Proc. of the 9th International Symposium on Geotechnical Safety and Risk (ISGSR)
Edited by Zhongqiang Liu, Jian Dai and Kate Robinson
Published by Research Publishing, Singapore.
doi: 10.3850/GRF-25280825_isgsr-247-P083-cd

ISGSR2025

STATISTICAL ANALYSIS AND INTERPRETATION OF THE UNCERTAINTY INHERENT TO THE EFFECTIVE FRICTION ANGLE OF NON-COHESIVE SOILS DETERMINED FROM SHEAR TESTS

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The effective shear strength is crucial for assessing the load-bearing capacity and serviceability of geotechnical structures. For a fully probabilistic assessment or to determine representative values to be used in the ultimate and serviceability limit state verifications according to EN 1997-1:2024, mean and variability of the shear strength parameters must be quantified. However, unlike other ground properties, the peculiarity of shear strength parameters lies in the fact that they are not directly measured in the laboratory. Instead, they are derived from the relationship between shear stress and normal stress, or effective mean stress, respectively. The interpretation of the results from shear tests is a particular case of a linear regression where the regression parameters are attributed a physical meaning. This makes the uncertainty analysis of $\tan\phi'$ nontrivial. Existing methods often fail to account for different sources of uncertainty. This study focuses on cohesionless soils, discussing a method to estimate the different contributors to the total uncertainty in $\tan\phi'$ using direct shear and triaxial test data from a preliminary ground investigation for an offshore wind farm in Northern Germany. The method is subsequently evaluated using varying sample sizes.

Keywords: uncertainty quantification; inherent variability; measurement uncertainty; limited number of samples.

1. Introduction

The effective shear strength is one of the most important parameters for assessing the load-bearing capacity and serviceability of a geotechnical structure. The shear strength parameters, effective friction angle $\tan\phi'$ and effective cohesion c', are used in the different limit state calculations, e. g., bearing capacity, sliding, tilting, etc.

A distinct challenge with shear strength parameters, unlike other ground properties, is that they are not directly measured in laboratory tests. Instead, they are derived from the relationship between shear stress (τ) and normal stress (σ'_n) or effective mean stress (σ'_m). This indirect determination adds complexity to the uncertainty analysis of shear strength parameters, making it a nontrivial task.

Several approaches for estimating the uncertainty of shear strength parameters from laboratory tests have been presented in the literature (e.g., Schoenemann and Piles 1990, Bond and Harris 2008, Bond 2011, Brzezinski 2021). However, these approaches often fail to explicitly account for individual sources of uncertainty. Yet, this differentiation is essential, since the different sources contribute differently to the total uncertainty of a geotechnical unit due to spatial averaging. To address this issue, this paper investigates the uncertainty quantification of $\tan\phi'$, initially limiting ourselves to cohesionless soils.

2. Methods

2.1. *Material*

The investigation area is characterised by a Holocene top layer of loose material with underlying Pleistocene deposits. For our analysis, we investigate four different sand layers abbreviated by IIa Sa, IIb Sa, IV Sa and V Sa. The direct shear tests were conducted as constant normal load direct shear (DS) tests according to DIN EN ISO 17892-10:2019 and the triaxial tests as isotropic consolidated drained triaxial (TX) tests according to DIN EN ISO 17892-9:2018 and BS 1377-8:1990. Details of the test procedures are given in BSH (2024). When evaluating the TX tests, it is important to emphasize that $\tan\phi'$ is directly calculated using Mohr's circles and the expression proposed by Bland (1980, 1983). This approach eliminates the need to transform $\sin\phi'$ to $\tan\phi'$, thereby ensuring that the assumption of a Gaussian distribution is preserved, which is essential for determining characteristic values in accordance with EN 1997-1:2024 or for integrating data from multiple sources, e. g., Müller et al. (2014). Although it is important to note that the assumption of a Gaussian distribution is only valid for small coefficients of variation due the non-negativity of $\tan\phi'$.

2.2. *Combining different uncertainty contributors to the total uncertainty*

According to ISO 2394:2015, the primary uncertainty sources of ground properties are the natural (inherent) variability, the measurement error, the statistical uncertainty, and, if applicable, the transformation uncertainty. Mathematically, the total uncertainty in the estimated ground property is quantified by its total variance $\varsigma^2_{\overline{X},\text{tot}}$ (e.g., Christian et al. 1994; Phoon und Kulhawy 1999):

$$\varsigma^2_{\overline{X},\text{tot}} = \varsigma^2_{X,\text{inh}} \cdot \Gamma^2 + \varsigma^2_{X,\text{stat}} + \varsigma^2_{X,\text{av, meas}} + \varsigma^2_{X,\text{trans}} \tag{1}$$

where $\varsigma^2_{X,\text{inh}}$, $\varsigma^2_{X,\text{stat}}$, $\varsigma^2_{X,\text{av, meas}}$, and $\varsigma^2_{X,\text{trans}}$ are variances of inherent variability, statistical uncertainty, average measurement error, and transformation uncertainty, respectively; and Γ^2 is the variance reduction factor that accounts for spatial averaging. Equation (1) assumes no correlation between the random error components. The coefficient of variation of a spatial averaged ground property $V^2_{\overline{X},\text{tot}}$ ($V_X = \varsigma_X/\mu_X$) is computed analogously.

In the case of $\tan\phi'$, the above equation simplifies as there is no transformation uncertainty ($V^2_{\tan\phi',\text{trans}} = 0$). Moreover, measurement uncertainty and statistical uncertainty are jointly accounted for in the average estimation error $V^2_{\overline{\tan\phi'},\text{est}}$ of the slope parameter in the linear regression (see also Section 2.3). The total uncertainty of the spatial averaged $\overline{\tan\phi'}$, $V^2_{\overline{\tan\phi'},\text{tot}}$, can then be expressed as:

$$V^2_{\overline{\tan\phi'},\text{tot}} = V^2_{\overline{\tan\phi'},\text{inh}} \cdot \Gamma^2 + V^2_{\overline{\tan\phi'},\text{est}} \tag{2}$$

Note that eq. (2) represents an adaptation of eq. (1) that can be applied to assess the uncertainty in $\tan\phi'$. However, it does not allow for the derivation of the uncertainty associated with the response variable of the linear regression. A notable distinction from the conventional uncertainty literature (e. g., JRC 2024) arises from the fact that no measurement uncertainty is subtracted from the inherent variability. From a mathematically viewpoint, all measurement (and statistical) uncertainty affecting $\tan\phi'$ is captured by $V^2_{\overline{\tan\phi',\mathrm{est}}}$.

2.3. *Assessment of uncertainties from shear tests*

The shear strength is commonly determined by plotting the peak shear stress τ_{peak} at failure versus the effective normal stress σ'_{n}. Although the shear strength depends nonlinearly on the effective stress, a linearization based on the Mohr-Coulomb criterion is applied in the stress range considered in the limit state analysis to obtain $\tan\phi'$:

$$\tau_{\mathrm{peak}} = \sigma'_{\mathrm{n}} \cdot \tan\phi' + \epsilon \tag{3}$$

assuming that the stress states (σ'_{n}, τ_{peak}) will cause local failure. Additionally, we introduce ϵ as a random error term with constant variance representing random sampling noise or the effect of variables not included in the model.

The ordinary least squares (OLS) method provides a single point estimate of $\tan\phi'_i$ and the standard error of the slope $SE_i(\tan\phi')$ for a triaxial test set i. As long as the individual samples within a test set of three samples are sampled spatially close to one another, with distances shorter than the autocorrelation length, spatial variability can be assumed to have no significant influence on the $\tan\phi'$ estimate obtained with a single regression. The inherent variability in $\tan\phi'$ is thus quantified using the $\tan\phi'$ estimates from n_{t} test sets to characterize the entire geotechnical unit based on the sample mean $m_{\tan\phi'}$ and sample variance $s^2_{\tan\phi'}$:

$$\overline{\tan\phi'_{\mathrm{inh}}} \approx m_{\tan\phi'} = \frac{\sum_{i=1}^{n_{\mathrm{t}}} \tan\phi'_i}{n_{\mathrm{t}}} \tag{4}$$

$$\varsigma^2_{\overline{\tan\phi'_{\mathrm{inh}}}} \approx s^2_{\tan\phi'} = \frac{\sum_{i=1}^{n_{\mathrm{t}}} \left(\tan\phi'_i - m_{\tan\phi'}\right)^2}{n_{\mathrm{t}} - 1} \tag{5}$$

The standard error of the slope $SE_i(\tan\phi')$ is denoted by:

$$SE_i(\tan\phi') = \sqrt{\frac{\sum_{j=1}^{n_{\mathrm{s}}} \left(\tau_{\mathrm{peak},j} - \hat{\tau}_{\mathrm{peak}}\right)^2}{n_{\mathrm{s}} - 1} \cdot \frac{1}{\sum_{j=1}^{n_{\mathrm{s}}} \sigma'^2_{\mathrm{n},j}}} \tag{6}$$

where $\hat{\tau}_{\mathrm{peak}}$ are the predicted values and n_{s} the number of samples available for the regression. $SE_i(\tan\phi')$ quantifies the expected variation in the estimated slope due to sampling variability if the regression were repeated with different samples from the same population. Given this definition, and under the assumption that a single regression does not capture inherent variability, $SE_i(\tan\phi')$ reflects a combination of statistical uncertainty and measurement uncertainty for each regression. The average estimation error $\overline{SE}(\tan\phi')$ and its coefficient of variation $V^2_{\overline{\tan\phi'},\mathrm{est}}$ are approximated in a manner analogous to the inherent variability.

$$\overline{SE}(\tan\phi') \approx m_{\overline{SE}(\tan\phi')} = \frac{\sum_{i=1}^{n_\mathrm{t}} SE_i(\tan\phi')}{n_\mathrm{t}} \tag{7}$$

$$V^2_{\overline{\tan\phi'},\mathrm{est}} = \left(\frac{\overline{SE}(\tan\phi')}{\overline{\tan\phi'_\mathrm{inh}}}\right)^2 \tag{8}$$

3. Results

3.1. *Total uncertainties*

Fig. 1 illustrates the breakdown of the two types of uncertainty as a proportion of the total uncertainty for different soil and test types. For $\Gamma = 1.0$, the inherent variability predominantly contributes to the total uncertainty, whereas with increasing spatial averaging ($\Gamma \to 0$), the estimation error becomes the dominant factor, cf. eq. (2), highlighting the critical importance of accurate uncertainty attribution.

Except for IIa, the total uncertainty is greater for the DS tests than for the TX tests. In the case of IIa (DS), the small sample size ($n = 3$) could lead to the underestimation of inherent variability (see Section 3.2). Moreover, it is expected that the inherent variability determined through TX and DS tests within the same geotechnical unit is comparable. In the investigation presented herein, however, the inherent variability of TX is approximately half that of DS, suggesting that, in the current formulation of eqs. (2 – 8), the inherent variability may still encompass a portion of measurement or statistical uncertainty that is not captured by $SE_i(\tan\phi')$ or $\overline{SE}(\tan\phi')$.

Compared to the inherent variability, the observed estimation error is small. For the DS tests it is consistently greater than that for the TX tests, which aligns with expectations. Triaxial tests are associated with lower measurement uncertainty. For the herein presented data, the coefficient of variation of the estimation errors range from $0.02 \leq V_{\overline{\tan\phi'},\mathrm{est,\,TX}} \leq 0.03$ for TX and $0.04 \leq V_{\overline{\tan\phi'},\mathrm{est,\,DS}} \leq 0.06$ for DS tests. These results are slightly smaller, but of the same order of magnitude as the measurement errors reported by Phoon and Kulhawy (1999), where $0.02 \leq V_{\overline{\tan\phi'},\mathrm{meas,\,TX}} \leq 0.22$ for "Sand, silt" and $0.06 \leq V_{\overline{\tan\phi'},\mathrm{meas,\,DS}} \leq 0.22$ for "Clay".

3.2. *Effect of sample size*

Theoretically, a smaller number of samples should exhibit larger statistical uncertainty or, in the case of shear tests, larger estimation errors. Ideally, this ensures that even when the inherent variability is underestimated, a conservative estimate of the total uncertainty is applied for geotechnical design purposes. To evaluate these practical engineering demands, subsets of varying sample sizes were drawn several times from the total population of TX tests with IIb sands and the uncertainties are assessed.

The findings indicate that the mean values of the estimation error (Fig. 2, center) remain robust, whereas the mean inherent variability (Fig. 2, left) fluctuates significantly

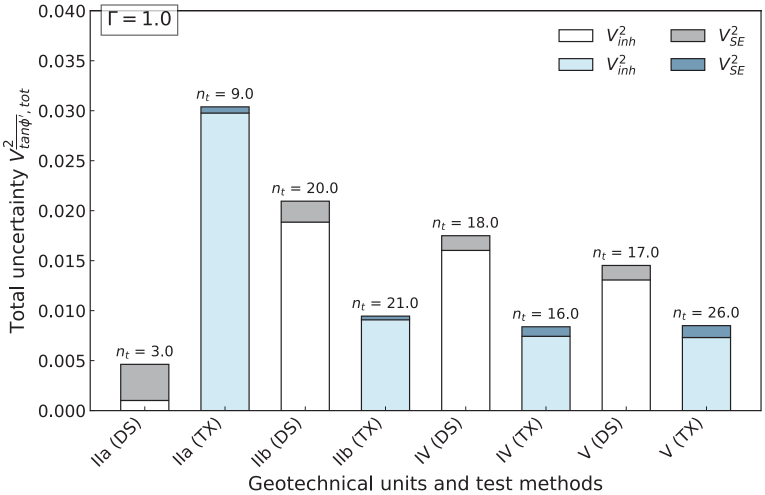

Fig. 1. Relative influence of different terms on the total uncertainty.

with a limited number of samples and is clearly underestimated. Significant deviations are only mitigated when the number of samples reaches more than six. When interpreting the required minimum number of samples, it is important to consider its dependence on the population's variability. Greater total uncertainty necessitates a larger number of samples. The estimation error exhibits comparable trends but with substantially smaller absolute values and a smaller number of required samples. As a result, the total uncertainty (Fig. 2, right) is observed to increase as the number of samples increases until it reaches a constant average. The results of the DS tests show a similar trend, albeit with significantly higher absolute values (cf. Fig. 1).

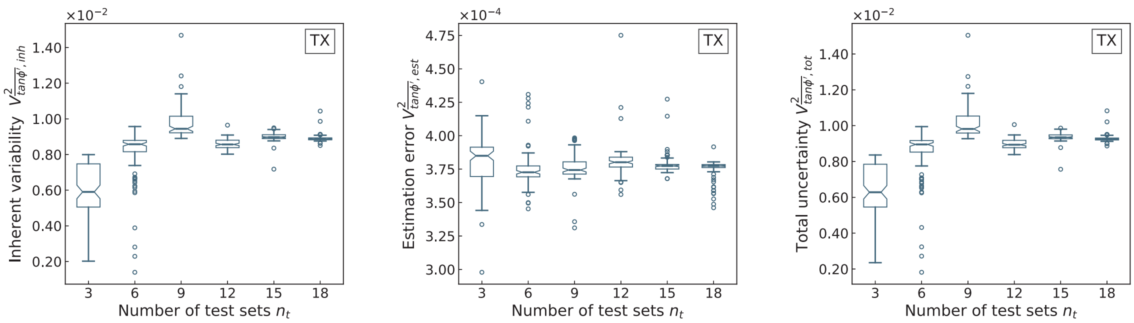

Fig. 2. Influence of number of samples on the inherent variability, estimation error and total uncertainty for TX / IIb Sa / $\Gamma = 1.0$.

4. Conclusions

The analyses highlight the challenges associated with assessing the various contributors to the total uncertainty of the effective friction angle, $\tan\phi'$. The results underscore the critical importance of obtaining a sufficient number of samples to ensure a robust estimation of total uncertainty within a strictly Frequentist framework. This is particularly significant in the case of shear tests, where inherent variability, unlike in other measurement methods, is the dominant contributor to total uncertainty. Future research should aim to extend the proposed approach, for example, by incorporating Bayesian methods to better address the uncertainties arising from a limited number of samples. Additionally, the accuracy of the current uncertainty attribution warrants further investigation. Specifically, the inherent variability within the same geotechnical unit should, in theory, produce consistent results regardless of the measurement method employed. Further analysis should also consider extending the approach to cohesive soils. Lastly, applying the method to the determination of characteristic values as specified in EN 1997-1:2024 would be highly beneficial.

Acknowledgement

The project is funded by the European Social Fund Plus (ESF Plus) and tax revenue on the basis of the budget approved by the Saxon state parliament.

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