

## Decoding Transition Mechanisms of Seismic Response via Dynamic Mode Decomposition

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This study focuses on geotechnical system identification and surrogate modeling for ground response analysis using only surface and subsurface time-series data, without relying on geotechnical information or ground modeling. By representing ground response in state-space form and applying Dynamic Mode Decomposition, a linear system is proposed to identify the ground system's inherent time evolution and predict its response as a data-driven approach. The proposed method's validity is confirmed through comparisons with ground response simulation results.

*Keywords:* dynamic mode decomposition; data-driven; seismic response analysis.

### 1. Introduction

Understanding ground dynamics, response mechanisms, and accurate seismic response predictions is vital for seismic design and risk assessment. However, ground response simulations face uncertainties as geotechnical data are often converted into simulation parameters via empirical rules based on generic databases, neglecting site-specific features. In addition, ground response is further complicated by material nonlinearity and wave propagation influenced by topography, making it challenging to incorporate these factors into simulation models and governing equations.

To mitigate these uncertainties, surrogate methods bypass traditional modeling, capturing complex phenomena while reducing errors. In Japan, a dense seismic sensor network provides extensive time-series data rich in ground response information. Integrating this data with advanced data-driven technologies promises unprecedented resolution in geotechnical dynamics monitoring.

This study adopts a data-driven approach to ground response analysis using seismic time-series data. Dynamic Mode Decomposition (DMD), based on singular value decomposition, was chosen for its ability to identify governing operators and decompose data into modes with characteristic frequencies and damping parameters. Unlike black-box methods, DMD offers interpretability and flexibility, making it broadly applicable to forecasting and control (Proctor 2016, Schmid 2022).

This study forms the basis for developing a DMD-based ground response analysis method through preliminary investigations using simulated seismic data as pseudo-observations. DMD analyzes surface and subsurface acceleration data to examine correlations between intrinsic parameters and ground dynamics. The operator's predictive accuracy is evaluated, and the data-driven surrogate model is validated against simulation results to assess its effectiveness.

## 2. Research Method

In this study, DMD with control (DMDc), which is an extension of DMD to a control system, is used. This is because it is thought that the ground response problem can be represented as a state-space model. That is, the ground system uses the input earthquake as the control external force and outputs the ground response.

The data structure of DMDc in this study considers a stacked high-dimensional space with time-delay, where  $\mathbf{u}_k$  is the ground earthquake and  $\mathbf{x}_k$  is the ground surface response at the  $k$  th time step. Then, the operators  $\mathbf{A}$  and  $\mathbf{B}$  are obtained using singular value decomposition such that the following equations approximately hold at all time steps:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k \tag{1}$$

Here,  $\mathbf{A}$  represents a linear operator describing the time evolution law of the geotechnical system, while  $\mathbf{B}$  evaluates how external forces disturb this linear time evolution. The complex eigenvalues of  $\mathbf{A}$  correspond to the eigenfrequencies and damping ratios as follows:

$$f_i = \frac{\arg(\lambda_i)}{2\pi\Delta t}, \quad h_i = -\frac{\log |\lambda_i|}{|\log \lambda_i|} \tag{2}$$

Here,  $\lambda_i$  is the  $i$  th eigenvalue of operator  $\mathbf{A}$ ,  $\arg(\lambda_i)$  is the argument of complex eigenvalue  $\lambda_i$ ,  $f_i$  and  $h_i$  are the eigenfrequency and damping constant, respectively. Using the learned operators  $\mathbf{A}$  and  $\mathbf{B}$  and the input earthquake, the state can be predicted sequentially from Eq. (1). The framework for geotechnical system identification and response prediction is shown in Fig. 1.

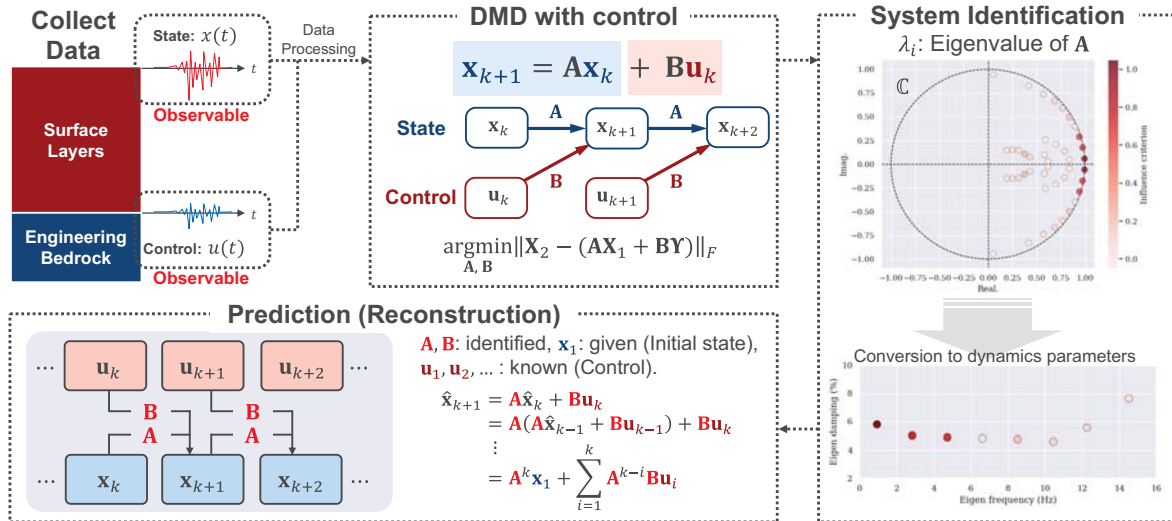


Fig. 1. Flow of geotechnical system identification and response prediction using DMDc: The operator  $\mathbf{A}$ , representing the inherent time evolution law of the geotechnical system, is derived, while operator  $\mathbf{B}$  accounts for system nonlinearity introduced by the input earthquake. The complex eigenvalues of  $\mathbf{A}$  are used for ground identification. A linear system is reconstructed from  $\mathbf{A}$  and  $\mathbf{B}$ , enabling sequential response prediction based on input earthquakes.

### 3. Validation of the proposed method

To validate the proposed method, it is compared with frequency response analysis. The assumed ground model is shown in Table 1. For the assumed ground model, three seismic records observed at a port wharf in Japan are used as input waves, and the response to each is simulated by a linear frequency analysis. All three sets of input and response waves are used for validation studies. The input seismic data are sourced from "Strong-Motion Earthquake Observation in Japanese Ports" by the Port and Airport Research Institute (PARI).

This verification expects two outcomes from DMDc: (1) the operator  $\mathbf{A}$  reflects the ground's inherent time evolution law, independent of the input waves, and (2) the dynamic parameters derived from  $\mathbf{A}$ 's eigenvalues correspond to the ground model's characteristics. Furthermore, since frequency response analysis uses time-invariant transfer functions, DMDc is expected to approximate the system as a linear surrogate model capable of highly accurate response prediction.

Table 1. Parameters of ground model for frequency response analysis.

Layer	Unit weight (kN/m <sup>3</sup> )	Thickness (m)	$V_S$ (m/s)	Damping (%)
1	20.0	10.0	150	2.00
2	20.0	10.0	130	2.00
3	20.0	10.0	180	2.00
4	20.0	10.0	200	2.00
5	20.0	-	400	2.00

Fig. 2 shows the distribution of eigenvalues of operator  $\mathbf{A}$  and the results of converting the eigenvalues to frequency and damping parameters. The color of the plots represents the importance of each eigenvalue calculated from the DMD modes. The parameter identification results by DMDc are in very close match with the parameters computed from the ground model. The major eigenvalues were identified as features specific to the ground, independent of the case.

Fig. 3 shows the results of surrogate model construction by DMDc. The surface response predicted from the operator  $\mathbf{A}$  identified by DMDc and the input earthquake reproduces the simulation results almost exactly.

### 4. Conclusion and Future Perspectives

In this study, the DMDc for surface and subsurface seismic waves was used for training. The results confirm that the effect of the input seismic wave on the ground-specific time evolution law can be isolated and that the ground time evolution law independent of the input seismic wave can be identified as a linear operator. It was also confirmed that the identified operators can be used to construct a surrogate model for ground response prediction.

However, since this discussion is based on frequency response analysis assuming linear ground materials, application to simulations that take into account ground nonlinearity and to actual observation records is a future issue.

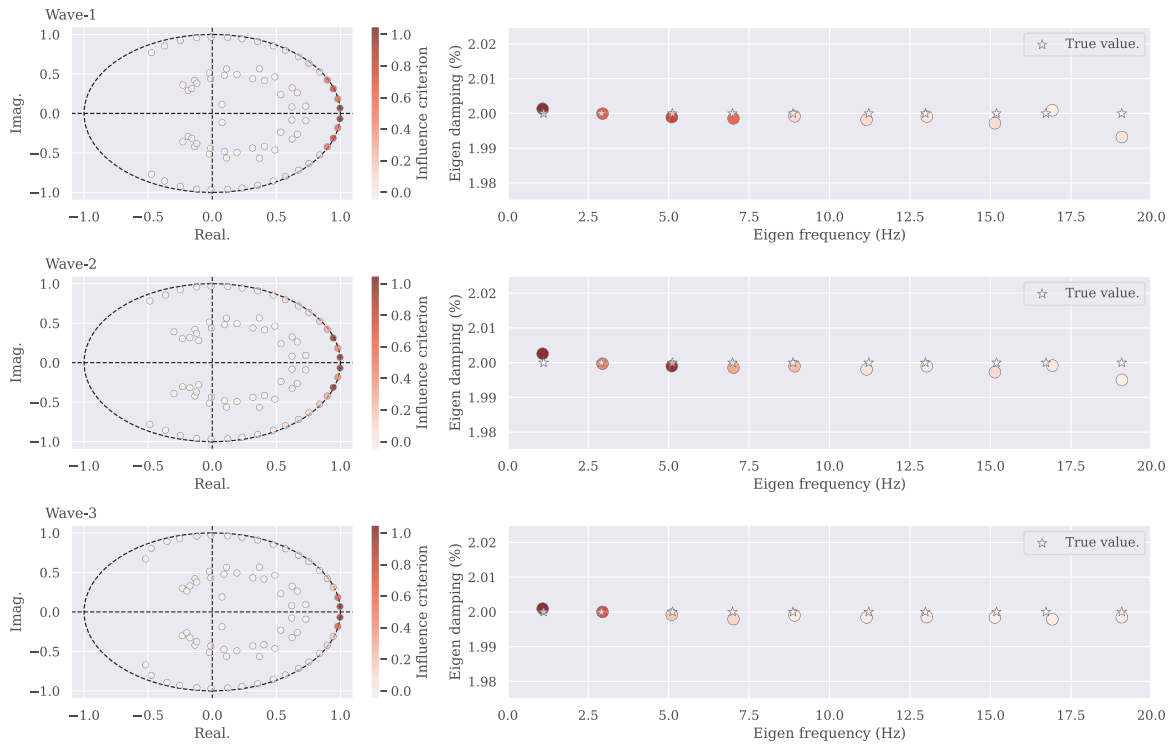


Fig. 2. Distribution of complex eigenvalues of operator A (left panel) and their transformation into dynamic parameters (right panel). It can be verified that the eigenfrequencies and eigendamping identified by the DMD are equivalent to those calculated based on the ground model. It shows that the major eigenvalues are obtained independently of the input earthquake.

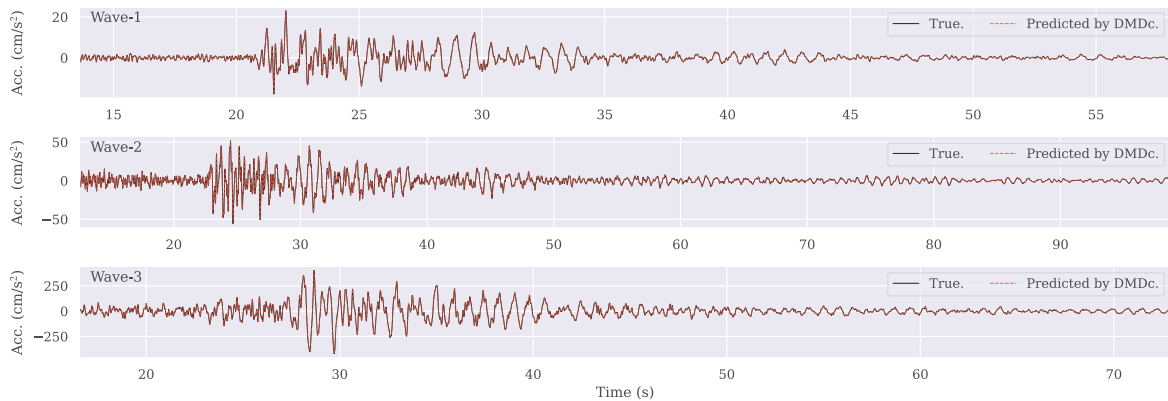


Fig. 3. Response prediction results using surrogate model constructed by DMDc. The DMDc properly constructed the linear system and achieved response estimates similar to the simulation results, even though it did not refer to any information of the ground model.

### References

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