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THE EFFECT OF COEFFICIENT OF VARIATION AND DISTRIBUTION FUNCTIONS IN DETERMINING CHARACTERISTIC VALUES

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Stratigraphy units and linear horizontal sections have associated variability of soil parameters even for a defined "uniform" section. The representative value used in analysis is based on either a nominal or characteristic value. When sufficient data is available a statistical goodness of fit test should be used as the normal probability distribution function (PDF) may not appropriately model the data. Assuming a normal PDF produces dubious results where large coefficient of variation (COV) occurs. Data from Dynamic cone penetrometer (DCP) tests and California Bearing Ratio (CBR) case studies show a normal PDF would have prediction errors at or above a COV = 30%. As the COV increases to 80%, the % of negative (and invalid) predicted values are approximately 10%. Similar findings were found for rock socket assessment based on point load index (PLI) strength data. During construction, the high COVs were associated with uncertainty of rock strength and with deepening of the socket lengths beyond the nominal length shown in the drawings. Using data from these and other geotechnical assessments, the COV value is shown to provide an indicator when to adopt a normal or non-normal PDF for statistical assessment of characteristic values.

Keywords: Probability Distribution Function, Coefficient of Variation, Characteristic Value, Normal Distribution, CBR, Subgrade Point load index, Rock Socket

1. Introduction to the Coefficient of Variation in Geotechnical Engineering

The safety and reliability of structures requires understanding and predicting the behavior of soil and rock materials under various environmental and loading conditions. However, soil properties can vary significantly, making it essential to quantify this variability. One of the most effective statistical tools used for this purpose is the Coefficient of Variation (COV), which serves as a measure of relative dispersion in a set of data. The COV is defined as the ratio of the standard deviation to the mean of a dataset, often expressed as a percentage.

Geotechnical material parameters typically have a large COV as compared with manufactured materials such as steel or concrete. A COV of 20% for concrete is considered poor (Phoon and Kulhawy 2008), while in soil and rock materials such a strength result would be considered unusual and values of 40% typical. The seminal paper by Phoon and Kulhawy (1999) evaluates geotechnical property variability and is considered a practical initial input for reliability-based analysis in the absence of sufficient project specific data. COVs of 90% are common for strength index tests such as CBR (Look, 2009) or Point Load index tests (Look and Wijeyakulasuriya, 2009). Such index tests and large COVs were shown to be not well modelled by the Normal distribution. The assumed distribution can affect the moderately conservative design value or simulation results considerably.

Goodness of fit tests are required to determine the appropriate probability density function (PDF). The normal distribution was comparable with the lognormal and best fit PDF at the 20% to 30% fractile (Look, 2015). The normal distribution model is not appropriate at the 10% fractile or below, when high COV occur.

The COV and it implications for design and construction are examined by case studies, which cover:

(i) Dynamic Cone Penetrometer (DCP) data with temporal, spatial and testing variation.
(ii) CBR data for a "uniform" site. The COV was used to assess site uniformity.
(iii) Rock socket foundations. Variability in test results is an early indicator that the geotechnical assessor on site may likely lengthen the rock socket length due to material uncertainty.

Using a normal PDF may lead to incorrect predicted values. These case studies highlight when the normal PDF is inappropriate.

2. Case Study 1 – DCP data with temporal, spatial and testing variation

Using data obtained over 1 year of monitoring Look (2024a) shows the DCP and CBR results obtained at the dry period can be twice the value obtained at the end of a wet period. Using the normal PDF would result in a prediction of negative values despite a measured value below zero not being possible. Using the 624 DCP tests with the COV and the % of negative values, the relationship was established in Figure 1. This shows that for this DCP data, a normal PDF is acceptable at or below a COV = 30%. As the COV increases from 50% to 60% and 80% the % of negative (and invalid) predicted values are 2.3%, 4.7% and 10.5%, respectively.

3. Case Study 2 – CBR data for "uniform" zones sites

Look (2024b) discussed two case studies where CBR data were used. One case study was for road widening sections along a major highway. The COV was used to assess between uniform and non-uniform sites. The other case study was for a major 13 km rail upgrade to illustrate how the COV can be used in site characterisation.

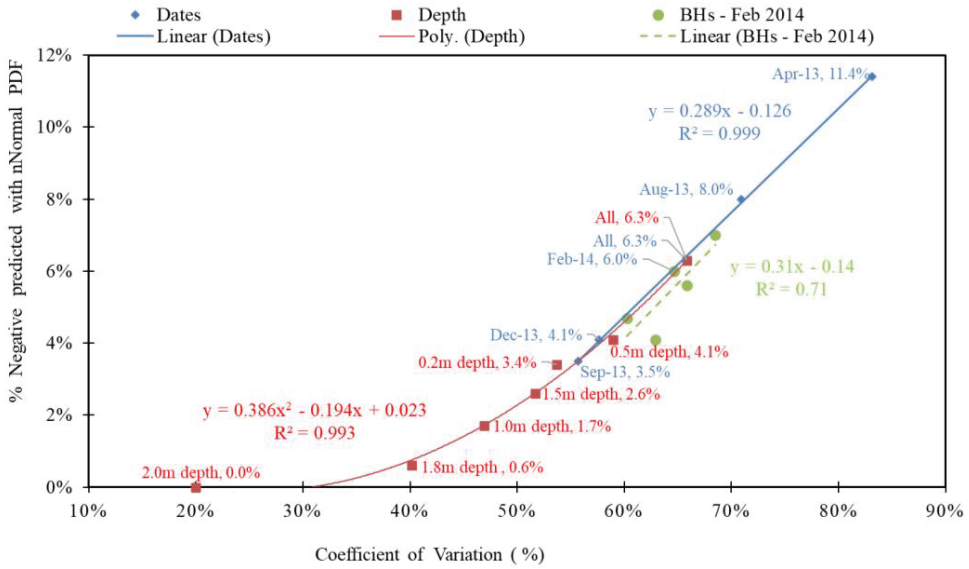


Fig. 1. COV compared with the negative values of the normal PDF for DCP tests.

Defining a "uniform" section of a site for both design and construction is a major decision which should anticipate potential contractual issues for a "uniform" section. Natural testing variability for CBR is typically 40% and could be up to 60%. Thus, sectioning based on CBR should have this COV as a considered factor. Sections with a COV > 80% is considered non uniform and likely to have contractual disagreement.

Figure 2 combines the data from those 2 CBR case studies presented in the Look (2024b) paper. For this CBR data, a normal PDF would not have prediction errors at or below a COV = 40%. As the COV increases from 60% to 80% the % of negative (and invalid) predicted values are 5.6% and 10.8%, respectively.

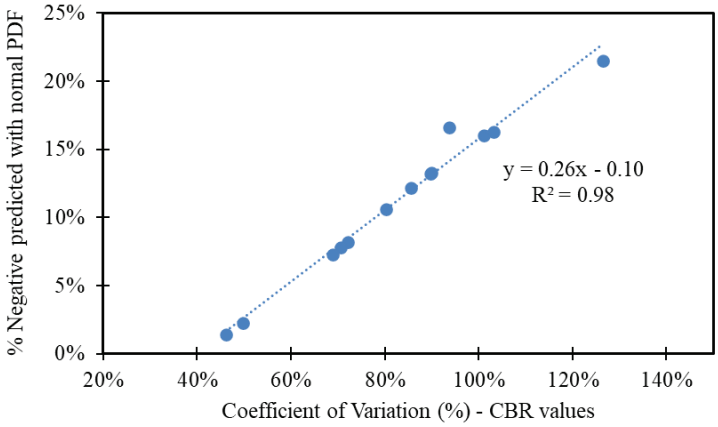


Fig. 2. COV compared with the negative values of the normal PDF (if used).

4. Case Study 3 – Point Load Index data for rock socketed piles

Rock strength data using a fitted Probability Density Function (PDF) at Pier 6 of the Gateway Bridge near the mouth of the Brisbane River was discussed in Look and Wijeyakulasuriya (2009). There was a high COV = 91% for diametral point load index ($I_s(50)$) tests at this location with number of tests (n) = 330. At this river pier, there were 24 Bored piles of 1.5m diameter, 50m length with a rock socket of approximately 6m. There were boreholes at each pile ~ 3.5m apart. The summary of test results is provided in Table 1.

All piles with an $I_s(50)$ COV above 80% were ultimately extended during construction several metres deeper than the design estimated toe levels. The requirement for fitting a non-normal PDF to geotechnical data was discussed in Look (2015) and that dataset is reproduced in Figure 3. The legend shows the best fit Log Logistic PDF from goodness of fit tests and compares with the lognormal and normal PDF. The PDFs shapes and comparison at the median and predicted by the various PDFs at 0.0 MPa are shown.

Table 1 Bridge pier 6 – BEST FIT of at each pile location for P I_s (50) diametral values

Pile	Rank of Anderson Darling Goodness of Fit		Best Rank	Using normal PDF	
	Normal	Log Normal	PDF	COV	% Negative
P6-21	5	3	Inv Gauss	37%	0.5%
P6-22	8	4	Log Logistic	113%	13.6%
P6-17	1	4	Normal	44%	0.0%
P6-18	6	13	Pearson V	156%	19.3%
P6-13	8	7	Inv Gauss	105%	14.4%
P6-14	6	7	Logistic	31%	0.1%
P6-9	8	1	Log Normal	46%	1.3%
P6-10	9	1	Log Normal	76%	8.0%
P6-5	6	13	Expon	39%	0.3%
P6-6	7	3	Loglogistic	151%	3.8%
P6-1			No test results		
P6-2			No test results		
P6-23	1	5	Normal	40%	0.6%
P6-24	8	4	Log Logistic	87%	14.4%
P6-19	8	3	Pearson V	127%	16.9%
P6-20	8	3	Log Logistic	73%	6.0%
P6-15	7	2	Log Logistic	135%	11.6%
P6-16	7	10	Uniform	65%	9.9%
P6-11	9	4	Log Logistic	67%	9.6%
P6-12	10	3	Log Logistic	98%	12.2%
P6-7	7	5	Weibull	56%	4.2%
P6-8	8	7	Inv Gauss	68%	6.3%
P6-3	8	3	Log Logistic	77%	8.0%
P6-4	1	4	Normal	55%	3.4%
All Values	9	3	Log Logistic	91%	9.0%

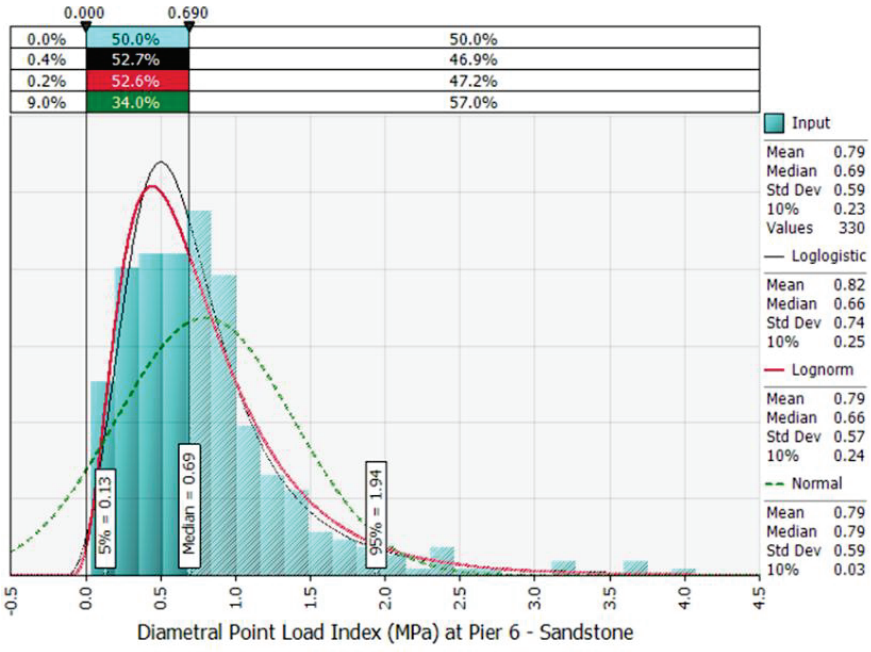


Fig.3. Comparison of distribution functions for diametral Is(50) at pier 6 of the Gateway Bridge (Look, 2015).

Comparing the percent of negative values (not possible) predicted if fitting a normal PDF to $I_s(50)$ values at each borehole is plotted against the COV in Figure 4a. This shows a COV < 30% is unlikely to have negative values. For a COV > 80%, there is greater than 10% predicted negative values and can lead to extended pile depths or contractual variations.

A similar finding was observed at the Centenary Bridge along this same Brisbane river, but 40km upstream of the Gateway Bridge site. Figure 4b shows the % negative PDF values with the COV analysis for this bridge site but with less pile data (n = 6) and for a different rock type. The piles where a high $I_s(50)$ COV occurred at this site typically also required deepening of the rock sockets in construction.

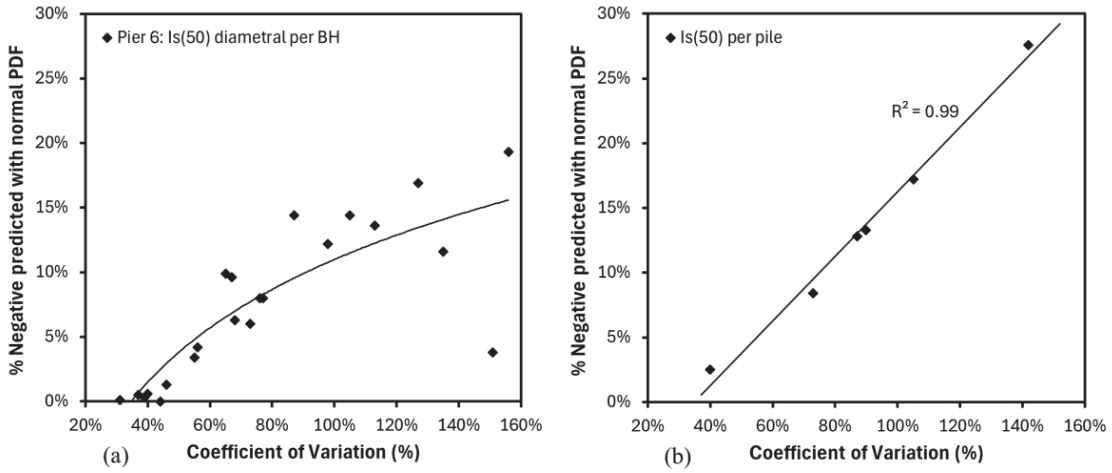


Fig. 4. Comparison of COV with negative values from normal PDF for (a) Gateway Bridge and (b) Centenary Bridge

5. Conclusion

In reliability analysis, probability of occurrence is combined with simulation models. The standard normal PDF should be avoided as an input at high COVs in simulation sampling (Look, 2022). Unreasonable negative or low results may be calculated if adopted, and truncation is then required. Case studies were presented to show when a normal PDF may be applicable and when design and contractual issues may arise.

Comparisons were shown between a normal PDF, log normal PDF and best fit PDFs. When COVs are greater than 30%, a normal PDF is unlikely to be the best fit PDF. The lognormal PDF, although not always the best fit, provides a more meaningful prediction of the site variability. Where COVs are greater than 80%, assessment difficulties for the geotechnical assessor or contractual issues may arise during construction. This 80% COV represents 10% or more of the (incorrect) negative values predicted by the normal PDF. This data provides the COV boundary when the normal PDF is applicable for analysis.

Using a normal PDF when the COV is greater than 30% may likely lead to prediction errors. The Log Normal generally provides an appropriate PDF, although not necessarily the number 1 ranked from goodness of fit tests.

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