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REVISITING OF LONDON CLAY SIMPLE CORRELATIONS USING A BAYESIAN APPROACH

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The London Clay (LC) is an Eocene stiff clay which underlies a large proportion of London with thickness up to 130 meters. As such it has been extensively studied, and there exists many empirical relationships that estimateits stiffness and strength properties based on some simpler parameters (e.g. depth, plasticity index, etc.). Such relationships are routinely used in practice, especially on small construction projects where ground investigation is often too limited for establishing site-specific correlations. However, there is an increasing trend for geotechnical practices, codes and standards to move towards reliability-based methods and wider use of statistical evaluation of the variations in the geotechnical parameters. It requires rigorous methods to quantify both the total uncertainty of the measured property (including inherent soil variability, measurement errors and statistical uncertainties) and the uncertainty of the transformation model.

This paper investigates the derivation of one of these simple correlations, the variation of undrained strength with depth, using a Bayesian statistical approach tobetter understand these uncertainties. The study relies on a large database gathered during the site investigation for the construction of a majorrail infrastructure project running East-West across London. The analyses explore the predictive capacity of different models including hierarchical models to investigate the variability between different LCsub-units. In practice, the obtained correlations may then be combined with site-specific data from an appropriate site investigation using a Bayesian approach.

Keywords: Stiff clay, London Clay, Undrained strength, Unconfined undrained, Bayesian analyses,Correlation; Uncertainties; Soil properties.

1. Introduction

The London Clay (LC) formation is an Eocene stiff clay which underlies a large portion of south-east England, and in particularcentralLondon. As such it has been extensively studied, and there exists many empirical relationships correlating its stiffness and strength properties with some simpler parameters. This paper focuses on carrying Bayesian analyses of one of such basic relationships, namely the variation in undrained strength (S_u) with depth below the top of the London Clay, using data from across central London (from Royal Oak in the west to Pudding Mill Lane and Plumstead in the east).

All analyses presented in this paper were carried out using the open-source program Python (Version 3.12.2) and in particular the following libraries: PyMC (Version v5.12, Abril-Pla et al. 2023) and Arviz (Version 0.17.1).

2. Datasetof Undrained Shear Strength

Crossrail, now known as the Elizabeth line, was a £14.8 billion project to construct a suburban railway serving London and its surrounding regions. Part of the route included 21 km of twin-bore tunnels under central London, along with new underground stations and shafts.This analysis focuses on the results of Unconfined Undrained (UU) triaxial compression tests carried out on U100 samples as part of the Crossrail site investigation. U100 samplers are likely to cause intense shearingaround the sides of the sample and thus disturbance to the sample (Vaughan et al., 1993). Therecorded S_u may not be as accurate as ones obtained from less disturbing methods (e.g. rotary coring, blocks sampling, etc.). However, in the UK, values of S_u for design were historically taken from UU tests on U100 samples, and consequently it constitutes a “standard” test against which a range of empirical design methods that are still used in today’s practice have been correlated. Therefore, it can be considered a type of “index” value (S_{u0}) and it is still of interest to investigate its variation.

Table 1. Basic statistics for the dataset of S_{u0}(in kPa) based on UU tests, broken down by London Clay sub-units.

LC unit	Count	Mean	Std dev	Minimum	Q_1 25%	Q_2 50%	Q_3 75%	Maximum
A2	877	230.5	106.2	12	153	222	298	762
A3	669	145.2	75.3	9	95	133	175	592
B	350	183.2	89.5	19	114.5	171.5	231.75	553
C	119	126.4	42.9	43	99.5	120	146	321
Unclassified	142	193.5	109.6	30	120.75	157.5	233.75	712
Weathered	35	90.0	39.9	25	58	86	124	186

Note: Std dev: standard deviation; QX: quartiles number X.

The dataset included a total of 2192 points which were subdivided into the main sub-units of LC as identified by King (1981), from oldest to youngest: A2, A3, B and C. The LC units result from cyclic changes in sea levels, and the material typically coarsens upwards within a sub-unit (Hight et al. 2003). Some points in the database were also either unclassified or marked as weathered material, and both groups were ignored in the following analyses, leaving a total of 2015 points from 288 boreholes.As seen in Table 1and Fig. 1, the older units constitute the larger part of the dataset (the younger units having been eroded in some areas). It appears reasonable to assume that S_{u0} increases linearly with depth in the top 30 or so meters, although showing large scatter (Fig. 1a). Consequently, difference in strength between the sub-units can be partly explained by the difference in depth range at which they are encountered: for example units A2 and B show higher strengths on average but are also encountered over a larger depth range than the other units(see Fig. 1b). It is also noteworthy that the height above the base of the London Clay is a good marker of where a sample is situated within the LC lithological sequence.

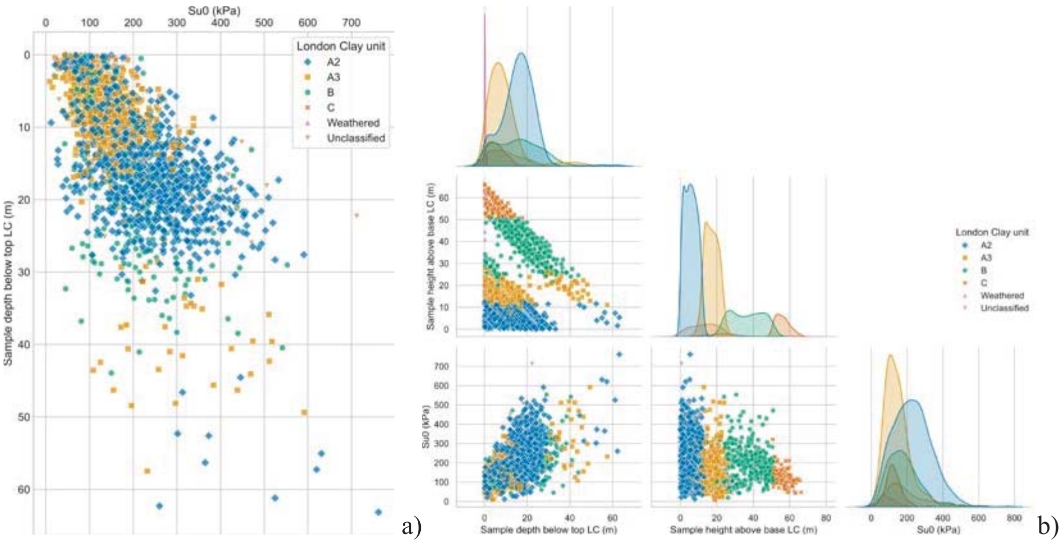

Fig. 1. Undrained strength variation with depth below top of LC and with height above base of LC.

3. Bayesiananalyses

To compare with existing correlations and how they are used in design, this analysis focuses on simple models of S_{u0} linearly increasing with depth below the top of the LC, as described in the sub-sections below.All modelswere fitted on the whole dataset using the No-U-Turn Sampler, a self-tuning variant of Hamiltonian Monte Carlo (Abril-Pla et al. 2023). All models presented here showed adequate convergence based on common checks e.g. effective sample size> 1000, Potential Scale Reduction Factor of 1 (Martin et al. 2021).

3.1.Model A -Complete poolingwith heteroscedastic variance

The first model (A) considered is a complete pooling linear model as presented in Eq.(1) to(3)below. A classical normal linear model was modified to account for the observed increase in variance with depth (see Fig. 1). For simplicity, the observations error Ã was also assumed to vary linearly with depth below the top of the LC, see Eq.(3). Additionally, models based on the normal distribution are easily influenced by outliers, and as such called “non-robust” (Gelman et al. 2013). This problem may be remediated by switching the likelihood to a distribution that can accommodate thicker tails (i.e. that allows for higher probability of extreme events). Consequently, the likelihood was set to the Student's t-distribution which is commonly use as replacement for a Normal distribution for such purpose (Gelman et al. 2013, Martin et al. 2021). As given in Eq.(1) below, the distribution takes an additional positive parameter ν which controls the amount of probability mass in the tails (i.e. how thick the tails are): as ν increases towards infinity, the Student’s t distribution converges to a normal distribution.

$$S_{u0} \sim \mathrm{StudentT}(\mu, \sigma, \nu) \quad (1)$$

$$\mu = \beta_0 + \beta_1 * z_{btLC} \quad (2)$$

$$\sigma = \delta_0 + \delta_1 * z_{btLC} \quad (3)$$

Where z_{btLC} is the depth below the top of the London Clay. Weakly informative priors were selected for the 5 parameters: for the ² parameters, the means were selected based on existing relationships(Crilly, 2024, Burland and Kalra, 1986) with relatively large standard deviations (see Table 2).Due to the large dataset, the choice of priors was found to have limited impact on the obtained posteriors.

Fig. 2a presents an overview of the posterior predictive distributions for the mean μ and the observations S_{u0}. The model indicates confidence in the predicted variation in mean μ, which at shallower depth (<30 m) also compares relatively well with existing correlations from the literature. The latter tends to lie below the predicted mean, but this is to be expected as cautious estimates are usually taken as input for design. The previously noted large scatter in the data results in large observation errors. Such predictive distributions may be used to quantify uncertainties of derived parameters (e.g. pile capacity), so it is important that the model capture accurately the posterior predictive distribution for the observations $S_{u0}$and not just for the mean. Fig. 2a shows that the posterior predictive distribution for S_{u0} match the dispersion of the data relatively well up to a depth of about 30 m, while it seems to slightly overestimate at deeper depth. The latter is likely a limit of a simple linear model that does not capture the likely curvature of the strength envelope as in-situ stress increases and any effect of locally common faulting offsetting sub-unit boundaries.

Table 2. Prior distributions selected for the various models.

Parameter	Weakly informative Prior	Parameter	Weakly informative Prior
² 0	Normal(75,10^2)	Ã 0	HalfNormal(5^2)
² 1	Normal(6, 10^2)	Ã 1	HalfNormal(5^2)
´ 0	HalfNormal(10^2)	R	LKJ(2)
´ 1	HalfNormal(10^2)	ν	HalfNormal(10^2)

LKJ: Lewandowski, Kurowicka, and Joe distribution (see Gelman et al. 2013)

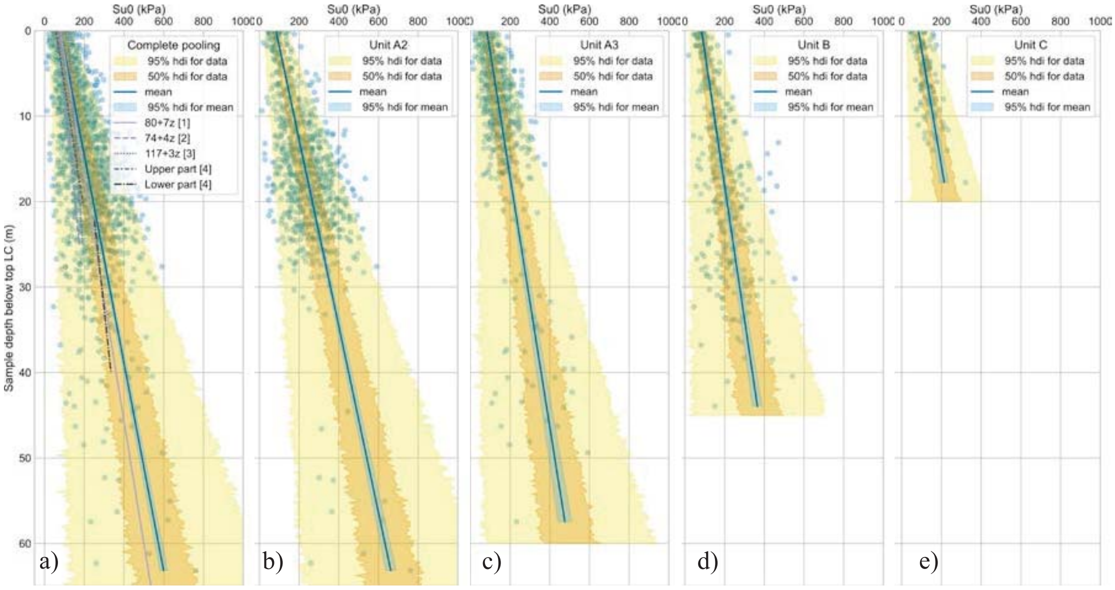

Fig. 2. Predicted variation of the mean with depthand uncertainties in the mean for the 2 models, alongside with the observation errors and existing relationships from the literature. HDI: Highest Density Interval, [1] Crilly, 2024, [2] Burland and Kalra, 1986 – Operational, [3] Burland and Kalra, 1986 – Mean, [4] Hight et al. 2003.

Table 3. Basic information for posterior distributions of Model B parameters

Param.	Mean	Std dev	HDI 3%	HDI 97%	Param.	Mean	Std dev	HDI 3%	HDI 97%
² 0	83.1	2.4	78.5	87.7	² 0[A2]	83.3	2.6	78.2	88.1
² 1	7.4	1.2	5.1	9.6	² 1[A2]	9.2	0.2	8.7	9.6
Á	0.0	0.4	-0.8	0.8	² 0[A3]	82.5	2.3	78.2	86.7
Ã 0	2.5	2.1	0.0	6.2	² 1[A3]	6.8	0.3	6.2	7.5
Ã 1	2.1	1.2	0.6	4.4	² 0[B]	83.7	2.6	79.0	89.0
´ 0	26.4	1.4	23.7	29.0	² 1[B]	6.4	0.3	5.8	7.0
´ 1	2.8	0.1	2.5	3.0	² 0[C]	84.0	3.0	78.9	90.3
ν	10.9	2.5	7.0	15.8	² 1[C]	7.3	0.8	5.9	8.8

Note: Param.: Parameter name, Std dev: Standard deviation, HDI: Highest density Interval

3.2.Model B – Hierarchical model

The above model was extended into a hierarchical model (partial pooling) to investigate if part of the observed variation may be explained by the differences between LC sub-units, as detailed in Eq.(4) to (6) below, keeping the same distribution for generating $S_{u0}$and a common Ã - see Eq.(1) and (3). The model makes provision for possible correlations between the intercept and slope (² 0 and ² 1) for each sub-unit.

$$\mu = \beta_{0,[unit]} + \beta_{1,[unit]} * z_{btLC} \quad (4)$$

$$\beta_{[unit]} \sim \text{MVNormal}(\beta, \Sigma) \sim \beta + (C \cdot Z)^T \quad (5)$$

$$\Sigma = \begin{bmatrix} \sigma_{\beta 0} & 0 \\ 0 & \sigma_{\beta 1} \end{bmatrix} R \begin{bmatrix} \sigma_{\beta 0} & 0 \\ 0 & \sigma_{\beta 1} \end{bmatrix} \text{ where } R = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \quad (6)$$

Where MVNormal is a 2-component multivariate Normal distribution with means $^{2} = [^{2}{}_{0}, ^{2}{}_{1}]$ and covariance matrix £. To ensure effective sampling, the non-centred parametrization of the model was used as presented in the second part of Eq.(5) where C is the Cholesky covariance matrix decomposition and Z is a 2-dimension Standard Normal random vector. Weakly informative priors were again selected (see Table 2). Table 3 summarises the posterior distributions of the 16model parameters. Overall, the intercept values are relatively constant across LC sub-units while the predicted slopes vary(from smallest to highest: B, A3, C and A2), showing that some of the variation observed are indeed linked to the difference in the sub-units. However, the model still appears to over-estimate the strength at depth deeper than 30 m, especially for unit A2.The lack of intercept variation also results in no correlation being predicted between $^{2}{}_{0}$ and $^{2}{}_{1}$, possibly because the strength near the top of LC is controlled by fissures.

4. Model comparison

This analyse aims at providingpriors that can be incorporated with site-specific data to estimate the uncertainty in S_{u0}. Consequently,it is important to evaluate model predictionon unseen data. The out-of-sample predictive accuracy was estimated usingthe expected log pointwise predictive density (ELPD) scoring rule. Estimation of out-of-sample accuracy would typically involves re-fitting the models many times through the Cross-Validation (CV) method. In this analyse, the models were compared using the Pareto smoothed importance sampling leave-one-out CV method which allows to make an estimate of the out-of-sample accuracy with only one fitting of the model (Vehtari et al., 2017). As seen on Fig. 3, Model B performsslightlybetter in terms ofout-of-sample accuracy, meaning that despite being a more complicated model, it does not seem to be over-fitting the data.

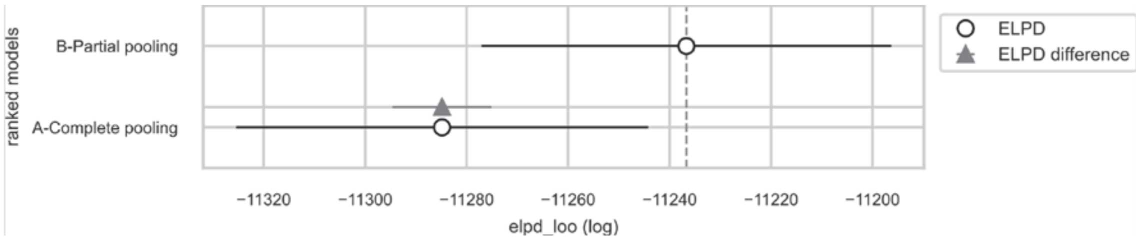

Fig. 3. Comparison of ELPD values obtained using Pareto smoothed importance sampling leave-one-out cross validation (higher ELPD is better), horizontal lines show standard errorin absolute values (black) and in the difference between the 2 values (grey).

5. Conclusion and further considerations

This paper investigated Bayesian modelling of a simple linear relationship between the undrained strength S_{u0} from UU tests and depth below the top of the London Clay (LC), through complete and partial pooling. The obtained mean linesfrom the models are generally in agreement with existing relationships from the literature. However, the Bayesian models provide not only additional information on the uncertainties in the regression parameters, but on the uncertainties in the predicted S_{u0}. It is recommended to use models that accurately represent the increasing uncertainties in $S_{u0}$with depth and that are robust against outliers. The study indicates that part of the observed uncertainties in S_{u0} may be explained by the difference betweensub-units. The proposed modelsmay then be used as a prior for future projects in the London area and combined with site-specific data from an appropriate site investigation using a Bayesian approach to estimate the uncertainties in S_{u0}. While simple models were considered here to compare with existing relationships, further studiesare underway to consider the variation in $S_{u0}$with LC within units using the height above the base of LC, and the horizontal spatial variation.

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