Proc. of the 9th International Symposium on Geotechnical Safety and Risk (ISGSR)
Edited by Zhongqiang Liu, Jian Dai and Kate Robinson
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doi: 10.3850/GRF-25280825_isgsr-047-P165-cd

PROBABILISTIC ASSESSMENT OF A CIRCULAR TUNNEL IN THE NON-ROTATED AND ROTATED ANISOTROPIC RANDOM FIELDS

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The spatial variability of soil properties is common in geotechnical engineering projects due to the complex geological and deposition processes.Spatial variation of soil property with rotational anisotropy is simulated, and its impact on tunnel convergence has been studied. This study employs the random finite difference method (RFDM) to assess the tunnel convergence influenced by both the soil property's transverse (non-rotated) anisotropy and rotated anisotropy. In the present study, Young's modulus of soil is considered as a spatially variable random parameter which is modelled using a rotated random field. Moreover, comprehensive parametric studies were also conducted regarding the effect of coefficient of variation (COV), the effect of different scales of fluctuations and rotation angles of the random fields on the stability of tunnel support. The results highlight the significance of accounting for rotational anisotropy of soil while assessing the mean of tunnel convergence and the probability of exceeding certain thresholds of deformation.

Keywords: Spatial variability; Tunnel convergence; Random finite difference method (RFDM); Rotational anisotropy; Random fields.

1. Introduction

The increasing demand for better underground transportation infrastructure has led to a rise in tunnel construction. Various techniques have been developed to predict tunnel deformation, including analytical and numerical approaches(Verma et al., 2022). Many studies frequently assume that soil properties are homogeneous and isotropic, which may result in inaccurate predictions of tunnel deformation and pose a risk to human safety(Miranda et al., 2022). The random field (RF) approach considers the spatial variability of soil properties resulting from geological processes(Chwała et al., 2023). Previous studies have often neglected rotated anisotropy, which is common in glacial terrains and areas with sedimentary deposits(Zhu & Zhang, 2013).The deformation of embedded geo-structures is mainly influenced by Poisson's ratio and Young's modulus of the soil. Young's modulus, which measures soil stiffness, plays a crucial role in soil deformation under load(Verma et al., 2024). While Poisson's ratio, which describes the ratio of lateral to axial strain, also contributes to deformation analysis, it is considered less variable and critical. Hence, this study introduces a probabilistic framework to model spatial variations in Young's modulus of soil and evaluates its effect on tunnel convergence, considering both non-rotated and rotated anisotropy. It investigates the effects of the coefficient of variation, fluctuation length, and rotation angle on tunnel convergence. Lastly, the results are analyzed, and conclusions are drawn.

2. Numerical model and Random Field methodology

2.1. *Numerical model of tunnel*

The presentstudy numerically models a circular tunnel in variable soil conditions. The tunnel has an outer diameter of 6.2 m and a cover depth of 3.2 times the diameter. The soil domain is 9.4 times the depth and 13.2 times the tunnel width to avoid boundary effects (Fig 1). The ground surface boundary condition is free, the bottom boundary is fixed, and only vertical displacement is allowed at the side boundaries. The study assumes plane strain conditions and uses the Mohr-Coulomb yield criterion to describe soil behaviour. Soil parameters include unit weight (^{3} = 18 kN/m³), cohesion (c = 25 kPa), friction angle ($\tilde{O}$ = 15°), Poisson's ratio ($\AA$ = 0.33), and Young's modulus (E = 20 MPa). The study evaluates long-term tunnel lining performance under drained soil conditions. The tunnel lining is modelled as an elastic continuous ring with reduced rigidity (reduction coefficient of 0.7). The interaction between the tunnel lining and the surrounding soil is not considered. The lining thickness is 0.35 m, and the longitudinal ring width is 1.2 m. The lining material properties include unit weight (25 kN/m³), elastic modulus

(34.5 GPa), and Poisson's ratio (0.2). The tunnel convergence in the deterministic case is 41.5 mm vertically (Dy), which is comparable to the findings of Wu et al. (2021)where Dy = 46.8 mm.

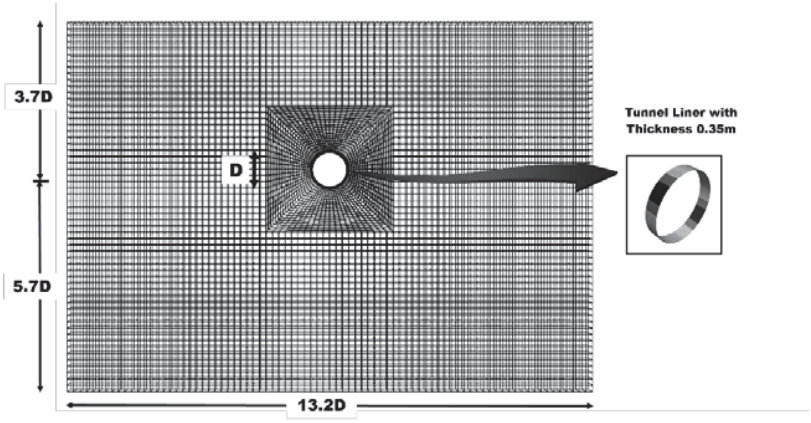

Fig. 1.Computational domain of finite difference numerical model of the circular tunnel.

2.2. *Random field theory*

A random field is a set of indexed random variables, characterized by mean, variance, and autocorrelation function(Phoon & Kulhawy, 1999). In this study, the Markovian autocorrelation function has been employed. Eq. (1) presents the Markovian autocorrelation function $\rho(\Delta_x', \Delta_y')$ for a rotation angle (^{2}), which is calculated based on the spanning distances Δ_x' andΔ_y' between centroid coordinates (x_i', y_i') and (x_j', y_j') of two elements. θ_1 and θ_2 are representing the major and minor fluctuation lengths. In the present study θ_1 and θ_2 are set to 20 m and 5 m, respectively, while the rotation angle (^{2}) is considered as ^{2}=0º (non-rotated RF), and ^{2}=30º, ^{2}=60º, and ^{2}=90º (rotated RF).

$$\rho(\Delta_x', \Delta_y') = exp\left[-2\left(\frac{|\Delta_x \cos\beta + \Delta_y \sin\beta|}{\theta_1} + \frac{|-\Delta_x \sin\beta + \Delta_y \cos\beta|}{\theta_2}\right)\right] \quad (1)$$

The FLAC3D numerical code can handle complex geotechnical problems like tunnel instability. Random fields are generated using Python and incorporated into a tunnel model to analyse stochastic finite-difference analysis. Steps include identifying uncertain soil parameters, constructing a deterministic tunnel model, calculating autocorrelation matrices, generating random variables, mapping fields onto the finite difference mess, and performing Monte Carlo simulations for statistical analysis with 300 simulations.

3. Result and discussion

Young's modulus of soil is considered as the random parameter. The mean value of Young's modulus is the same as the deterministic value (μ_E = 20 MPa). However, three different cases are considered for variance level COV_E =5%, COV_E =10%, and COV_E =15%. Fig. 2 presents the realizations of different combinations of rotation angle and COV_E of young's modulus of soil. It is clear from the random field realizations that as the coefficient of variation (COV) of the Young's modulus increases, variation in Young's modulus RF also increases. Running the numerical tunnel model with Monte Carlo samples of these random fields yielded a range of vertical displacement (Dy) values. The obtained Dy values at different rotation angles are presented as boxplots for COV_E =5%, COV_E =10%, and COV_E =15% in Fig. 3(a), 3(b), and 3(c), respectively.

Fig 4 illustrates the statistical measures (maximum, minimum, and mean) for vertical tunnel convergence (Dy). The mean values show minimal fluctuation as the COV_E increases from 5% to 15%. However, the minimum and maximum statistics display significant changes with an increase in COV_E. Specifically, as the COV_E value rises, the maximum value increases while the minimum value decreases.

3.1. *Effect of fluctuation length*

A parametric study was carried out to evaluate how the different fluctuation lengths impact the response of a numerical model of tunnel convergence at a particular rotation angle (^{2}=60º) and constant COV_E (=10%). The engineering parameters exhibit more significantspatial variability vertically than horizontally, this parametric study of rotational random fields focused on changing the minor fluctuation length only while maintaining the major fluctuation length constant. The major and minor fluctuation lengths for different anisotropic cases in the study are as follows:

(i) ANIS 20-5: Major fluctuation length 20 m, Minor fluctuation length 5 m.
(ii) ANIS 20-10: Major fluctuation length 20 m, Minor fluctuation length 10 m.
(iii) ANIS 20-15: Major fluctuation length 20 m, Minor fluctuation length 15 m.
(iv) ANIS 20-20: Major fluctuation length 20 m, Minor fluctuation length 20 m.

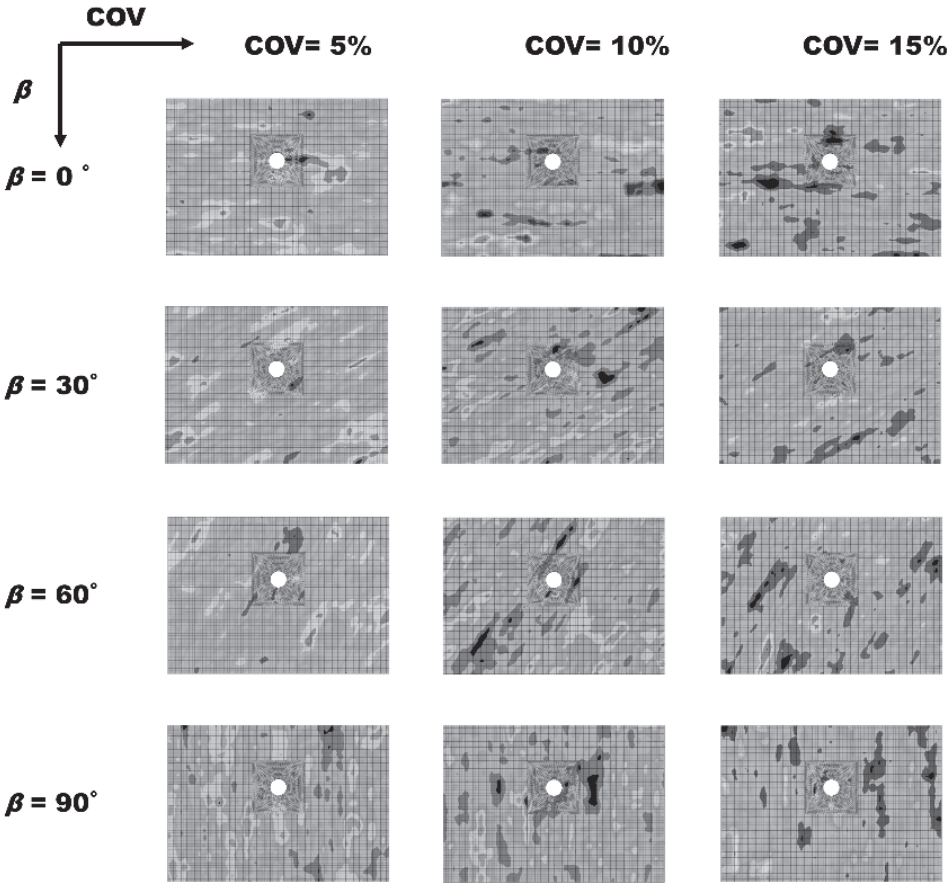

Fig. 2.RF realization of Young's modulus of soil for different rotation angles (²=0º, 30º, 60º, and 90º) and coefficient of variation (COV_E =5%, 10%, and 15%).

Table 1 shows the tunnel model responses under different anisotropic conditions (ANIS 20-5, ANIS 20-10, ANIS 20-15, ANIS 20-20) at various rotation angles in terms of probability of exceedance (PE). Probability of exceedance refers to the likelihood of a value surpassing a specified threshold (In the present study, deterministic tunnel convergence is considered as threshold value). It can be expressed mathematically as:

$$PE = P(X > x_{deterministic\ \ value}) = \frac{N_{runs\ \ exceeding\ \ deterministic\ \ value}}{N_{total\ \ runs}} \quad (2)$$

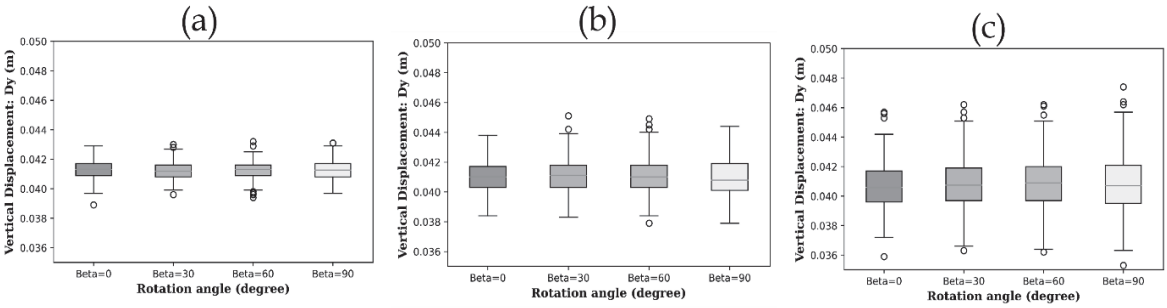

Fig. 3. Boxplots of displacements at differentCOV_E of Young's modulus: (a) COV_E=5%, (b) COV_E=5%, (c) COV_E=15%.

The data in Table 1 displays that as the minor fluctuation length increases at a specific rotation angle, the probability of exceeding vertical displacement (Dy) also increases.

Table 1. Effect of correlation length on exceedance probability.

Rotation angle	Anisotropic case	Exceedance probability (%)
60°	ANIS 20-5	38.00
	ANIS 20-10	40.67
	ANIS 20-15	42.00
	ANIS 20-20	41.33

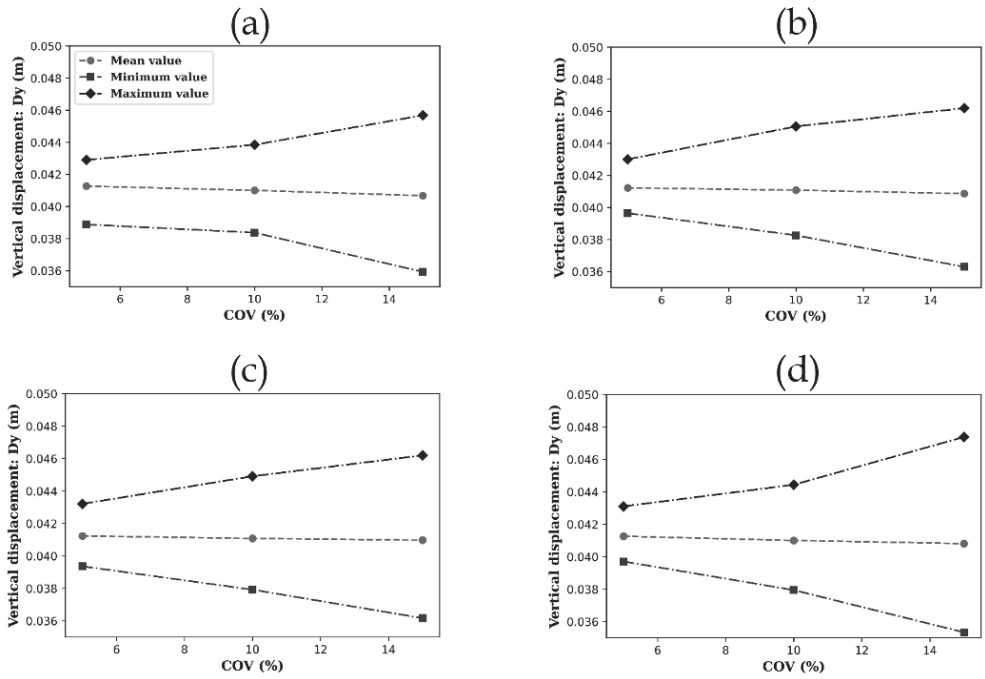

Fig. 4.Comparison of line graphs for different statistics of tunnel responseatCOV_E of 5%, 10%, and 15% of Young's modulus for rotation angles (^{2}): (a) ^{2} =0°, (b) ^{2} =30°, (c) ^{2} =60°, (d) ^{2} =90°.

4. Conclusion

Various parametric studies were conducted to examine the effects of coefficient of variation (COV), rotation angles, and fluctuation length of spatial variability on tunnel convergence. Key findings include:

- The rotation angle of random fields has a significant impact on probabilistic analysis of tunnel convergence.
- As the COV_E rises, the variance in vertical tunnel displacement also increases significantly.
- The vertical tunnel displacement shows an increase in the probability of exceedance (PE) as the fluctuation length increases.

Acknowledgement

Ajeet Kumar Verma acknowledges AcSIR, India and RMIT University, Australia for providing an opportunity to carry out this doctoral research. Ajeet Kumar Verma also acknowledges CSIR India for the financial support provided as a fellowship under Direct-SRF Scheme [File No: 31/0024(19417)/2024-EMR-I].

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