Proc. of the 9th International Symposium on Geotechnical Safety and Risk (ISGSR)
Edited by Zhongqiang Liu, Jian Dai and Kate Robinson
Published by Research Publishing, Singapore. ©2025 The Author(s), CC-BY 4.0 licence.
doi: 10.3850/GRF-25280825_isgsr-026-P064-cd

OBSERVATIONS ON PROBABILISTIC TWO-LAYER UNDRAINED SLOPE STABILITY ANALYSIS

Desheng Zhu

Institute of Geotechnical Engineering, Yangzhou University, P. R. China. E-mail: deshengzhu@yzu.edu.cn

D. V. Griffiths

Department of Civil and Environmental Engineering, Colorado School of Mines, USA. E-mail: d.v.griffiths@mines.edu

Gordon. A. Fenton

Department of Engineering Mathematics, Dalhousie University, Canada. E-mail: gordon.fenton@dal.ca

In geotechnical engineering a two-layer undrained slope may be encountered in which an embankment or levee rests on a different foundation soil. This paper will use the random finite-element method (RFEM) to conduct probabilistic stability analyses of two-layer undrained slopes in which each layer has different statistical properties. The RFEM results show that, if both layers of the slope consist of spatially variable soil, the critical failure mechanism may go through the lower foundation layer, which would not occur in a deterministic analysis. This is explained by the "seeking out" effect of RFEM, which permits the deeper failure mechanisms to form naturally because of possible lower undrained strengths in the foundation layer. The RFEM results also demonstrate the worst-case phenomenon and show that the worst-case effect becomes less obvious as the mean strength ratio of the two layers increases.

Keywords: RFEM, two-layer, undrained strength, failure mechanism, worst-case, probabilistic anaysis.

1. Introduction

Construction of embankments or levees sometimes leads to a two-layer undrained slope (e.g., Duncan et al. 2008). Recently, in the framework of deterministic analyses, this problem has been investigated by Qian et al. (2015) using finite-element limit analysis methods, and by Guo and Griffiths (2020) adopting finite-element strength reduction methods. It is well known however, that soil properties can vary 'from point to point' (Terzaghi 1948), and that the classical deterministic approaches cannot account properly for this spatial variability in geotechnical analysis. The random finite-element method (RFEM) provides a systematic way of tackling spatial variation of soil properties. This advanced numerical method combines finite-elements with random fields, which were generated by the local averaging subdivision (LAS) method (Fenton and Vanmarcke, 1990) to account for spatial variation.

The objective of this study is to investigate the reliability of two-layer undrained slopes by RFEM. Figure 1 shows the profile for a two-layer undrained slope, where s_{u1} is the undrained strength of the embankment layer; s_{u2} is the undrained strength of the foundation layer ($\phi_u = 0$); β is the slope angle; H is the embankment height and D is the foundation depth ratio. For simplicity in this study, the s_{u1} and s_{u2} are spatially variable, while other parameters are deterministic.

In this study, the RFEM program `twosided`, which can consider two soil layers with their own statistical properties, has been applied to the reliability analyses of two-layer undrained slopes in random soil. The undrained strength s_u is modelled as a spatially varying random variable, with a lognormal distribution. For simplicity, the coefficient of variation $v_{s_u} = v_{s_{u1}} = v_{s_{u2}}$ is assumed in this study. Because of soil sedimentary deposition processes, the spatial correlation length may be anisotropic. Furthermore, the spatial correlation lengths in the embankment layer and foundation layer may be different. However, for simplicity an isotropic spatial correlation length $\Theta = \Theta_{s_{u1}} = \Theta_{s_{u2}} = \theta / H$ is used. The probability of failure p_f is calculated as the proportion of the 2000 Monte-Carlo analyses where slope failure occurred.

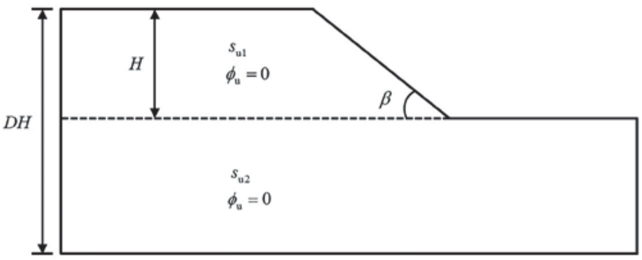

Fig. 1. A profile for the two-layer undrained slope.

2. Failure mechanisms

An example slope used by Guo and Griffiths (2020) is used here to investigate the failure mechanism of two-layer undrained slopes in random soil. The example slope has parameters $H = 18$ m, $D = 2.0$, $\cot\beta = 2.0$ ($\beta \approx 26.6°$), $s_{u1} = 60$ kPa, unit weight $\gamma = 20$ kN/m^3, Young's modulus $E = 10^5$ kPa and Poisson's ratio $\nu = 0.3$. Figure 2 shows the calculated factor of safety FS for different values of s_{u2} / s_{u1} by finite-element method (FEM) with strength reduction. It is obviously shown in Figure 2 that there exists a sharp transition occurring at $s_{u2} / s_{u1} \approx 1.5$. Guo and Griffiths (2020) defined this transition point as the critical strength ratio (referred to as R_{crit}), at which the failure mechanism transitions from shallow to deep. The R_{crit} is influenced by β and D, and the value is between 1.2 and 1.8 in all the cases considered in Guo and Griffiths (2020).

Figure 3 shows the typical failure mechanisms for the example slope. For two-layer undrained slopes, when $s_{u2} / s_{u1} < R_{crit}$ the failure mechanism is deep (Figure 3a), while when $s_{u2} / s_{u1} > R_{crit}$ the failure mechanism is shallow (Figure 3c). Due to the "seeking out" effect (the failure mechanism seeks the path of least resistance through the geotechnical system) in FEM (non-random), both mechanisms will be seen when $s_{u2} / s_{u1} = R_{crit}$ (Figure 3b).

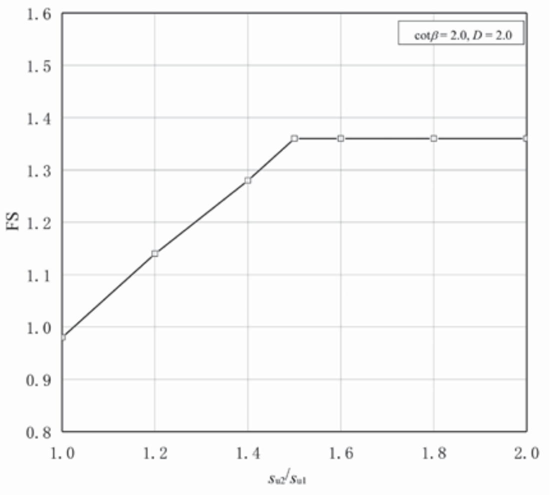

Fig. 2. Caclulated FS for different values of s_{u2} / s_{u1}.

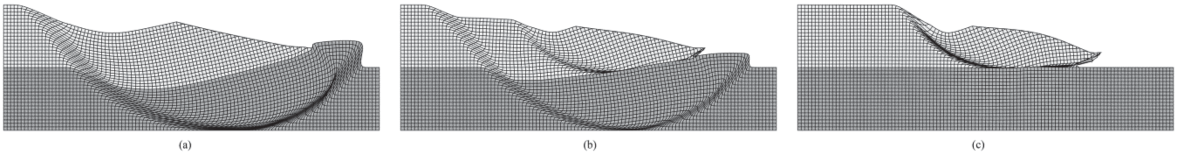

Fig. 3. Typical failure mechanisms for the example slope with: (a) $s_{u2} / s_{u1} = 1.2$, (b) $s_{u2} / s_{u1} = 1.5$, (c) $s_{u2} / s_{u1} = 1.8$.

Figure 4 shows the probability of slope failure p_f plotted against mean strength ratio $\mu_{s_{u2}} / \mu_{s_{u1}}$ with different coefficients of variation v_{s_u} when $\Theta = 0.5$. It can be seen from Figure 4 that when $v_{s_u} \geq 0.3$ and $\mu_{s_{u2}} / \mu_{s_{u1}} \geq 1.5$, as the mean strength ratio increases, the p_f still decreases. This phenomenon indicates that when considering soil spatial variation for two-layer undrained slopes, even with a mean strength ratio $\mu_{s_{u2}} / \mu_{s_{u1}}$ greater than the R_{crit}, the failure mechanism may be deep. If failure mechanisms are all shallow in the Monte-Carlo process, the probability of failure is constant and increasing the mean strength ratio will not decrease the p_f.

Figure 5 shows a typical failure mechanism for the example slope with $\mu_{s_{u2}} / \mu_{s_{u1}} = 1.8$, $v_{s_u} = 0.5$ and $\Theta = 0.5$. The failure mechanism will be shallow when $s_{u2} / s_{u1} > 1.5$ by deterministic finite-element analysis (Figure 3c), however, the two-layer undrained slope can also exhibit a deep failure mechanism when considering soil spatial variation.

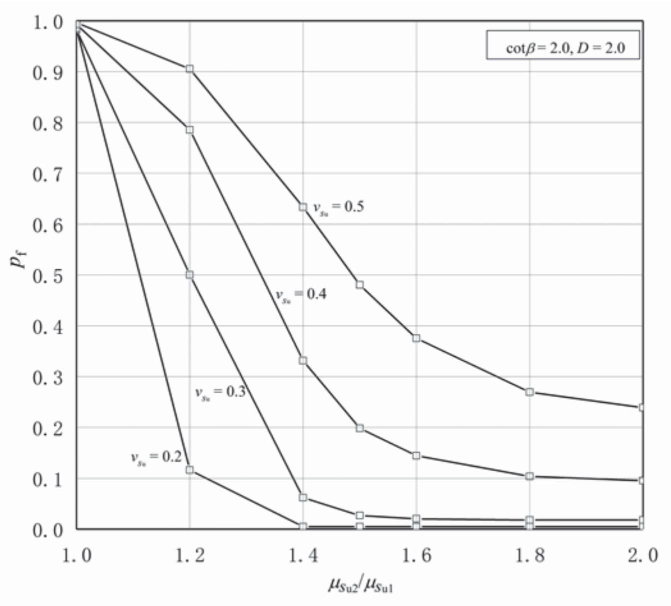

Fig. 4. p_f plotted against $\mu_{s_{u2}} / \mu_{s_{u1}}$ with different coefficients of variation.

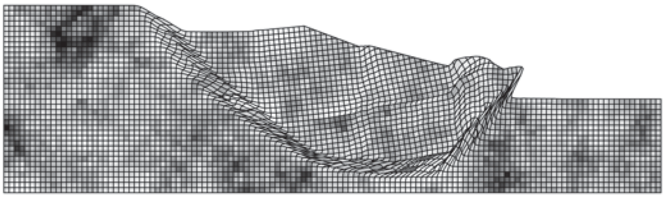

Fig. 5. A typical failure mechanism for the example slope with $\mu_{s_{u2}} / \mu_{s_{u1}} = 1.8$, $v_{c_u} = 0.5$ and $\Theta = 0.5$.

3. Worst-case spatial correlation length

To investigate the worst-case phenomenon in two-layer undrained slope reliability, a test slope with $s_{u1} = 50$ kPa, $\gamma = 20$ kN/m^3, $H = 10$ m, $\cot\beta = 1.0$ ($\beta = 45°$), $D = 2.0$, $E = 10^5$ kPa and $\nu = 0.3$ is considered here. Figure 6 shows the deterministic factor of safety FS for different values of s_{u2} / s_{u1} , with a sharp transition at $s_{u2} / s_{u1} \approx 1.2$ and the factor of safety $\text{FS} \approx 1.55$.

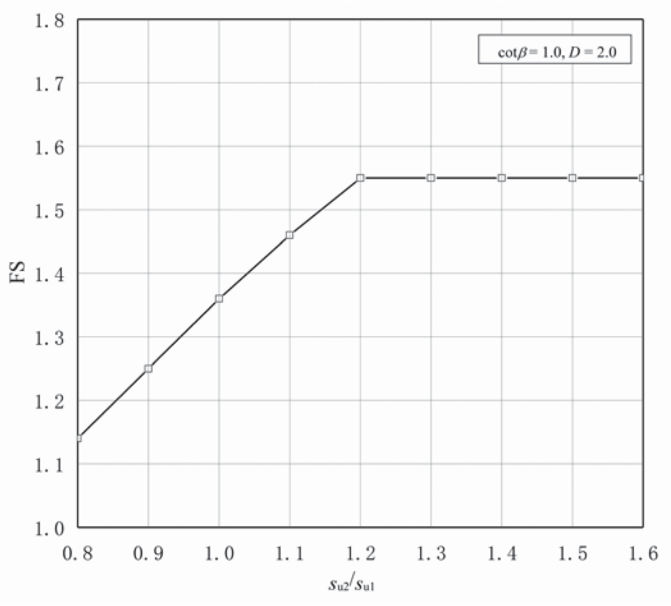

Fig. 6. Calculated FS for different values of s_{u2} / s_{u1} .

Figure 7 shows the probability of slope failure p_f plotted against the non-dimensional spatial correlation length Θ with different mean strength ratios when $v_{s_u} = 0.7$. The following non-dimensional spatial correlation length values are selected: $\Theta = 0.01,\ 0.1, 0.2, 0.5, 1, 2, 5, 10, 100$. $v_{s_u} = 0.7$ is selected herein as a higher value of the coefficient of variation for undrained strength within the range suggested by Phoon and Kulhawy (2008). According to the results by deterministic analyses in Figure 6, the factor of safety assuming the mean strength is uniform throughout is $\mathrm{FS} \approx 1.55$. However, it can be seen from Figure 7 that the worst-case phenomenon is most obvious when $\mu_{s_{u2}} / \mu_{s_{u1}} = 1.2$. With the increase of mean strength ratio, the worst-case effect becomes less pronounced. This phenomenon can also be expected, because with the increase of mean strength ratio, the p_f will still decrease as shown in Figure 4.

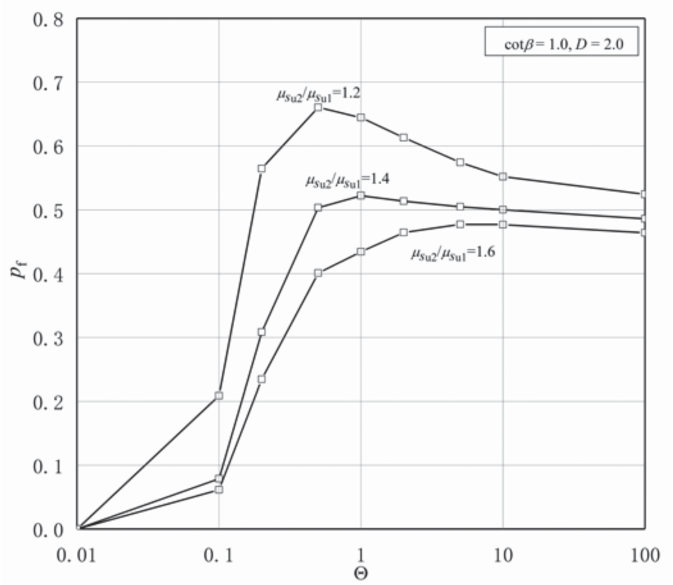

Fig. 7. p_f plotted against Θ with different mean strength ratios when $v_{s_u} = 0.7$.

4. Concluding remarks

This paper has investigated the reliability of two-layer undrained slopes by the RFEM. Typical applications might include short-term stability of embankments or levees resting on foundation soils with different statistical properties. As demonstrated in this paper, if a two-layer undrained slope consists of spatially variable soil, the failure mechanism can pass through the foundation layer in some cases even if the lower soil is stronger on average. This phenomenon is most obvious when the coefficient of variation of undrained strength is relatively high (e.g., $v_{s_u} \geq 0.3$). This is in contrast to deterministic analyses in which mechanisms will never go deep once a critical strength ratio is exceeded. This contrary phenomenon can be attributed to the "seeking out" advantage of RFEM, which allows the deeper failure mechanism to form naturally due to the low strengths in the foundation layer. This paper has also discussed the worst-case phenomenon, and found the worst-case effect becomes less pronounced with increasing mean strength ratio.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (Grant No. 51808484).

References

Duncan, J.M., Brandon, T.L., Wright, S.G., and Vroman, N. (2008). Stability of I-walls in New Orleans during Hurricane Katrina. *J. Geotech. Geoenviron. Engng.* 134 (5), 681-691.

Fenton, G.A., and Vanmarcke, E.H. (1990). Simulation of random fields via local average subdivision. *J. Engng. Mech.* 116 (8), 1733-1749.

Griffiths, D.V., and Lane, P.A. (1999). Slope stability analysis by finite elements. *Géotechnique* 49 (3), 387-403.

Guo, S., and Griffiths, D.V. (2020). Failure mechanisms in two-layer undrained slopes. *Can. Geotech. J.* 57 (10), 1617-1621.

Phoon, K.K., and Kulhawy, F.H. (2008). Serviceability limit state reliability-based design. Chap. 9 in *Reliability-based design in geotechnical engineering: computations and applications*. Taylor & Francis, London.

Qian, Z.G., Li, A.J., Merifield, R.S., and Lyamin, A.V. (2015). Slope stability charts for two-layered purely cohesive soils based on finite-element limit analysis method. *Int. J. Geomech.* 15 (3), 06014022.

Terzaghi, K. (1948). Foreword to the inaugural volume of Géotechnique. *Géotechnique* 1 (1), 3-5.