Proc. of the 9th International Symposium on Geotechnical Safety and Risk (ISGSR)
Edited by Zhongqiang Liu, Jian Dai and Kate Robinson
Published by Research Publishing, Singapore.
doi: 10.3850/GRF-25280825_isgsr-083-P090-cd

ISGSR2025

INVESTIGATING THE POTENTIAL OF DATA-DRIVENSEISMIC RESPONSE ANALYSIS:INTEGRATING NEURAL NETWORKS WITH DYNAMIC MODE DECOMPOSITION

Kotaro Asano

Department of Civil and Environmental Engineering, Tohoku University, Japan. E-mail: asano.kotaro.r6@dc.tohoku.ac.jp

Yu Otake

Department of Civil and Environmental Engineering, Tohoku University, Japan. E-mail: yu.otake.b6@tohoku.ac.jp

Akihiro Shioi

Disaster Prevention Solution Department, KOZO KEIKAKU ENGINERRING Inc.,Japan. E-mail: akihiro.shioi.eng@gmail.com

Hiroki Kamada

Institute of Technology, SHIMIZU CORPORATION, Japan. E-mail: Hiroki.kamada@shimz.co.jp

Stephen Wu

Research Organization of Information and Systems, The Institute of Statistical Mathematics, Japan. E-mail: stewu@ism.ac.jp

This study introduces a new approach to seismic response analysis that integrates Dynamic Mode Decomposition (DMD) and Neural Networks (NN) takes advantage of the strengths of each method. NNautonomously capturing complex nonlinear phenomena, but their lack of interpretability limits their use in geotechnical engineering, where transparency and reliability are critical. DMD, on the other hand, provides interpretability through the use of linear operators, but has limited adaptability when dealing with nonlinear dynamics. By combining these methods, we aim to create an interpretable data-driven model that balances adaptability and reliability. The proposed hybrid approach effectively decomposes the seismic response characteristics into a linear term (captured by the DMD) and an input-dependent nonlinear term (captured by the NN). We will examine whether the model learns the unrepresentable relationships of the DMD in one-dimensional seismic response, and whether DMD-based analysis is feasible. This integrated framework will not only advance the use of NN in engineering applications, but also establish the basis for reliable and interpretable ground response analysis. By compensating for the limitations of each method, the hybrid model allows for efficient seismic response modelling and more accurate inverse analysis of subsurface properties based on surface-based observations. The findings of this study elucidate the fundamental mechanisms underlying the high performance of NNs and highlight the essential considerations for developing data-driven models that can be applied effectively and confidently in an engineering context.

Keywords: Neural Networks,Dynamic Mode Decomposition; Machine Learning, Seismic Response Analysis.

1. Introduction

Seismic response analysis involves unsteady input waves traveling through heterogeneous ground to the surface, and simulation requires prior investigation of ground parameters and expensive numerical calculations. Computational cost can also be an important issue when dealing with uncertainties, such as in the MonteCarlo Simulation, which involves many iterations of such simulations.With significant recent advances, the ability of Machine Learning (ML) to automatically learn systems from real data is becoming applicable to these complex problems, requiring a large cost to train the model initially, but once trained, it can be used as a fast surrogate model, yielding long-term benefits.

While ML can be a powerful tool in geotechnical engineering, the black box nature of the models created and the limited amount of data that can be collected during field investigations are major challenges when applying ML to risk assessment in geotechnical engineering.On the other hand, the existing data-driven approach called Dynamic Mode Decomposition (DMD) learns the characteristics of real data while the model represents amplified or damped oscillations, allowing analysis of the learned model and learning with fewer data. However, the generalization performance is not high, and the benefit for increasing the number of data is small.Building on this background, this study proposes an integrated model combining Neural Networks (NNs) and DMDs to develop alternative models for simulating ground seismic response.We focus on the effect of training with a small number of data and the analyticity of the trained model.In this study, a one-mass point nonlinear elastic response model is used to verify feasibility.

2. Method

2.1. *DMD*

DMD is a method for determining an operator matrix $\boldsymbol{A}$ such that a snapshot$\boldsymbol{x_k}$ at$k$th time step gives $\boldsymbol{x_{k+1}}$ at the next step, based on stationary spatio-temporal data.

$$\boldsymbol{x_{k+1}} = \boldsymbol{A}\boldsymbol{x_k}\#(1)$$

The operator matrix $\boldsymbol{A}$ allows the period and amplification/decay characteristics of each decomposed mode to be analyzed by finding its eigenvalues and eigenvectors.Eq. (1) represents the relationship in discrete time, but if theeigenvalues$\lambda_1, \cdots, \lambda_n$ of $\boldsymbol{A}$are converted to $\omega_k = \ln(\lambda_k)/\Delta t$ in the continuous time system, the complex eigenvalue ω in the continuous time system expresses the characteristics of amplification or attenuation by its real part and the frequency of vibration by its imaginary part. (Steven Brunton et al., 2022)

$$\boldsymbol{X}(t) = \sum_{k=1}^{r} \boldsymbol{\phi}_k \exp(\omega_k t) b_k = \boldsymbol{\Phi} \exp(\boldsymbol{\Omega} t)\, \boldsymbol{b}\#(2)$$

In addition to the steady-state quantity $\boldsymbol{x}$, DMD with control can consider the control quantity u, which represents the action of non-stationary external forces, etc., can handle a wider range of problems.

2.2. *NNs*

There are various classes of Neural Networks with different methods of coupling between layers, but here we deal with the most basic Fully Connected Neural Networks.The input to the NNs is output through alternating computations as in the following equation

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{W_L} \cdot \boldsymbol{\sigma} \odot \boldsymbol{W_{L-1}} \cdot \ldots \odot \boldsymbol{W_2} \cdot \boldsymbol{\sigma} \odot \boldsymbol{W_1}\boldsymbol{x}, \#(3)$$

where W is the weight matrix (including the bias vector) and σ is the activation function. $\sigma \odot z$ denotes the operation of applying a scalar function σ to each element of vector $\boldsymbol{z}$ and returning a vector of the same length.NNs is known to have a problem with significantly deteriorating output for small noise of a kind called Adversarial Example, and Rim et al. (2024) used their proposed low-rank expansion

$$f(\boldsymbol{x}) = [\boldsymbol{A_0} + \boldsymbol{A_\sigma}(\boldsymbol{x})] \cdot \boldsymbol{x}\#(4)$$

of NNs to isolate the noise-responsive part of the NNs activated by the Rectified Linear Unit (ReLU), which has the property of significantly worsening the prediction.

As a result, the matrixA_0 is an input-independent element, obtained simply as the total product of the weight matrices, while$A_\sigma(x)$ is input-dependent and expresses a nonlinear relationship, the rank of $A_\sigma(x)$ must be less than the layers of the NN.

Based on the above developments, when NNs are viewed as a time evolution problem, like DMDs, NNs may also be viewed as generators that generate operator matrices, like DMDs.

2.3. Time-Delay Embedding

Normally, it is desirable to use data with high-dimensional spatial information for training DMDs and NNs, but in many cases, sufficient spatial dimension is not available when actual observation data is used.However, the learning process can be accelerated by increasing the dimensionality of the data using time-delay embedding. The number of modes acquired by DMD is limited by the spatial dimension of the data, so more modes can be acquired by increasing the dimensionality of the data.

Consider a scalar state quantity$\boldsymbol{v} = \{v_1, v_2, \ldots, v_n\}$ at discrete time $\boldsymbol{t} = \{t_1, t_2, \ldots, t_n\}$, and use time-delay embedding to construct the following matrix $\boldsymbol{H}$ to lift the state quantity to d dimensions The time-delay embedding constructs the following matrix $\boldsymbol{H}$ and lifts the state quantities to d dimensions.

$$\boldsymbol{H} = \begin{bmatrix} v_1 & v_2 & \cdots & v_{n-d+1} \\ v_2 & v_3 & \cdots & v_{n-d+2} \\ \vdots & \vdots & \ddots & \vdots \\ v_d & v_{d+1} & \cdots & v_n \end{bmatrix}\#(5)$$

Each column of the time-delay matrix $\boldsymbol{H}$ can be thought of as a snapshot of the d-dimensional time series data at a given time, as well as a time series data cut from the original time series by a length d.

The state and control quantities$\boldsymbol{x}$ and $\boldsymbol{u}$, which are one-dimensional respectively, are time-delayed to a higher dimension and used as inputs to DMDs and NNs.

When training DMDs and NNs, both models are trained to predict $\boldsymbol{x}$ and u in the next step from $\boldsymbol{x}$ and u at a certain time, but when reconstructing and predicting after training, only $\boldsymbol{x}$ in the next step is estimated from $\boldsymbol{x}$ and $\boldsymbol{u}$, since the control quantity $\boldsymbol{u}$is considered to be known.

2.4. *Suggested method*

In this study, we focus on the fact that the operator matrix of DMD and the matrix obtained by low-rank expansion of NNs, which is originally a noise separation method, have a similar structure, and perform an integration to introduce the analyticity by modes that DMD has to NNs.

The integration is performed by using the following loss function in addition to the usual Mean Squared Error (MSE), and inducing A_0 obtained by the low-rank expansion of NNs in 3.2. with the operator A_{dmd} determined by DMD.

$$loss = \alpha/d \cdot \|\boldsymbol{x_{k+1}} - f(\boldsymbol{x_k})\|_F + \beta \cdot \|\boldsymbol{A_0} - \boldsymbol{A_{dmd}}\|_F \#(6)$$

where, α and β are the weight for MSE loss and DMD loss, respectively.

If the system can be fully represented by DMD and the operator $\boldsymbol{A_{dmd}}$ is obtained that strictly satisfies Eq. (1), then by comparison with NN of Eq.(5),$[\boldsymbol{A_0} + \boldsymbol{A_\sigma}(\boldsymbol{x})]$ is expected to match $\boldsymbol{A_{dmd}}$ for NNs trained on the same system, and since $\boldsymbol{A_{dmd}}$ is independent of input, $\boldsymbol{A_\sigma}(\boldsymbol{x})$ is expected to be a zero matrix. $\boldsymbol{A_{dmd}}$ is independent of the input, $\boldsymbol{A_\sigma}(\boldsymbol{x})$ is expected to be a zero matrix.However, real systems contain various non-stationary components, i.e., components that switch depending on the input $\boldsymbol{x}$. The objective of the proposed method is to separate non-stationary relationships that cannot be represented by DMD into $\boldsymbol{A_\sigma}(\boldsymbol{x})$ by learning NNs while inducing $\boldsymbol{A_0}$with $\boldsymbol{A_{dmd}}$, which represents stationarity.

3. Problem Settings

We verify the capability of the proposed model by training the acceleration response by a one-mass point nonlinear elastic response model that obeys the following differential equation, using an artificially generated one-dimensional input wave based on the equation of motion of a one-mass point system as the control quantity.

$$\frac{dx}{dt} = v, \qquad \frac{dv}{dt} = \frac{-(k_1 x + k_2 x^3) - cv + m}{m} \ddot{x}_g \#(7)$$

where m is the mass, c is the damping coefficient, k_1 is the linear stiffness coefficient, k_2 is the nonlinear stiffness coefficient, and $\ddot{x}_g$ is the input acceleration. The stiffness coefficients are assumed to have nonlinear elastic behavior depending on displacement x, as referenced in Zhang et al. (2020).

The input data is lifted from one dimension to 32 dimensions by time-delay embedding for each of $\ddot{x}$and $\ddot{x}_g$. $\ddot{x}$ is considered as a state quantity and $\ddot{x}_g$ as a control quantity, and the time evolution of the state quantity is estimated using known $\ddot{x}_g$. The NNs used for training have five layers, and the dimensions of the intermediate layers are all set to 128. all activation functions are ReLU, and Adaptive Moment Estimation (Adam) is used for optimization.

4. Result

Fig. 1 shows the reconstruction results using various methods. The upper panel shows the conventional DMD, the middle panel shows the proposed integration model, and the lower panel shows the conventional NN. It can be seen that the proposed method correctly evaluates the maximum amplitude for the DMD only and the conventional NN.

Fig. 2 shows the eigenvalues of the DMD operator and the operator $\boldsymbol{A_0}$ by low-rank expansion of the learned NNs, and Fig. 3 shows the time-series variation of the eigenvalues of the operator $[\boldsymbol{A_0} + \boldsymbol{A_\sigma}(\boldsymbol{x})]$generated by the proposed method.

In DMD, the absolute value of the eigenvalues is proportional to the amplitude amplification factor and the arguments are proportional to the frequency, enabling mode-by-mode analysis of complex waveforms.When DMD is considered, the eigenvalues appear around the unit circle, indicating that the waveform is represented by a small number of modes that generally maintain a constant amplitude; when DMD is not considered, very large real eigenvalues appear, making it difficult to examine the waveform in a modal framework.The eigenvalues of the operator derived using the proposed method exhibit periodicity and likely reflect eigenfrequencies caused by changes in the model's stiffness.

The Mean Squared Error of the model is 0.374 for DMD, 0.0148 for the proposed model, and 0.0195 for NNs, indicating that the NNs model can learn NNs with the same accuracy by considering DMD, it can be analyzed by mode.

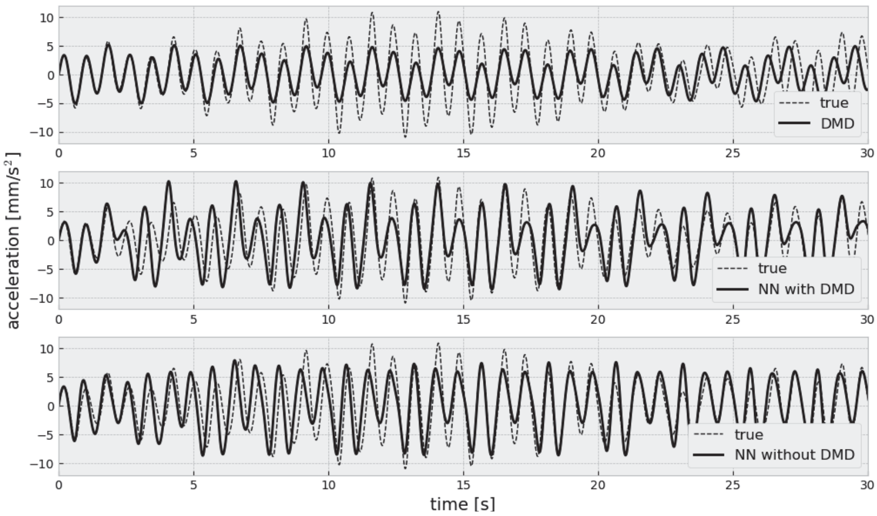

Fig. 1. Reconstruction results by DMD, proposed method and NNs

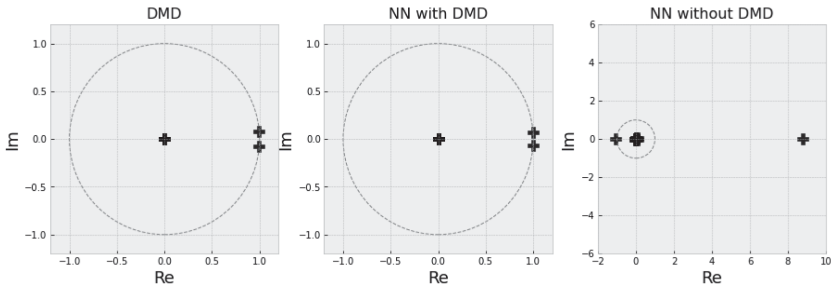

Fig. 2. Eigenvalues of A_{dmd} and A_0 of NNs

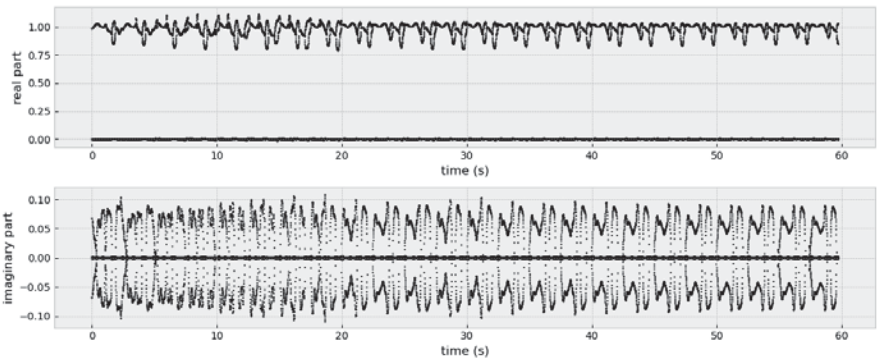

Fig. 3. Time series variation of operator eigenvalues from the proposed method

5. Conclusion

This study presents a new approach to evaluate and analyze learned models by proposing a loss function that incorporates the properties of DMD into NNs. Furthermore, whereas DMDs constitute a single operator matrix for the entire model, in the proposed method, the operators are constantly changing within the range where a low-rank matrix can be influenced by the input. By tracking the time-series changes of the operators, it becomes possible to more effectively analyze the points in time when nonlinear effects are dominant.

References

Brunton, S. and Kutz, N. (2022). *Data-driven science and engineering: Machine Learning, Dynamical Systems, and Control.* Cambridge University Press.

Rim, D., Suri, S., Hong, S., Lee, K. and LeVeque, R. (2023). A Stability Analysis of Neural Networks and Its Application to Tsunami Early Warning. *AGU Fall Meeting Abstracts, 2023,* S42B-02.

Shioi, A., Otake, Y., Yoshida, I., andMuramatsu, S. (2024):Data-Driven Seismic Response Analysis Based on Dynamic Mode Decomposition with Time-Delay Embedding.*Japanese Journal of JSCE, Vol. 80, Issue 15, 80(15),* 23-15006.

Zhang, R., Liu, Y. and Sun, H. (2020). Physics-Guided Convolutional Neural Network (PhyCNN) for Data-Driven Seismic Response Analysis. *Engineering Structures, Vol. 215, 2020,*article 110704