

## QUASI-REGION-SPECIFIC MODEL UNCERTAINTIES OF LIQUEFACTION POTENTIAL ANALYSIS

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In earthquake-prone regions, assessing soil liquefaction potential is indispensable for contemporary seismic design. Various procedures for liquefaction triggering analysis have emerged over the past decades. However, most of them are derived from generic liquefaction databases, such that the model uncertainties in liquefaction potential assessments applied to a specific region of concern remain unknown, which poses a challenge for engineers to evaluate the liquefaction risks of target sites. This study aims to propose a hierarchical Bayesian model (HBM) to learn the inter-region characteristics of model uncertainties of the traditional simplified liquefaction potential evaluation methods based on a database containing global case histories of liquefaction categorized into several regions where those triggering events occurred. The learning outcomes can yield the model uncertainty of the target region, and the liquefaction probability at the target site under a given ground motion condition. For an illustration of the proposed model, a case history of liquefaction from a specific region is adopted to construct a quasi-region-specific model uncertainty and evaluate the liquefaction probability in the target soil. The illustration shows that the constructed quasi-region-specific model uncertainty with liquefaction histories in the target region can improve liquefaction occurrence prediction in comparison with the prediction without any histories, which is believed to benefit the engineering practice.

*Keywords:* Soil liquefaction, soil database, hierarchical Bayesian model, liquefaction potential analysis, model uncertainty, quasi-region-specificity.

### 1. Introduction

Assessing soil liquefaction potential is a cornerstone for contemporary seismic design. Diverse approaches for assessing soil liquefaction potential have been developed over the past decades (e.g., Seed and Idriss 1971; Youd et al. 2001; Boulanger and Idriss 2016; Cetin et al. 2018). Among them, the stress-based simplified procedures that exploit the measurements of in situ tests, such as the blows of standard penetration tests (SPT-N), the cone resistance of piezocone penetration tests (CPT- $q_c$ ), and the shear wave velocity ( $V_s$ ) for liquefaction potential evaluations, are the most prevalent in practice and accepted by seismic design codes of various countries.

Although simplified procedures are prevailing in practice, since they do not perform perfectly for predicting the occurrence of liquefaction, their model uncertainties become a further focus of studies in geotechnical engineering (e.g., Juang et al. 2003; Cetin et al. 2018; Moss et al. 2006; Boulanger and Idriss 2016). However, most of the simplified procedures and their probabilistic versions were driven based on generic liquefaction potential databases, such that their quantified model uncertainties are also generic. Namely, the model uncertainties of simplified procedures for liquefaction potential assessments applied to a specific region of concern remain unknown, which poses a challenge for engineers to evaluate the liquefaction risks of target sites.

Therefore, this study aims to propose a hierarchical Bayesian model (HBM) for two objectives. One is to learn the inter-region characteristics of model uncertainties of the traditional simplified liquefaction potential evaluation procedures. The other one is to utilize the learning outcomes to acquire the quasi-region specific model for predicting liquefaction occurrence. This model can yield the quasi-region-specific model uncertainty that incorporates the information contained in a generic database and the target region data. Accounting for quasi-region-specific model uncertainty, this model yields the liquefaction probability of the target site adapted to the concerned region under a given ground motion condition. Developing the aforementioned HBM requires two items: a database containing global case histories of liquefaction categorized into several regions where those triggering events occurred, and a hierarchical model that can accommodate the inter-region characteristics of liquefaction triggering and an indicator to determine whether the soil liquefies. The former is introduced in Sec. 2, and the latter is elaborated on in Sec. 3, respectively.

### 2. Soil Liquefaction Potential Database

A soil liquefaction potential database is compiled in this paper. Most of the data in this database were collected from 22 studies published in scientific journals or investigation reports (e.g., PEER, 2000; Green et al., 2014, Boulanger and Idriss, 2016; Hwang et al., 2021), and the others came from Next Generation database of Liquefaction (NGL) (Ulmer et al., 2023). This database contains 2833 site-investigation records from 36 regions where historical earthquake-induced liquefaction hazards occurred. Those site-investigation data cover 6 parameters that are adopted in common for liquefaction risk evaluation, including the ratio of overburden stress ( $\tilde{\sigma}_v'/\tilde{\sigma}_v'$ ), clean-sand normalized SPT-blows  $((N_1)_{60})$ , normalized cone tip resistance ( $q_{t1N}$ ), soil behavior index ( $I_c$ ), normalized shear wave velocity ( $V_{S1}$ ), and fines content (FC). The distribution of the dataset is shown in Fig. 1 via histograms.

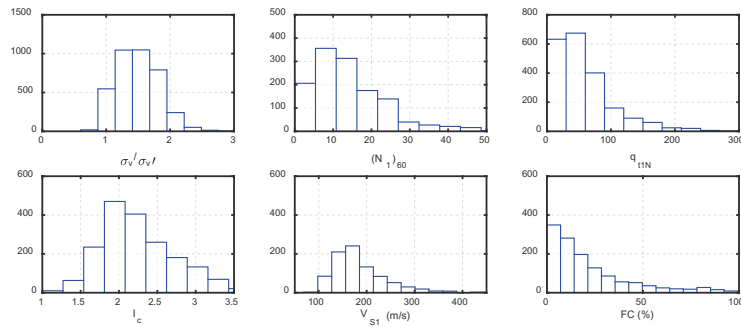


Fig. 1. Histograms of soil data in liquefaction potential database.

Along with these investigation data, the corresponding liquefaction histories (i.e., the soil liquefied or not) given the event conditions (i.e., moment magnitude and peak ground acceleration), are also included in this database. This database is utilized to develop an HBM for region-specific liquefaction risk evaluation.

### 3. Quasi-region-specific Model Uncertainty of Liquefaction Potential Analysis

To capture the statistical uniqueness of each data group in a generic soil/rock database, a hierarchical Bayesian model (HBM) was formulated by Ching et al. (2021). HBM can characterize the inter-group and intra-group correlation of each group of soil/rock property data. The structure of the HBM shown in Fig. 2(a) is a two-level hierarchical tree. The top level contains hyper-parameters ( $\underline{\mu}_0, C_0, \underline{\xi}_0, \tilde{A}_0$ ) which are modeled to characterize the inter-group correlation. The second level comprises the statistical parameters of each group (the mean vector,  $\underline{\mu}_i$ , and the covariance matrix,  $C_i$ ) in the database, which is modeled the intra-group uniqueness. The bottom level is houses the soil/rock data in which are modeled as Gaussian distributed:

$$f(\underline{x}_{ij} | \underline{\mu}_i, C_i) = |C_i|^{-\frac{1}{2}} (2\pi)^{-\frac{n}{2}} \exp\left[-\frac{1}{2}(\underline{x}_{ij} - \underline{\mu}_i)^T C_i^{-1} (\underline{x}_{ij} - \underline{\mu}_i)\right] \quad (1)$$

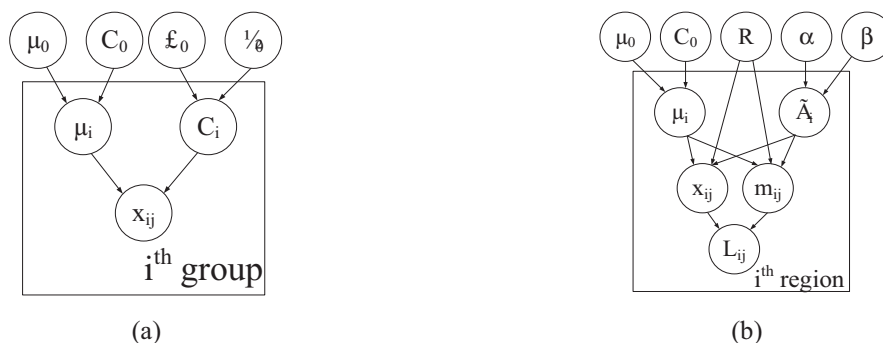


Fig. 2. (a) model structure of HBM (modified from Ching et al., 2021a); (b) model structure of GR-Liq-HBM.

However, HBM cannot accommodate categorical parameters (e.g., liquefied soils or non-liquefied soils), such that it cannot learn the data group uniqueness of liquefaction triggering in our liquefaction potential database. As it transpired, this paper proposes a modified HBM considering a “group” as a “region” for liquefaction potential analysis. This model is called as “GR-Liq-HBM”. Its model structure is shown in Fig. 2(b). The main modifications can be enumerated below:

- (i) A model factor (denoted by  $m$ ) modelled log-normally distributed is introduced into GR-Liq-HBM.  $m_{ij} = FS_a/FS_c$ , where  $FS_a$  is the actual value of anti-liquefaction factor of safety (FS) which indicates the soil actually liquefies or not.  $FS_c$  is the computed FS via a deterministic liquefaction potential analysis procedure.
- (ii) An indicator ( $L_{ij}$ ) is also introduced in GR-Liq-HBM.  $L$  can be derived from the  $FS_c$  incorporated with  $m$ :  $P(L_{ij}|FS_{c,ij},m_{ij}) = (FS_{c,ij} \times m_{ij} < 1)^{L_{ij}} (FS_{c,ij} \times m_{ij} \geq 1)^{1-L_{ij}}$ .  $L_{ij} = 1$  as the soil liquefies, whereas  $L_{ij} = 0$  as the soil does not liquefy.
- (iii) The region-specific variance of soil properties ( $x_{ij}$ ) and the model factor ( $m_{ij}$ ) are modelled separately in GR-Liq-HBM. It is worth noting that the values of model factors are unobservable in reality, whereas the soil properties can be acquired via site investigation, such that the uncertainties of soil properties and the model factor can be significantly different, which cannot be considered by the original HBM. This is because the original HBM models the variance of soil properties and the model factor in one covariance matrix which is governed by one parameter  $\tilde{\Lambda}_0$ .
- (iv) Due to the independence of modelling the variance of soil properties ( $x_{ij}$ ) and the model factor ( $m_{ij}$ ), the region-specific covariance matrix should be modified as follows:

$$\mathbf{C}_i = \mathbf{S}_i \mathbf{R} \mathbf{S}_i = \begin{bmatrix} \sigma_{i,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{i,m} \end{bmatrix} \begin{bmatrix} 1 & \cdots & \rho_{1,m} \\ \vdots & \ddots & \vdots \\ \rho_{m,1} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \sigma_{i,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{i,m} \end{bmatrix} \quad (2)$$

where  $\mathbf{S}_i$  is a diagonal matrix composed by the  $i$ -th region-specific standard deviation (denoted by  $\tilde{\Lambda}_{i,k}$ ,  $k = 1, 2, \dots, 6$ ) of each uncertain variables of  $x_{ij}$  and  $m_{ij}$ ;  $\mathbf{R}$  is the correlation matrix of  $x_{ij}$  and  $m_{ij}$ . The region-specific variance  $\tilde{\Lambda}_{i,k}^2$  are inverse-gamma distributed with the shape parameters  $\pm_k$  and the scale parameters  $^2_k$  respectively.  $\mathbf{R}$  is modelled as an inter-region parameter and denotes a correlation matrix universally applicable across all regions in the database. It is modelled following LKJ distribution (Lewandowski et al. 2009):  $\mathbf{R} \sim \text{LKJ}(\cdot = 1)$ .

Based on the model settings mentioned above, the quasi-region-specific model uncertainty of a simplified liquefaction triggering analysis procedure applied to the region of concerns can be obtained by Gibbs samples from the following posterior PDF:

$$f(\mu_m, \sigma_m^2 | \Theta, \mathbf{D}_g, \mathbf{D}_R) \propto f(\Theta | \mathbf{D}_g) f(\mu_m, \sigma_m^2 | \Theta) f(\mathbf{D}_R | \mu_m, \sigma_m^2) \quad (3)$$

where  $\tilde{\cdot} = \{\mu_0, \mathbf{C}_0, \pm, ^2, \mathbf{R}\}$  which is learned from  $\mathbf{D}_g$  = the generic liquefaction database;  $\mathbf{D}_R$  denotes the data of the liquefaction histories of the target region;  $\mu_m$  and  $\tilde{\Lambda}_m^2$  are the mean value and the variance of the quasi-region-specific model factors, respectively.

#### 4. A Real Case Illustration

A site located in North Kaiapoi, Canterbury, New Zealand is indicated to be a liquefied site given the shock of the 2011/2/22 Canterbury earthquake (Shen et al. 2016). For demonstration and validation, let us consider this liquefaction information is unknown, and a construction project aims to evaluate the liquefaction potential of this site. Its critical liquefiable layer is located at 3.5-4.5m in depth. The CPT data of that stratum: the average cone resistance  $q_c$  H 5.55MPa, the average sleeve friction  $f_s$  H 0.028MPa, and the soil behavior index  $I_c$  H 1.83 (Shen et al. 2016). Let us consider the scenario the 2011/2/22 Canterbury earthquake occur: the peak ground acceleration H 0.18g, and the moment magnitude H 6.3. Based on the above information, the  $FS_c$  can be evaluated via a CPT-based simplified procedure (Juang et al. 2003) H 1.45, which says that the stratum of concerns does not liquefy (but in fact it liquefied). Now the model uncertainty of the liquefaction potential should be further inferred.

To infer the concerned model uncertainty, the project engineer can perform the proposed model in either two options: conditioning liquefaction history data of other sites in this region, or conditioning no liquefaction information. These two conditioned inference results of model uncertainty via the proposed model along with

the Gibbs sampler are shown in Fig. 3: the posterior mean of the quasi-region-specific model factor conditioned on the regional liquefaction histories is evidently lower than zero, which points out that the bias of the liquefaction potential evaluation in this target region exists.

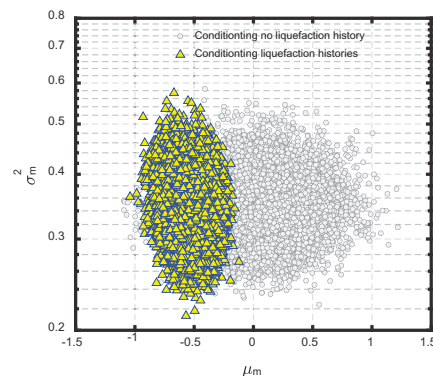


Fig. 3. Quasi-region-specific model uncertainty.

The posterior samples of the model factor of the target stratum can yield the liquefaction probability ( $P_L$ ) of the layer by evaluating the sample mean of posterior samples of  $\mathbf{1}(m_{ij} \times FS_{c,ij} < 1)$  where  $\mathbf{1}(\cdot)$  is the indicator function. The evaluation of  $P_L$  conditioning regional liquefaction histories is 0.642, whereas the one conditioned no region histories is 0.326. The estimated  $P_L$  by the traditional probabilistic model (Juang et al., 2003) is 0.016. It is evident that only the one by the GR-Liq-HBM with regional liquefaction histories is consistent with the true observation.

## 5. Conclusion

This paper aims to propose a hierarchical Bayesian model, GR-Liq-HBM, for data-centric liquefaction triggering analysis. The model characterizes the inter-region and the intra-region correlations between the soil properties and the model uncertainties of the simplified procedures for liquefaction triggering analysis. The characterizing outcomes can assist engineers yielding the model uncertainty of the target region adaptively, and the liquefaction probability of the target site under a given ground motion condition. Also, the model can accommodate the regional liquefaction history data and the target site investigation data to infer the quasi-region-specific model uncertainty and the liquefaction probability of the target site.

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