

## RELIABILITY-BASED OPTIMIZATION FOR THE DESIGN OF GREENLANDFILL COVER SYSTEMS CONSIDERING STRESS-DEPENDENT HYDRAULIC PROPERTIES

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**Abstract:** Landfill cover systems are crucial for preventing rainwater infiltration into waste. Although the effectiveness of three-layer landfill cover systems using recycled concrete aggregates (RCAs) under extreme weather conditions has been validated, there is limited research on optimizing their design. Furthermore, vegetation is a vital component of landfill covers, enhancing aesthetics and eco-friendliness while improving system performance. However, factors such as spatial variability in cover material properties and root characteristics are generally ignored. The impact of thickness-induced stress on hydraulic properties is also typically neglected, even though stress effects can significantly affect hydraulic properties. Therefore, this study aims to provide reliability-based optimization design recommendations for vegetated three-layer landfill covers with RCAs by considering these factors simultaneously. Stress effects are captured using a stress-dependent soil-water retention curve (SDSWRC), with hysteresis considered. Uncertainties in soil void ratio ( $e_b$ ) and root volume ratio ( $R_v$ ) within the root zone are characterized by copula-based cross-correlated random fields, while uncertainties in the bare zone are depicted by  $e_b$  random fields. The spatial variability of the SDSWRC and the water permeability function are further characterized based on empirical equations. Subsequently, the effects of planting strategy on the failure probability of the landfill cover system are analyzed. Correspondingly, reliability-based optimization recommendations for vegetated landfill covers, focusing on vegetation planting strategy, are provided.

**Keywords:** Landfill cover, stress-dependent hydraulic properties, vegetation, copula theory, random finite element method, optimization design

### 1. Introduction

A three-layer landfill cover system has been proven to effectively withstand intense rainfall in humid climates (Ng et al. 2024a; Min et al. 2024). This system consists of a fine-grained soil layer overlaying a coarse-grained layer, which in turn overlies a low-permeability soil layer. According to unsaturated soil mechanics, water can be stored and diverted in the fine-grained soil layer during light rainfall. Once the capillary barrier formed by the upper two layers is breached, the low-permeability layer prevents further percolation. Nevertheless, previous studies have primarily focused on analyzing performance under different climate conditions through experimental tests, with limited attention to optimization design, particularly considering the effects of thickness-induced stress in the design. Recently, Ng et al. (2024b) conducted a reliability-based design for non-vegetated three-layer landfill covers that incorporated stress effects, emphasizing that neglecting stress effects can cause unsafe designs. Therefore, stress effects should be considered in optimization design.

To achieve ecological restoration of landfills, plants have become an important component. Vegetation affects landfill cover system performance by reducing water content through transpiration, which decreases soil permeability or infiltration rate (Ni et al. 2017; Ng et al. 2024a). Moreover, soil water permeability can be further reduced by roots occupying the void ratio (Ni et al. 2019). Correspondingly, the soil water retention curve (SWRC) can also be affected due to the existence of roots (Ni et al. 2019). However, Ni et al. (2017) reported that the saturated water permeability ( $k_s$ ) of vegetated soil can increase due to decayed roots. Ng et al. (2024a) also found the increased  $k_s$  resulting from root decay through long-term field monitoring. Therefore, the effects of decayed roots on the behavior of landfill covers should be considered. Similar to soils, plants also exhibit

inherent spatial variability (Ni et al. 2017; Ng et al. 2024a). Ignoring uncertainties in plant characteristics may lead to unreasonable designs for vegetated landfill covers. Additionally, current design criteria for landfill cover systems lack specific recommendations for planting schemes. Thus, there is a need to optimize the design of vegetated three-layer landfill cover systems by incorporating uncertainties in plant properties.

To promote sustainability, recycled concrete aggregates (RCAs) have been increasingly used as cover materials in landfills because they possess hydraulic properties similar to those of natural aggregates (Ng et al., 2024a). However, due to the non-uniform particle size during RCA preparation, uncertainties in RCA properties should be considered when conducting optimization design of vegetated covers constructed with RCAs.

This study aims to carry out a reliability-based optimization design for the vegetated three-layer cover with RCAs by accounting for stress effects and spatial variability of cover materials and plant properties. The influence of root architecture, depth, and plant spacing on the failure probability of the cover is investigated. Based on the findings, the reliability-based optimization design recommendations for vegetation planting strategy are provided.

## 2. Transient seepage analysis under rainfall infiltration

Analyzing transient seepage in unsaturated vegetated soils involves accounting for changes in suction caused by transpiration, which differs from the approach utilized in unsaturated bare soils. Here, a modified Richard's equation that incorporates a sink term,  $S(y')$ , is employed (Ng et al. 2024a):

$$\frac{\partial}{\partial x} \left( k \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial h}{\partial y} \right) = \frac{\partial \theta}{\partial t} + S(y') H(y' - L'_1) \quad (1)$$

in which  $k$  implies unsaturated water permeability;  $h$  denotes the total head;  $\theta$  means volumetric water content;  $t$  is elapsed time;  $y'$  represents the perpendicular coordinate to the landfill cover system; and  $H(y' - L'_1)$  is given by:

$$H(y' - L'_1) = \begin{cases} 1 & L'_1 \leq y' \leq L'_1 + L'_2 \\ 0 & 0 \leq y' \leq L'_1 \end{cases} \quad (2)$$

in which  $L'_1$  and  $L'_2$  mean the external root zone thickness and the root zone thickness. The  $S(y')$  is calculated as:

$$S(y') = T_p G(y') \alpha(\psi) \quad (3)$$

where  $T_p$  indicates transpiration rate;  $\psi$  implies matrix suction;  $\alpha(\psi)$  represents transpiration reduction function proposed by Feddes et al. (1976);  $G(y')$  stands for the root architecture function. To solve Eq. (1), it is essential to incorporate SWRC and water permeability function (WPF). However, these properties differ in vegetated soils compared to bare soils due to modifications in soil hydraulic properties induced by plant roots, like  $k_s$  and SWRC. To capture the effects of plants on soil hydraulic properties, a simplified void ratio model for vegetated soils that simultaneously accounts for both root growth and decay, proposed by Ni et al. (2019) is used:

$$e_v = \frac{e_b - (1 - \eta) R_v (1 + e_b)}{1 + (1 - \eta) R_v (1 + e_b)} \quad (4)$$

where  $e_b$  and  $e_v$  mean the void ratio of bare soil and vegetated soil;  $R_v$  implies root volume ratio ( $\text{mm}^3/\text{mm}^3$ );  $\eta$  denotes root decay ratio (%). Both are dimensionless parameters. Subsequently, a void ratio-dependent SWRC model by Zhou and Ng (2014), which captures stress effects, is utilized to characterize the impact of plant-induced changes in the void ratio on the stress-dependent water retention curve (SDSWRC):

$$S(\psi) = \left[ 1 + \left( \frac{(e_v - \alpha_p \ln(1 + p/p_r) - \alpha_s \ln(1 + \psi/p_{\text{atm}}))^{m_4}}{m_3 (1 + p/p_{\text{atm}})^{m_5}} \psi \right)^{m_2} \right]^{-m_1} \quad (5)$$

where  $\alpha_p$  and  $\alpha_s$  are soil compressibility with respect to stress and matric suction, respectively;  $p$ ,  $p_{\text{atm}}$ ,  $p_r$  imply stress, atmospheric pressure, and reference pressure (1 kPa);  $m_1$ ,  $m_2$ ,  $m_3$ ,  $m_4$ ,  $m_5$  indicate soil parameters. The WPF,  $k(\psi)$ , is shown as (van Genuchten, 1980):

$$k(\psi) = k_{s,0} S^{0.5} \left[ 1 - (1 - S^{1/m_1})^{m_1} \right]^2 \quad (6)$$

in which  $m_1$  corresponds to the soil parameter specified in Eq. (5). It is also important to note that plant roots can significantly affect  $k_s$ . To remain consistent with the SDSWRC of vegetated soils, the variation in soil void ratio caused by roots is utilized to reflect the associated change in  $k_s$ . As a result, the empirical equation that defines the correlation between  $e$  and  $k_s$  is adopted (Kozeny, 1927):

$$k_s = k_{s,0} \left( \frac{e}{e_0} \right)^3 \left( \frac{1 + e_0}{1 + e} \right) \quad (7)$$

where  $k_{s,0}$  and  $e_0$  indicate the initial  $k_s$  and void ratio of vegetated soil. Eq. (7) is also suitable for bare soils.

### 3. Effects of planting schemes on the failure probability of landfill covers

In the vegetated three-layer landfill cover system, the materials from top to bottom are fine recycled concrete aggregate (FRC), coarse recycled concrete aggregate (CRC), and completely decomposed volcanic rock (CDV). The respective thicknesses are 1.0, 0.2, and 0.8 m, based on the design results from Ng et al. (2024b). The model length is set to 3.0 m to approximate an infinite slope, enhancing the generalizability of the optimization design. A  $3^\circ$  inclination angle, close to the lowest value recommended by GB 51220-2017 (2017), is employed to allow more rainwater infiltration into the cover, thereby representing a more unfavorable scenario. The drying and wetting fitting parameters of SDSWRCs for FRC, CRC, and CDV are summarized in Table 1. For vegetated FRC (VFRC), the SDSWRC fitting parameters are the same as those for FRC, except for the void ratio (Ng et al., 2024a). The  $R_v$  and  $\eta$  for the vegetation are set as  $0.075 \text{ mm}^3/\text{mm}^3$  and 2%.

Table 1. Summary of material parameters in numerical parametric studies.

Parameter	CDV		CRC		FRC		VFRC	
Initial void ratio, $e_0$	1.08		0.59		0.64		0.46	
$\alpha_p$	0.02		0.01		0.018		0.018	
$\alpha_s$	0.005		0.21		0.04		0.04	
$k_s$ (m/s)	$1.1 \times 10^{-7}$		$1.64 \times 10^{-1}$		$1.85 \times 10^{-4}$		$7.73 \times 10^{-5}$	
Drying/Wetting	Drying	Wetting	Drying	Wetting	Drying	Wetting	Drying	Wetting
$m_1$	0.08	0.085	0.80	0.75	0.55	0.40	0.55	0.40
$m_2$	1.45	1.38	2.68	2.52	1.30	1.29	1.30	1.29
$m_3$ (kPa)	33.30	16.38	0.74	0.52	2.50	0.71	2.50	0.71
$m_4$	8.62	8.53	0.47	0.53	1.40	1.94	1.40	1.94
$m_5$	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50

Fig. 1 shows the numerical model and boundary settings. A mesh size of  $0.05 \text{ m} \times 0.025 \text{ m}$  is applied. The numerical procedures consist of four steps: (i) an initial steady state with a  $0.01 \text{ mm/d}$  precipitation intensity; (ii) a 5-day transient drying period with a transpiration rate of  $10 \text{ mm/d}$ ; (iii) a 4-hour transient wetting stage with a rainfall intensity of  $75.5 \text{ mm/h}$ , equivalent to a 100-year return period in Hong Kong; (iv) an additional 5-day transient drying period with a transpiration rate of  $10 \text{ mm/d}$  to further balance water components. In the first stage, the bottom boundary BC is set at  $-0.2 \text{ m}$  total head to account for the underlying wet waste impact. In the second stage, a potential seepage face is established at BC, which remains unchanged for the last two stages. The right boundary CD and left boundary AB are configured as a potential seepage surface and no flow, respectively, for all stages. The top boundary AD is assigned as the flux boundary for the wetting phase, while the root water uptake is simulated during the drying period by utilizing the transpiration rate in the root zone. Notably, when the negative PWP rises above  $0 \text{ kPa}$ , the flux boundary is replaced by a pressure boundary to avoid water ponding on the surface. Moreover, different drying and wetting processes employ the corresponding drying and wetting paths of SDSWRCs and WPFs.

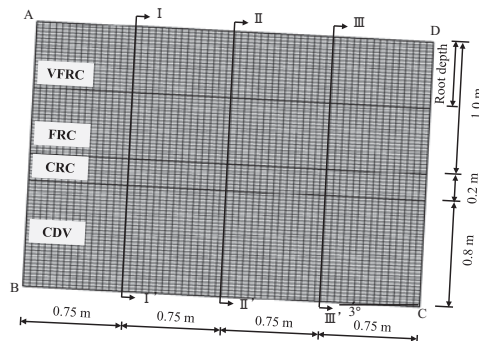


Fig. 1. The geometry and mesh size of the numerical model.

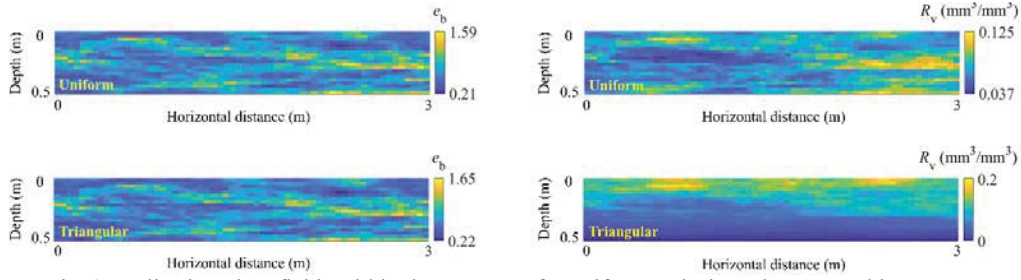


Fig. 2. Realized random fields within the root zone for uniform and triangular root architectures.

Referring to Eqs. (4-7), the spatial variability of  $e_b$  and  $R_v$  can induce spatial variability of SDSWRC and WPF. Thus, the spatial variability of  $e_b$  and  $R_v$  within the root zone is simulated by Clayton copula random fields (Ng et al. 2024b), while the spatial variability of  $e_b$  outside the root zone is modeled by conventional random fields. In the root zone, the vertical scale of fluctuation (SOF) is set to 0.1 m, and the horizontal SOF is set to twice the plant spacing. Outside the root zone, the horizontal and vertical SOFs are set to 1 m and 0.2 m, respectively. The coefficients of variation (COVs) of  $R_v$  and  $e_b$  are assumed to be 0.2 and 0.4. The cross-correlation coefficient between  $R_v$  and  $e_b$  is set to 0.6 based on previous experimental tests. Fig. 2 provides typical random field examples in the root zone for different root architectures, where both  $e_b$  and  $R_v$  are assumed to be lognormally distributed.

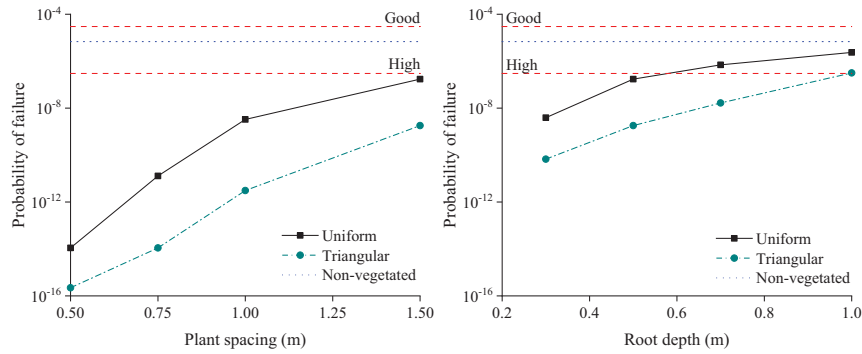


Fig. 3. Effects of planting schemes on the  $p_f$  of the three-layer landfill cover system: (a) plant spacing; (b) root depth.

Fig. 3 demonstrates the impact of planting schemes on the probability of failure ( $p_f$ ) of the three-layer landfill cover system. Here,  $p_f$  refers to the probability that percolation through the bottom exceeds the recommended limit (i.e., 30 mm). Detailed methods for calculating  $p_f$  can be found in Ng et al. (2024b), and the  $p_f$  is determined based on 500 Monte Carlo simulations (MCS) to ensure computational convergence and accuracy. The effects of root depth are evaluated under the same root biomass assumption for a fair comparison (Zhu et al., 2018). It is observed that the  $p_f$  of a vegetated cover system is consistently lower than that of a non-vegetated one, regardless of planting schemes. The triangular root is more effective in reducing  $p_f$ , irrespective of plant spacing and root depth. Additionally, decreasing plant spacing and root depth can significantly reduce  $p_f$ , regardless of root architecture. For example, with the triangular root, when the root depth increases from 0.3 m to 1.0 m, the  $p_f$  rises from  $6.7 \times 10^{-11}$  to  $3.2 \times 10^{-7}$ . Similarly, increasing the plant spacing from 0.5 m to 1.5 m leads to an increase in  $p_f$  by 7 orders of magnitude.

#### 4. Summary and conclusions

This study systematically investigates the influence of planting schemes on the  $p_f$  of a vegetated three-layer landfill cover system with RCAs. Stress-dependent and spatially variable hydraulic properties are considered. It is found that, with the same root biomass assumption, increasing plant spacing or root depth leads to an increased  $p_f$ , regardless of root architecture. The triangular root architecture proves more effective in reducing  $p_f$ , irrespective of plant spacing and root depth. Therefore, to achieve a high-performance level while minimizing costs, it is recommended to use plants with a 1.5 m spacing, a 0.3 m root depth, and a triangular root architecture. Notably, the influence of root depth on the performance of landfill covers is related to the root biomass assumption, which further affects design results. Therefore, future research should investigate the effects of different root biomass assumptions on design results.

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