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UNCERTAINTY IN THE NATURAL FREQUENCY OF WIND TURBINES SUPPORTED ON MONOPILES IN SPATIALLY-VARIABLE CLAYS

Ahmad Kahiel

College of Engineering and Technology, American University of the Middle East, Kuwait. E-mail: ahmad.kahiel@aum.edu.kw

Salah Sadek

Department of Civil and Environmental Engineering,
American University of Beirut–Mediterraneo (AUB Mediterraneo), Pafos, Cyprus;
Department of Civil and Environmental Engineering, American University of Beirut,
P.O. Box 11-0236, Riad ElSolh, Beirut 1107-2020, Lebanon. E-mail: salah@aubmed.ac.cy

Shadi Najjar

Department of Civil and Environmental Engineering, American University of Beirut, P.O. Box 11-0236,
Riad ElSolh, Beirut 1107-2020, Lebanon. ORCID: https://orcid.org/0000-0003-1824-4540.
E-mail: sn06@aub.edu.lb

The contribution of wind energy in the global energy mix has reached 1TW milestone in the past year with half of this power achieved in that year. It is planned to triple this capacity by 2030 (Global Wind Report). The major loads experienced by wind turbines are cyclic in nature (wind, wave, and operational loads); therefore, the dynamic analysis of these structures could be a governing design step. In this paper, the uncertainty in the natural frequency of a wind turbine supported on monopiles in clay is quantified using a random finite element model that considers soil variability. 1-Dimensional idealization of the monopile, tower and nacelle with p-y and M-¸ springs representing the surrounding clayey soil is adopted. The recently developed PISA model (Byrne et al. 2020) that is calibrated specifically for monopiles was used to develop the soil reaction curves. The effects of overconsolidation, correlation length, undrained shear strength, soil modulus and amplitude of vibration were investigated. Results indicated that the uncertainty in the natural frequency of the turbine was found to be low even for high soil variability because of variance reduction due to averaging along the monopile depth to fixity and relative soil to pile stiffness.

Keywords: Wind turbines; natural frequency; soil variability; random finite element, resonance.

1. Introduction

Dynamic loads experienced by wind turbines include wind, wave (offshore), and operational loads (mass imbalance “1P” and blade passing “3P”). Each of these loads may operate at a different frequency range. The 1P range ($f_{P1,min} \rightarrow f_{P1,max}$) results from the operational speed of the blades while the 3P range ($f_{P3,min} \rightarrow f_{P3,max}$) is a multiple of the 1P range by the number of blades (typically 3).Structural causes stand for 9% of failure cases in wind turbines (Ma et al. 2019). Some of the structural failures are attributed to fatigue. To avoid unfavourable dynamic amplification and eventually fatigue failure, the wind turbine-soil system must have the first-mode natural frequency (termed natural frequency later in the paper) not overlapping with any excitation frequency (Kühn 1997). The soft-stiff design approach places the natural frequency in the gap between the 1P and 3P frequency ranges. Therefore, accurate estimation of the natural frequency with the associated uncertainty is a critical design step.

Several approaches exist to estimate the natural frequency of wind turbines. 3D nonlinear dynamic finite element analysis is the most accurate method which comes at the cost of extensive computational effort. Pseudo 3D finite element modelling of the soil-monopile system allows the calibration of four springs (vertical, lateral, rotational, and cross coupling) that represent the soil effects (Bouzid et al 2018). This approach resulted in comparable estimates of measured natural frequency for 6 wind turbines.Several analytical models were developed to estimate the natural frequency of wind turbines supported on monopiles (Zania 2014). One analytical model modifies the fixed base natural frequency of the tower to account for the flexibility in the supporting soil (Arany et al 2016, Varghese et al 2022). A comparison of the methods used to estimate the natural frequency and have low computational cost shows that the stiffness matrix at the mudline approach is the best for monopiles (Zaaijer et al. 2006).

The PISA P- y and M- θ family of curves modelled the soil-structure interaction using a conic function (Byrne et al., 2020). Since these curves are specifically developed for monopiles, limitations that are related to slenderness are avoided (API and DNV). ISO/API will issue an updated $P- y$ framework for offshore piles (Jeanjean et al. 2022). Despite that the PISA model is new and not widely tested in the industry, its credibility relies on 3- dimensional finite element simulations that were calibrated by field scale tests.

Probabilistic approaches are increasingly used for the ultimate and serviceability limit state design of wind turbines (Lin et al 2023 and Sujawat and Kumar 2023). A probabilistic estimation of the natural frequency showed that its variability is low for overconsolidated clays even if the coefficient of variation for the clay reaches 40% (Andersen et al. 2012). The authors represented the pile-soil stiffness with two uncoupled springs at the ground level using the finite difference method with the DNV $P- y$ curves.The objective of this paper is to quantify the uncertainty in the prediction of the natural frequency of wind turbines supported on monopiles in normally and overconsolidated clays (Thomas 1989). The study covers various combinations of clay stiffness and strength, vertical correlation lengths, and lateral load magnitudes through a 1-Dimensional random finite element model developed in R software.

2. Random Finite Element Formulation

The natural frequency of the wind turbine-soil system is determined by solving the Eigen equation:

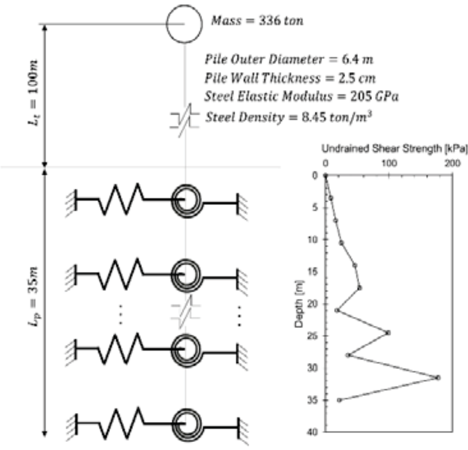
$$|\mathbf{K} - w^2\mathbf{M}| = 0 \quad (1)$$

where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness and mass matrices for the wind turbine-monopile-soil model.

2.1. Finite Element Model

Timoshenko beam elements represent the basic construction block of the tower and pile despite using Euler - Bernoulli model wouldn't result with significant difference (Arany et al. 2015). The stiffness matrix of the pile/tower and the consistent mass matrix that represents the mass distribution were developed following the procedure described in Cook et al. (2002).

The stiffnesses of the translational and rotational springs representing the soil reaction along the pile depth were included as the secant stiffness extracted from the $P- y$ and $M- \theta$curves. The displacements and rotations, at which the secant stiffnessesareobtained, are selectedby applying a static lateral load at the nacelle level. The deformed shape under this load is similar to the eigen vector of the first vibration mode. Newton-Raphson's method is utilized to converge to a solution when the tower is loaded statically to get the lateral displacements and rotations at each node along the monopile and tower. The resulting displacements and rotations at each spring level are fed into the $P- y$ and $M- \theta$ relationships to extract the secant stiffness values. The monopile is discretized into 100 elements to guarantee that the element length is smaller than the vertical correlation length. Coarser discretization for the tower (5 elements) is adopted. Figure 1shows a sketch of the finite element model. A realization of a normally consolidatedclay undrained shear strength profile is shown also. The studied parameters are displayed within Figure1. The overconsolidation is represented by assuming the clay was consolidated under a laterally infinite deterministic surcharge stress at the surface(σ'_p).



Correlation length θ_v [m]	Mean s_u/σ'_v	Surcharge stress σ'_p [kPa]	Lateral load [kN]
1, 35	0.15, 0.3	0, 10, 20, 30 ,40	10, 100, 1000

Figure 1. Schematic illustration of the finite element model of the monopile and the supporting clay. A clay undrained shear strength profile sampleand the varied parameters are also shown.

2.2. Modelling Soil Variability

The clay undrained shear strength and its low-strain shear modulus are the required parameters for the development of the $P-y$ and $M-\theta$ curves. $(s_u/\sigma_v^{'})$ random profiles are generated following a lognormal distribution. 500 realizations were performed to quantify the uncertainty in the natural frequency. The Markovian spatial correlation structure defines the correlation matrix:$\boldsymbol{C} = \exp(-2 \cdot \mathbf{d}/\theta_v)$ where $\mathbf{d}$ is matrix that consists of the separating distances between nodes in the discretized random field. The low-strain shear modulus is defined as$G_0 = 1100 s_u$.

3. Results and Discussion

Theresulting mean natural frequency was normalized by the fixed-base natural frequency which is equal to:

$$f_{FB} = \frac{1}{2\pi}\sqrt{3E_sI_t/(M_{RNA}+\frac{33}{140}\rho_sA_tL_t^4)} = 0.489Hz \tag{3}$$

where $E_s, I_t, M_{RNA}, \rho_s, A_t, L_t$are the steel elastic modulus, second moment of area, mass of the rotor-nacelle assembly, density of steel, cross-sectional area of the tower, and length of tower. The variation of the normalized mean natural frequency and the coefficient of variation under various surcharge stresses, vertical correlation lengths and load amplitudes is presented inFigure 2 (left 4x4 cascade). The surcharge stress is normalized by the vertical effective stress at one pile diameter depth. Themean natural frequency varies between 43% and 58% of the fixed based natural frequency. Despite the highly variable clay soil profile (COV=50%), the coefficients of variation of the natural frequency varied between 0.5% and 8%. This comparably low variability is related to the relative stiffness of the pile and soil in addition to the averaging effect of the soil properties along the pile length.

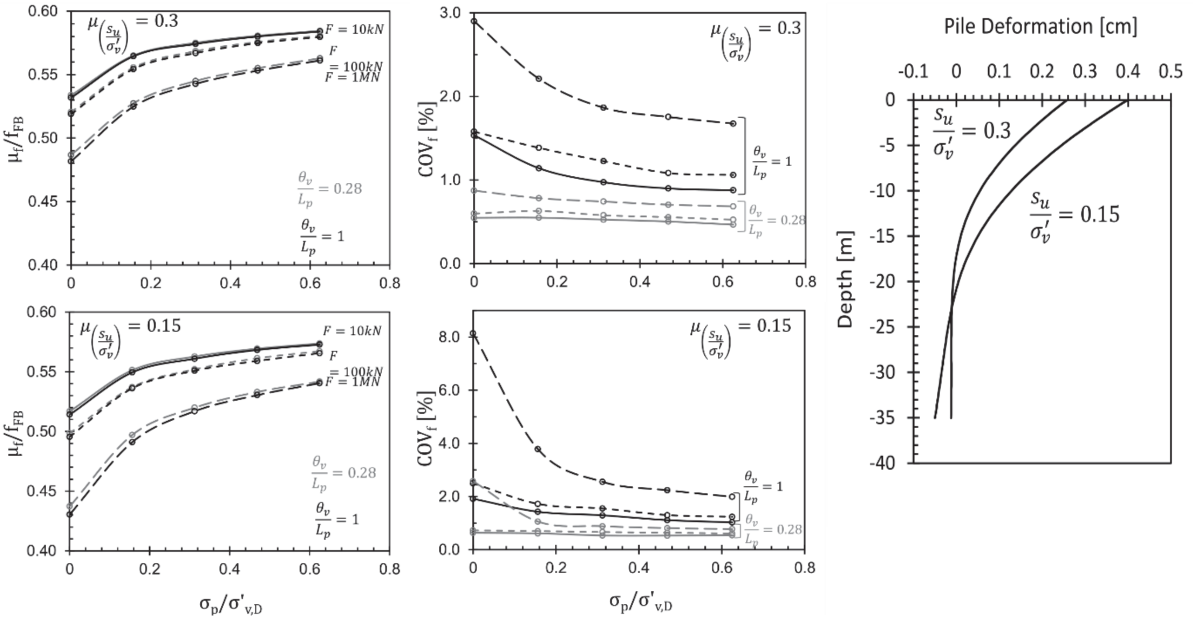


Figure 2. The variation of the normalized mean natural frequency and the coefficient of variation as function of preconsolidation stress, mean soil strength/stiffness, vertical correlation length and load amplitude. Right plot: lateral pile deformation versus depth for two soil profiles with different strength/stiffness due to a lateral force at the top of the tower (100kN)

The natural frequency increases as the undrained shear strength/shear modulus increases. This increasing trend also prevails as the surcharge load on the soil increases. The vertical correlation length does not have any impact on the mean natural frequency.Moreover, the mean natural frequency decreases as the load magnitude increases.This is because as the lateral deformation increases, the secant stiffness of the soil decreases. Thistrend was validated by field data measurements from Jiangsu offshore wind farm where wide-shallow composite bucket foundation was used (Fan et al. 2023).

The coefficient of variation of the natural frequency COV$_f$increases as the vertical correlation length increasesdue to the variance reduction as a result of the averaging effect.As the soil becomes stiffer or when the lateral load is small, the depth to fixity is shallower; therefore, smaller body of the soil contributes to uncertainty in the natural frequency. This explains why COV_f increases as (1) the strength/stiffness of the soil decreases, (2) the surcharge load decreases, and/or (3) the lateral force increases. The right side of Figure 2 shows that for $s_u/\sigma'_v = 0.3$, the depth to fixity was approximately 25m while No fixity depth appears with the strength ratio reaches 0.15.

4. Conclusion

The estimation of the natural frequency of wind turbines is a soil-structure interaction problem. The uncertainty in the natural frequency remains low even when the clay is highly variable. This is a result of (1) the relative contribution of the soil to the global stiffness of the soil-monopile-tower stiffness and (2) spatial averaging that takes place along the monopile depth to fixity instead of the its total length. This paper demonstrates an additional advantage of monopile foundations which is its low sensitivity to soil variability. The results of this paper can be extended to perform a probabilistic assessment for fatigue failure in wind turbines. The effect of the variability in the parameters of the structural components is to be investigated as a future step in the project.

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