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PROBABILISTIC ANALYSIS OF TUNNEL STABILITY IN SPATIALLY VARIABLE COHESIVE-FRICTIONAL SOIL

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This paper explores a probabilistic study of plane-strain tunnel stability in a spatially variable cohesive-frictional soil medium. The cohesion, friction angle, and the unit weight of soil are considered as random variables, which are assumed to be log-normally distributed. First, the multivariate cross-correlated random fields are discretized using the Karhunen–Loève expansion method in MATLAB for isotropic and anisotropic scale of fluctuation. A two-dimensional finite element model of a circular tunnel is then created in PLAXIS2D. FEM-based pressure-deformation approach is used for the determination of critical tunnel support pressure. This approach ensures the determination of the critical tunnel support pressure consistently in a repeatable manner. Using Monte-Carlo simulations, stochastic analysis is performed by repeatedly mapping random fields onto the finite element mesh in PLAXIS2D for multiple realizations. The input parameters include a mean cohesion of 10 kPa, a mean tangent of friction angle of tan 22°, and a mean unit weight of 16 kN/m³, each with coefficients of variation of 10%. Two cases were considered with ratios of horizontal to vertical scale of fluctuation as 1 and 10, respectively. The study assesses the effect of isotropic and anisotropic scale of fluctuation on the critical tunnel support pressure. The limiting tunnel support pressures from the random analyses are compared with the results of deterministic analyses to study the impact of spatial variability on tunnel stability. The mean critical tunnel support pressure and the failure probability of the tunnel for all the cases are presented.

Keywords: Tunnel, Cohesive-frictional soil, Spatial Variability, Anisotropic scale of fluctuation, Critical Tunnel support pressure.

1. Introduction

Soil exhibits inherent variability due to complex geological, environmental, and physical-chemical processes (Phoon and Kulwahy,1999). Ignoring these variations can lead to unfavourable, cost-ineffective outcomes, affecting construction expenses, failure risks and safety. Thus, reliability-centered approaches are crucial for assessing uncertainties and improving decision-making (Babu, 2024). The impact of spatial variability on tunnel stability has received growing attention. Various authors (Mollon et al. 2009; Mollon et al. 2011, Ji et al. 2021) attempted to address spatial variability and treated soil properties as random variables without considering spatial correlation lengths. Cheng et al. (2018) showed that neglecting soil spatial structure can overestimate failure probabilities. Additionally, while the spatial variability of cohesion (c') and friction (ϕ') is often considered, variability in unit weight (γ) is usually overlooked (Ji et al. 2021; Mollon et al. 2011; Cheng et al. 2018; Pan and Dias 2017).The anisotropic nature of soil spatial variability is well-documented, with vertical auto-correlation distance (l_y) typically being much shorter than horizontal auto-correlation distance (l_x) due to geological processes. The l_y/l_x ratio of 1:10 has been adopted by past researchers (Baecher and Christian 2003; Cho and Park 2009). However, several researchers have made simplifying assumptions by ignoring this anisotropic nature (Mollon et al. 2011; Ali et al. 2017; Cheng et al. 2019). The cross-correlation between soil parameters is another significant yet often overlooked factor in stability analyses. Studies widely report a negative correlation between c' and ϕ' or $\tan\phi'$ (Forrest and Orr 2010; Cherubini, 1997; Lumb, 1970). Contrary to this, a positive correlation is observed between γ and shear strength parameters (c' and ϕ'). Denser soils with higher γ tend to exhibit greater shear strength due to reduced void ratios (Roy and Dass 2014; Garcia et al. 2012). Empirical studies further establish this positive correlation, as higher SPT N-values indicative of higher γ are usually associated with increased ϕ' in granular soils (Peck et al. 1974) and higher undrained cohesion in clayey soils (Terzaghi and Peck 1967). Studies like Babu and Srivastava (2007) and Chowdhury and Xu (1992) also assumed positive correlations between γ and shear strength parameters in reliability assessments of shallow foundations and slope stability respectively. Despite this, many studies neglect cross-correlations. For instance, Cheng et al. (2019) ignored the cross-correlation between c' and ϕ' in their reliability study of shield tunnel face in multilayered soils, while Pan and Dias (2017) overlooked the cross-correlation between undrained cohesion and γ in their tunnel stability analysis in cohesive soil. These simplifications can lead to overly conservative tunnel support pressures (Ji et al. 2021).

The present study aims to bridge the gap and conduct a probabilistic analysis of a two-dimensional unlined circular tunnel stability in spatially varying cohesive-frictional soils. The analysis considers c', $\tan\phi'$ and γ as random fields. Cross-correlated random fields are discretized using the Karhunen-Loève (KL) expansion method in MATLAB for isotropic and anisotropic autocorrelation distances. PLAXIS2D is used to create a two-dimensional finite element model of a circular tunnel. Spatial variability is integrated into the finite element mesh using Python built-in module. Monte-Carlo simulations generate statistical responses. The study evaluates the impact of spatial variability on the mean normalized critical tunnel support pressure relative to deterministic results. The methodology employs FEM-based pressure-deformation analysis to determine the normalized critical tunnel support pressure. Additionally, the study presents failure probabilities and recommends safety factors for the target probability of failure under both isotropic and anisotropic spatial variability conditions.

2. Finite element analysis with spatially variable c'-ϕ' soils

The study analyses shallow unlined circular tunnel stability using PLAXIS2D (Version 2023.1.0.136) with a plane strain, random finite-element model discretized with 15-noded triangular elements (Fig. 1a). The tunnel diameter, B_t is taken as 6 m and cover-to-diameter ratio (H_t/B_t) is taken as 1. The study focuses on the normalized critical tunnel support pressure to determine tunnel stability, p_{nc} (%), determined as the ratio of tunnel support pressure at failure (p_f) to the initial geostatic pressure (p_o), expressed as a percentage. Value of p_{nc} (%), is determined through a staged simulation approach, progressively reducing radial pressure, (p_i) applied at tunnel periphery until solver non-convergence or zero applied support pressure. It is identified using a double tangent method on a graph of normalized applied pressure (p (%) = $(p_i/p_o)\times 100$), versus normalized crown deformation, (δ/B_t). Validation of this FEM-based pressure-deformation approach to obtain p_{nc} (%) is detailed in Mondal and Tyagi (2024). Fig.1b shows the curves corresponding to deterministic and Monte-Carlo simulations (anisotropic case) where, the p_{nc} (%) for each case is determined by the double tangent method. The soil domain dimensions are chosen accordingly to minimize boundary effects. Mohr Coloumb yield criterion is chosen to model the soil. The soil is characterized by spatially varying properties with an underlying log-normal distribution with mean, μ_M and standard deviation, σ_M where, M can be c', tan ϕ', γ. A log normally distributed random field ensures non-negative geotechnical parameters while, using it for tan ϕ' instead of ϕ' restricts generated values to the range of 0° to 90° (Griffiths et al. 2011). In PLAXIS2D, the spatial variability is integrated into the finite-element mesh using Python built-in module by assigning M to the j^{th} element (Eq.(1)). The value of M^j is determined by the M_{KL} term KL Expansion method (Spanos and Ghanem, 1989).

$$M^j \approx \exp\left[\mu_M^{\ln} + \sigma_M^{\ln}[L]\left\{\sum_{k=1}^{M_{KL}} \sqrt{\lambda_k}\,\xi_k \psi_k(x)\right\}\right] \tag{1}$$

L is derived as a lower triangular matrix by Cholesky decomposition algorithm to matrix R, representing the cross-correlation between the three random variables.

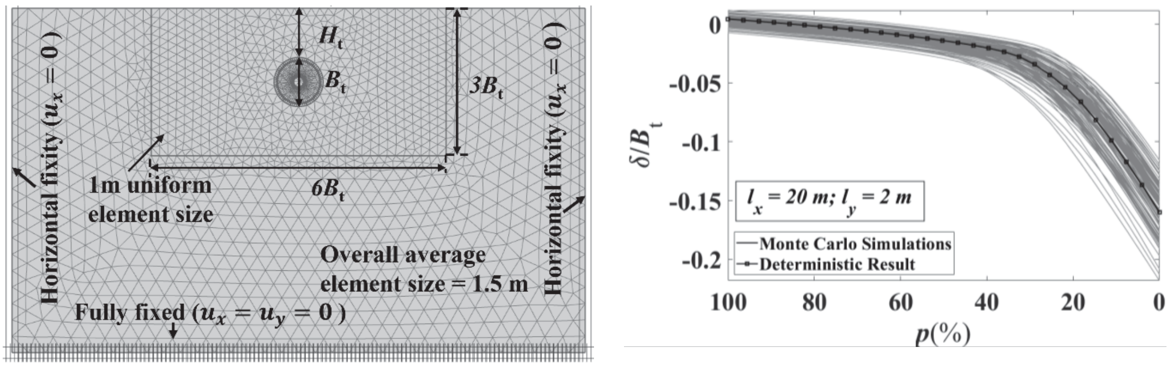
$$R = \begin{pmatrix} 1 & \rho^{\ln}_{c'\tan\phi'} & \rho^{\ln}_{c'\gamma} \\ \rho^{\ln}_{c'\tan\phi'} & 1 & \rho^{\ln}_{\gamma\tan\phi'} \\ \rho^{\ln}_{c'\gamma} & \rho^{\ln}_{\gamma\tan\phi'} & 1 \end{pmatrix} = LL^T \tag{2}$$

Where, $\rho^{\ln}_{c'\tan\phi'}$ between ln c' and ln (tanϕ'), $\rho^{\ln}_{c'\gamma}$ between ln c' and ln γ and $\rho^{\ln}_{\gamma\tan\phi'}$ between ln γ and ln (tanϕ') are the cross- correlation coefficients. These coefficients can be obtained from cross-correlation coefficients, $\rho_{c'\tan\phi'}$, $\rho_{c'\gamma}$, $\rho_{\gamma\tan\phi'}$ respectively (Fenton and Griffiths 2008) which characterize the dependence between the strength parameters. In the present study, $\rho_{c'\tan\phi'}$ is considered -0.5 (Ji et al. 2021), while each of $\rho_{c'\gamma}$ and $\rho_{\gamma\tan\phi'}$ has a value 0.25 (Babu and Srivastava 2007).

Here, ξ_k represents independent standard normal distribution variables, and λ_k and ψ_k denote the eigen values and eigen functions, respectively, of the autocorrelation function represented by Eq.(3). Convergence and accuracy of the expansion rely on M_{KL}. M_{KL} is chosen to ensure that the proportion of retained variance, exceeds 95% where, λ_k are sorted in descending order (Jiang et al. 2015; Pan and Dias 2017).

$$\rho(\Delta x, \Delta y) = \exp\left[-\frac{|x_i - x_j|}{l_x} - \frac{|y_i - y_j|}{l_y}\right] \tag{3}$$

$l_y = l_x$ is for isotropic case while $l_y > l_x$ for anisotropic case. Here, (x, y) refers to the centroidal coordinates of the finite elements. A code is written in MATLAB to generate the random fields.



(a) (b)

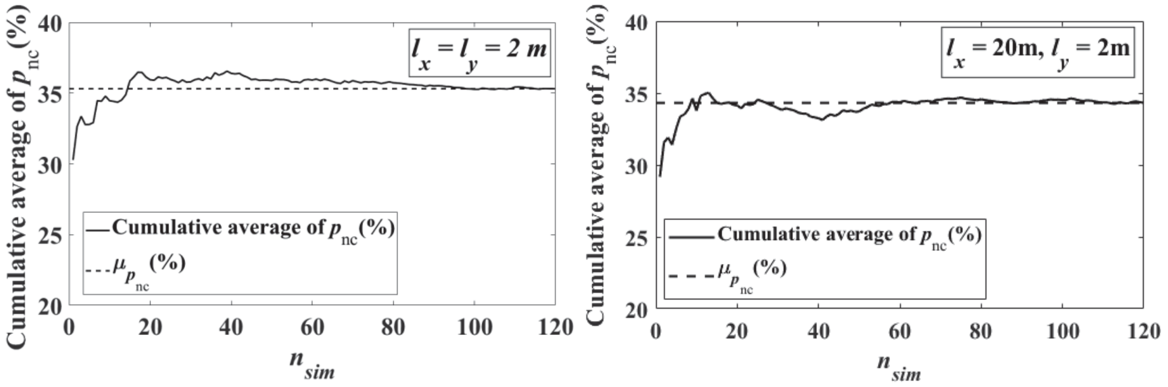
Fig.1 (a) Typical finite element mesh chosen for the analysis (b) normalized applied pressure vs normalized crown deformation plot for the tunnel in homogenous and spatially variable soil.

Tunnel stability is analysed assuming, $\mu_{\tan\phi'}$ = tan 22°, μ_{γ} =16 kN/m^3, $\mu_{c'}$ = 10 kPa each with a 10% coefficient of variation (COV), for two spatial variability cases with l_x/l_y = 1 and 10 with l_y = 2.

3. Results

As, a precursor to random analysis, the deterministic analysis is carried out by assigning the mean c', tan ϕ' and γ to all the elements of the mesh and the p_{nc}^{det} is determined to be approximately 32 %. Thereafter, probabilistic analysis is carried out through Monte Carlo simulations. 120 simulations are found to be sufficient based on cumulative average of the normalized critical support pressure values and computational efficiency (Fig. 2). The isotropic case resulted in $\mu_{p_{nc}}$ of 35.3% (10.3% above deterministic), while the anisotropic case shows 34.3% (7.3% above deterministic). The 95th percentile values are nearly identical at 44% for both cases (27.27% above deterministic). The higher (2.9% difference between isotropic and anisotropic cases) and 95th percentile values compared to deterministic results, indicates that probabilistic analysis provides more conservative results.

Additionally, the distribution of p_{nc}(%) is analysed using a goodness of fit test by fitting the data to a lognormal distribution through the cumulative distribution function (CDF) as shown in Fig. 3 for soil with isotropic and anisotropic correlation length. A Kolmogorov-Smirnov (KS) at 5 % significance level yielded values of 0.108 for the isotropic case and 0.119 for the anisotropic case, both below the critical threshold of 0.124, confirming that the lognormal model is an appropriate fit for the data.

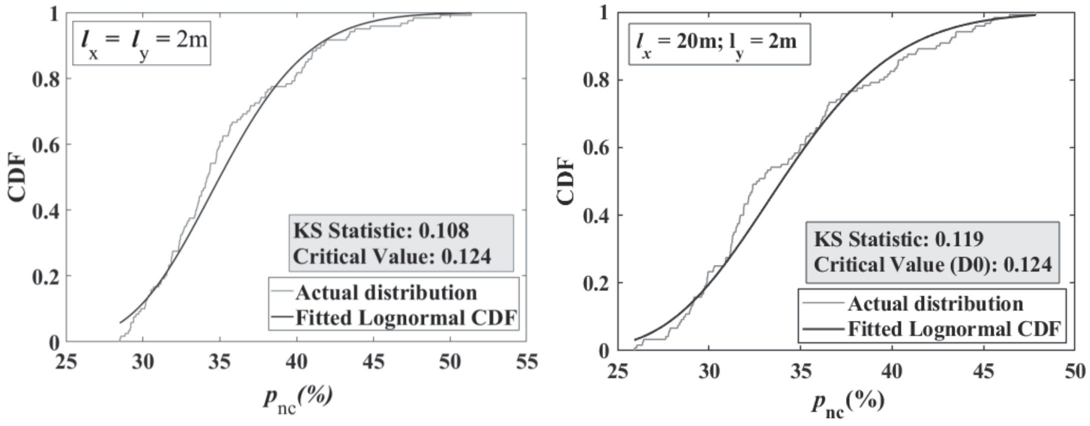


(a) (b)

Fig.2 (a) Plot of cumulative average normalized critical tunnel support pressure vs. number of simulations for (a) isotropic case and (b) anisotropic case.

3.1. *Probability of failure*

The probability of tunnel during excavation can be mathematically expressed as Eq. (5). Monte Carlo simulations estimate P_f by running the numerical model multiple times (n_{sim}), generating values of p^{r}_{nc} (%), and comparing



(a) (b)

Fig.3 Comparison between empirical and fitted distribution of p_{nc} (%) using CDF for (a) isotropic case (b) anisotropic case.

them to the factored $p^{\det}_{nc}$ (%). Here, $N_{failure}$ is the number of times p^r_{nc} (%) is greater than or equal to $p^{\det}_{nc}$ (%), which is defined as a failure event. F_s is the factor of safety. For the present case the calculated P_f for isotropic correlation case is 0.725 and for anisotropic case is 0.567 for $F_s = 1$.

$$P_f = P[p^r_{nc}(\%) \geq p^{\det}_{nc}(\%) \times F_s] = \frac{N_{failure}}{n_{sim}} \tag{5}$$

According to USACE (1997), a target P_f of 0.001 is recommended for geotechnical projects to achieve above-average performance. To meet this target P_f, the required F_s are computed as 1.49 for anisotropic correlation case and 1.61 for isotropic case.

4. Conclusions

Tunnel stability is analyzed assuming, $\mu_{\tan\phi'} = \tan 22°$, $\mu_\gamma = 16$ kN/m^3, $\mu_{c'} = 10$ kPa each with a 10% COV, for two spatial variability cases with $l_x/l_y = 1$ and 10 with $l_y = 2$ m. The study showed that deterministic approaches overestimate tunnel stability during excavation by ignoring spatial variability in soil properties, which can potentially lead to failure consequences. Probabilistic analysis revealed that calculated P_f for isotropic correlation case is 0.725 and for anisotropic case is 0.567 for $F_s = 1$. A significant difference in P_f is observed between the isotropic and anisotropic correlation length cases.

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