

PROBABILISTIC BACK ANALYSIS OF FAILED LATERITIC SOIL CUTTING USING BAYESIAN APPROACH

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Abstract: In this paper, the back analysis is performed on the failed lateritic soil cutting using a probabilistic Bayesian approach, updating uncertain spatially varying parameters based on observed slope failure behaviour due to rainfall. This study demonstrates that coupled-flow deformation (CFD) analysis and Bayesian approach, combined with random field theory, are effective for back-analysing slope parameters. The prior probability density functions (PDFs) of cohesion (c'), friction (ϕ') and unit weight (γ) are obtained by generating normal random fields for the prescribed coefficient of variation and anisotropic scale of fluctuation using the Karhunen-Loève expansion method. The factor of safety obtained from CFD analysis and prior PDFs are then used to generate the posterior PDFs of random variables using the Bayesian approach. It is observed that the posterior PDFs are narrower than the prior ones with reduced mean for c' and ϕ' , whereas the posterior PDFs for γ have a higher value of the mean, overall indicating a reduction in shear strength of soil leading to instability of slopes. This method enhances the understanding of geotechnical strength parameters through probabilistic inverse analysis. The updated information on strength and unit weight parameters from this approach can be utilized for improved slope stability assessments, addressing the significant issue of uncertainty in soil parameters and offering a robust tool for geotechnical engineering applications.

Keywords: laterite soil cutting, Bayesian method, random fields, anisotropic scale of fluctuation

1. General Appearance

The Western Ghats region of India faces a significant threat from soil instability, leading to numerous landslides, rockfalls, and soil erosion problems each monsoon season. This issue is particularly acute in the Konkan region, a part of the Western Ghats, characterized by steep lateritic soil slopes, high rainfall, and intensive land use practices. The Konkan region, with its numerous lateritic soil cuttings, serves as a corridor for highways and railways. However, the heavy to very heavy rainfall during the monsoon season often triggers frequent landslides, resulting in considerable loss of life and property. The highly heterogeneous nature of these soil cuttings makes slope stability analysis particularly challenging due to the inherent uncertainty in soil properties. Back analysis is a widely used technique to investigate the causes of slope failure and refine soil parameters for slope remediation design (Wesley and Leelaratham, 2001; Tiwari et al., 2005). Back analysis can be conducted using deterministic and probabilistic approaches (Feng et al., 2004; Li et al., 2021). Deterministic methods yield a single set of stability parameters corresponding to an observed failure (Feng et al., 2004; Zhang et al., 2010). In contrast, probabilistic methods account for the inherent uncertainties in soil properties by identifying multiple sets of parameters that may explain the same failure (Ering and Babu 2016; Shadani 2024). Probabilistic approaches also quantify the relative likelihoods of these parameter combinations using probability distributions (Zhang et al., 2010). Past researchers (e.g., Ering and Babu, 2016) used the Bayesian approach with random field theory. However, the effect of autocorrelation distance, i.e., scale of fluctuation, was excluded in most of these previous studies. Ering and Babu (2016) used the Bayesian approach with random field theory using the K-L expansion method. They assumed an isotropic scale of fluctuation of 2 m in both cohesion and friction angle.

In this study, first, the coupled flow-deformation (CFD) analysis is performed on the Kondavi cutting subjected to rainfall infiltration using Plaxis 2D software. The factor of safety (FOS) values is computed for input c' , ϕ' , γ and each day of rainfall infiltration. The normal random fields for c' , ϕ' , and γ with a COV of 10% using the K-L expansion method with an anisotropic SOF ($SOF_x/SOF_y = 10$), were then generated using OPTUM G2 software to obtain the prior distributions of c' , ϕ' , and γ . A probabilistic Bayesian approach proposed by Tarantola (2005) is then employed for posterior PDFs using MATLAB.

2. Site Description and material properties

The Kondavi cutting, located in the Konkan region of Maharashtra within Ratnagiri District, comprises five berms, each with a slope angle of 45° and a height of 6 m. The upper three berms, totalling 23 m in height, are composed of lateritic soil, while the lower two berms, with a combined depth of 11 m, consist of hard basalt rock. In mid-July 2022, the top three berms failed due to the heavy monsoon rainfall that year. Rahul and Tyagi (2024) conducted geotechnical testing on soil samples collected from the Kondavi cutting. The soil was classified as high-plasticity silt, and its effective strength parameters were determined through the consolidated undrained triaxial test and reported effective cohesion c' and effective friction angle ϕ' as 25 kPa and 22°, respectively. Table 1 summarises the material properties of the soil from the Kondavi cutting.

Table 1: Soil properties of Kondavi cutting (Rahul and Tyagi, 2024)

Parameters	Values
Soil classification	MH
Effective cohesion (c')	25 kPa*
Effective friction angle (ϕ')	22°*
Bulk unit weight	16.4 kN/m ³

*CU triaxial test

3. Methodology

First, the coupled flow-deformation (CFD) analysis is performed on the Kondavi cutting subjected to rainfall infiltration using the soil properties listed in Table 1, using Plaxis 2D software. The factor of safety (FOS) values is computed for each day of rainfall infiltration applied for the monsoon months of June and July 2022 on the top surface through the infiltration boundary (more details on CFD methodology, refer Rahul and Tyagi, 2024). The normal random fields for c' , ϕ' , and γ with a COV of 10% using the Karhunen-Loève (K-L) Expansion method with an anisotropic SOF ($SOF_x/SOF_y = 10$), are then generated using OPTUM G2 software to obtain the prior distributions of c' , ϕ' , and γ .

A probabilistic Bayesian approach proposed by Tarantola (2005) is then employed for back-analysing slope stability parameters using MATLAB. In this approach, the slope stability model is represented as $g(x)$, where x is a vector of uncertain input parameters, defined as $x = [x_1 \ x_2 \ \dots \ x_M]^T$. The parameter x considered in this study includes c' , ϕ' , γ . The probabilistic back-analysis method iteratively updates the probability density function of x based on observed slope behaviour. A multivariate normal distribution is employed to represent the prior probability distribution of x , characterised by a mean vector $\mu_x = [\mu_1 \ \mu_2 \ \dots \ \mu_M]^T$ and a covariance matrix C_x .

For instance, for c' , the prior distribution is expressed as:

$$f(c') = \frac{1}{(2\pi)^{\frac{M}{2}} \cdot \sqrt{|C_{c'}|}} \exp\left[-\frac{1}{2}(c' - \mu_{c'})^T C_{c'}^{-1}(c' - \mu_{c'})\right] \dots \dots \dots Eq1$$

The probability density function $f(c')$ must be validated using a normalisation constant. Observational uncertainty causes the observed data (FOS_{obs}) to differ from the actual system response (FOS_{act}). Furthermore, model uncertainty introduces discrepancies between the predicted response $g(c')$ and the FOS_{act} . The updated distribution of c' is then updated accordingly.

Finally, the improved distribution of c' taking in account the prior distribution $f(c')$ based on observed data FOS_{obs} can be represented as follows based on all the above assumptions (Tarantola, 2005):

$$f(c' | FOS_{obs}) = \frac{1}{(2\pi)^{\frac{M}{2}} \cdot \sqrt{|C_{c'}|}} f(c') \exp\left[-\frac{1}{2}(g(c') - FOS_{obs})^T C_M^{-1}(g(c') - FOS_{obs})\right] \dots \dots Eq 2$$

The posterior distribution of c' given FOS_{obs} on expansion of $f(c')$ in Eq. (2), can be given as:

$$f(c' | FOS_{obs}) = \frac{1}{(2\pi)^{\frac{M}{2}} \cdot \sqrt{|C_{c'}|}} \exp\{[g(c') - FOS_{obs}]^T C_M^{-1}[g(c') - FOS_{obs}] + (c' - \mu_{c'})^T C_{c'}^{-1}(c' - \mu_{c'})\} \dots \dots Eq 3$$

where FOS_{obs} is the observed FOS value from CFD analysis, $C_{c'}$ is the prior covariance matrix of c' and, $C_M = C_y + C_z$, such that C_y is the covariance of the actual system response (FOS_{act}) and C_z is the covariance of the model uncertainty (z).

The solution for a linear model $g(c')$ is a closed-form solution which is given as follows (Tarantola, 2005):

$$\mu_{(c'|y)} = \mu_{c'} + C_{c'}G^T(GC_{c'}G^T + C_M)^{-1}(FOS_{obs} - G\mu_{c'} - \mu_z) \dots \dots Eq 4$$

$$C_{(c'|y)} = (G^TC_M^{-1}G + C_{c'}^{-1})^{-1} \dots \dots \dots Eq 5$$

where $\mu_{(c'|y)}$ and $C_{(c'|y)}$ are the posterior mean and covariance of c' respectively. G is the sensitivity of $g(c')$ with respect to c' at point $\mu_{c'}$ which is defined as a row vector. For a slope stability model, the model uncertainty may be taken as a random variable characterized by mean (μ_z) = 0.05 and standard deviation (σ_z) = 0.07 (Zhang et al., 2016, Ering and Babu, 2016). Similarly, the posterior pdfs were computed for ϕ' and γ .

4. Results

The result of CFD analysis showed that after applying rainfall as a flux for 43 days, the factor of safety (FOS) decreased to 1.206. However, field observations indicate that failure occurred after 43 days of rainfall, implying that the FOS should have reduced to unity at that point. Therefore, the soil parameters have to be updated to reflect actual failure conditions. Further, figures 1 (a), (b), (c) show the random fields generated for c' , ϕ' , and γ , respectively. The mean and standard deviation, and hence the calculated output COV for the simulated random fields, are tabulated in Table 2.

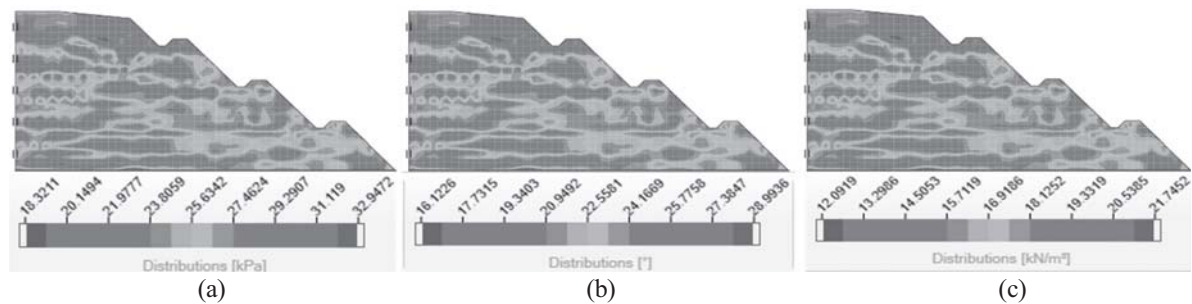


Figure 1: Random fields generated for (a) cohesion (b) friction angle (c) unit weight using OPTUM G2

Table 2: Mean, Standard deviation, and COV obtained for Simulated Random Fields

Parameters	SOF_x/SOF_y	Input Mean value	Input COV (%)	Output Mean Value	Output Standard Deviation	Output COV (%)
Cohesion (kPa)		25		25.36	2.36	9.4
Friction Angle (°)	10	22	10%	22.31	2.08	9.4
Unit weight (kN/m ³)		16.4		16.74	1.56	9.4

On the basis of the prior mean and covariance of x (Table 2), the prior mean row vector and a prior covariance matrix are formed as follows:

For $SOF_x/SOF_y = 10$;

$$\mu_x = \begin{bmatrix} 25.36 \\ 22.31 \\ 16.74 \end{bmatrix} \quad C_x = \begin{bmatrix} 2.36^2 & 0 & 0 \\ 0 & 2.08^2 & 0 \\ 0 & 0 & 1.56^2 \end{bmatrix}$$

Sensitivity (G) analysis is performed on the slope for c' , ϕ' , and γ , and the range of material parameters is defined as offsets derived from the mean and standard deviation of the generated random fields. This analysis was carried out with $n = 3$ steps, where n represents the number of variable parameters. The result of G is calculated as $G = [0.0244 \ 0.0282 \ -0.0278]$ for c' , ϕ' , and γ respectively. Now, the posterior mean and covariance are calculated by solving equations 4 and 5 using prior mean, prior covariance, and G .

$$\mu_{x(posterior)} = \begin{bmatrix} 22.7763 \\ 20.0168 \\ 18.0161 \end{bmatrix} \quad C_{x(posterior)} = \begin{bmatrix} 4.2322 & -1.2326 & 0.6835 \\ -1.2326 & 3.2291 & 0.6085 \\ 0.6835 & 0.6085 & 2.0962 \end{bmatrix}$$

Using the prior and posterior mean and standard deviation, the PDFs are plotted for c' , ϕ' , and γ and are shown in Figure 2 (a, b, and c).

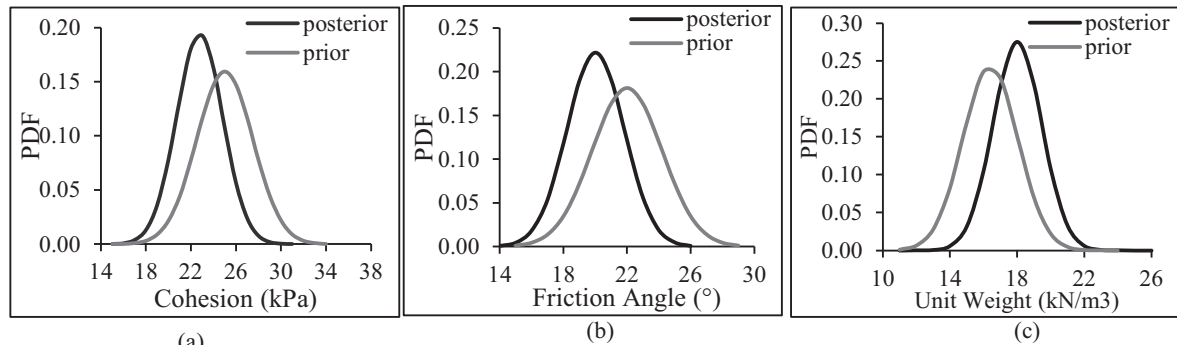


Figure 2: Prior and posterior PDF for (a) Cohesion, (b) Friction Angle, and (c) Unit Weight

The PDFs of the prior and posterior distributions for c' , ϕ' , and γ show notable changes in the mean values. The mean value of c' decreases from 25.36 kPa to 22.77 kPa (10.2% reduction), ϕ' decreases from 22.31 $^{\circ}$ to 20.92 $^{\circ}$ (6.2% reduction), and γ increases from 16.74 kN/m 3 to 18.02 kN/m 3 (7.6% increase). Thus, the updated parameters should be considered in the probabilistic-based design and numerical modelling of the rainfall-induced Kondavi slope failure to ensure accurate stability assessment. As shown in Figure 1, the random fields generated from OPTUM G2 for c' , ϕ' , γ are perfectly correlated. This may be one of the limitations of the present study. However, it should be noted that for generating posterior PDFs using the Bayesian method, the random variables are treated uncorrelated. Hence, the effect of c' , ϕ' , γ are considered independent of slope failure.

Conclusion

The present study highlights the importance of CFD analysis and probabilistic back-analysis with random field theory to accurately back-analyse the slope parameters in the failed laterite Kondavi cutting under rainfall infiltration. The CFD analysis performed in Plaxis 2D initially estimated an FOS of 1.206 after 43 days of rainfall, whereas field observations suggested failure, indicating the need for parameter refinement. The prior distributions of c' , ϕ' and γ for given COV and anisotropic SOF were then obtained by using K-L expansion method in OPTUM G2. Using Bayesian approach, the posterior distributions of c' , ϕ' , and γ were obtained. The mean values of c' and ϕ' decreased by 10.2% and 6.2%, respectively, while γ increased by 7.6%. These updated parameters provide realistic representation of soil properties for Kondavi cutting. The probability distributions generated through this approach enable more reliable slope stability assessments, effectively addressing the uncertainties in soil parameters and offering a robust tool for geotechnical engineering applications.

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