechnical scenarios, such as fissured samples and their mechanical behaviour, influenced by the presence of anisotropies and discontinuities in the soil (Cui et al. 2020). This approach will not only improve the understanding of mechanical properties of fissured soils, but contribute to the development of more reliable models for engineering applications involving challenging soils.

References

- Cui, W., Potts, D. M., Pedro, A. M. G. & Zdravković, L. (2020), 'Numerical assessment of the effects of end-restraints and a pre-existing fissure on the interpretation of triaxial tests on stiff clays', *Géotechnique* **71**(9), 765–780.
- Nishimura, S. (2022), 'Characterisation of soil deformation over wide strain ranges in triaxial test with high-precision stereophotogrammetry', *Géotechnique* **73**(12), 1071–1086.
- Roscoe, K. H. & Burland, J. B. (1968), 'On the generalized stress-strain behaviour of wet clay'.
- Saltelli, A., Chan, K. & Scott, E. M. (2000), 'Wiley series in probability and statistics', *Sensitivity analysis*.
- Smith, M. (2009), *ABAQUS/Standard User's Manual, Version 6.9*, Dassault Systèmes Simulia Corp, United States.
- Sobol, I. (1989), 'Computational methods and algorithms: On sensitivity estimation for nonlinear mathematical models'.
- Wagner, P. R., Nagel, J., Marelli, B. & Sudret, B. (2024), 'Uqlab - bayesian inversion user manual'.