

PROBABILISTIC SETTLEMENT PREDICTION FOR NEIGHBOURING FOOTINGS AT DIFFERENT SPACING DISTANCES IN ROTATED SPATIAL ANISOTROPIC MULTI-LAYERED SOIL

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This study develops a probabilistic settlement prediction model for neighboring footings in multi-layered soil under a rotated anisotropic spatial variability. The Random Finite Element Method (RFEM) is applied to predict settlement behavior over two layers, a strong clay layer overlying a weak clay layer, combining spatial uncertainty and anisotropic properties. Anisotropic random fields are introduced as rotated with various orientation angles and fluctuation scales to simulate variable soil conditions. Using the Mohr-Coulomb failure criterion within a Monte Carlo simulation, the settlement response is evaluated, modelling friction angle (ϕ) and stiffness (E) as positively cross-correlated random variables employing Cholesky decomposition techniques with log-normal distribution patterns. The results show significant variability in settlement behavior due to rotated anisotropic fields, with the coefficient of variation (COV) increasing along rotation angles while mean settlement remains stable. These findings highlight the importance of anisotropic properties and random field orientations in predicting settlement for spatially variable layered soils subjected to different spacing of neighboring footing.

Keywords: Neighboring footing; Settlement; RFEM; Rotated Anisotropy Random Field

1. Introduction

Although probabilistic methods, soil anisotropy, and footing interactions have each been studied independently, their combined effects on foundation settlement are still insufficiently studied (Sekhar et al., 2020). Most studies address one or two aspects, such as spatial variability or anisotropy, without integrating them into a comprehensive framework. In practice, interactions between neighboring footings, the orientation of the soil profile, and the spatially variable nature of soils often lead to non-uniform settlement patterns, significantly impacting engineering decision-making (Ghazavi et al., 2021; Jamshidi Chenari et al., 2019). The study by (Wang et al., 2020) emphasized that the lack of integrated approaches can lead to conservative or unsafe designs, underscoring the urgent need for research into these interconnected phenomena. Most studies simplify soil modelling by assuming horizontally or vertically stratified layers, yet actual soil layers are often perfectly horizontal nor vertical (Cami et al., 2020; Lloret-Cabot et al., 2014). To address this, the current study proposes a probabilistic framework incorporating the effects of rotated anisotropic random fields where the SOF varies in the horizontal and vertical directions according to soil layer orientation (Cao et al., 2024; Griffiths and Fenton, 2009; Luo and Luo, 2021). The framework considers a range of rotation angles to simulate varying orientations of the anisotropic field relative to the major and minor principal direction axes. In this study, the random variables of the young modulus (E) and friction angle (ϕ) are modelled under the Mohr-Coulomb failure criterion, with a positive cross-correlation ($\rho = +0.5$) assumed between them. This implies that increases in E correspond to increases in ϕ and vice versa. Both variables follow a log-normal distribution (Fenton & Griffiths, 2002; Kawa et al., 2021).

2. Research Methodology and Tools

The footings were modelled as linear elastic concrete strips with a width (B) of 1.2m, subjected to a total load of $Q=400$ kN/m. To minimize boundary effects on settlement behavior, the soil domain was set to extend 20m in width and 10m in depth, as shown in Fig. 1. The soil model consisted of a double-layer structure, with a 3m strong layer overlying a 7m weak layer. The interface between soil layers was modelled by accounting for the elastic stiffness in the soil-to-soil interaction, determined by the E and ϕ properties of adjacent soil elements (Fig. 1). The soil parameters were assumed based on (Zaskórski et al., 2017). Table 1 shows the details of the soil parameters devised on a two-layered model.

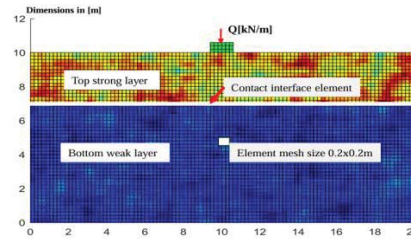


Fig. 1.: Geometry boundary model of the study problem at isolated footing

Table 1: Summary of soil parameters assumed in the study.

No	Parameter	Symbols/ Units	Random variable	Top strong clay layer	Bottom weak clay layer	Coefficient of variation (COV)	Probability distribution
				Mean	Mean		
1	Friction angle (ϕ)	[$^\circ$]	Random	21	10	10%	Log-normal
2	Young modulus (E)	[MPa]	variable	35	15		
3	Unit weight	[kN/m ³]		21	20	Deterministic	
4	Cohesion	[kPa]	-	38	20		
5	Poisson ratio	ν [-]		0.3	0.3		

Spatially correlated E and ϕ random fields were generated using the Fourier series method and Cholesky decomposition to model spatial variability. These random fields were oriented at various angles (β) to simulate rotations of major and minor principal directions in spatial correlation lengths. This approach helps us understand how changes in the orientation of principal directions affect the spatial correlation anisotropic properties, which influence settlement response (Fig. 2a). Fig. 2 also illustrates random field rotations at 0° (Fig. 2b) and 45° (Fig. 2c) for major correlation lengths $SOFx=1.0m$ and minor $SOFy=10.0m$. Therefore, the layered and anisotropic soil structure is expected to interact between neighboring footings at various distances (e.g., isolated, 1B, 2B, 3B). The analysis assumed the following scales as a study parameter: major $SOFx = 1, 5, 10, 50$ and 100 m, minor $SOFy = 1.0$ and 10 m with varying orientation angles ($\beta= 0^\circ, 5^\circ, 10^\circ, 15^\circ, 20^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, \text{ and } 90^\circ$).

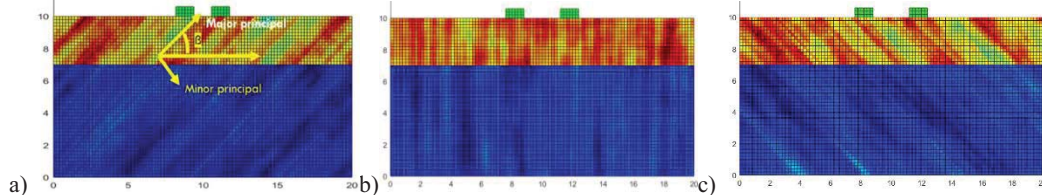


Fig. 2.: Random field rotation (a) major and minor principal direction of SOF of the anisotropic random field at β orientations; rotation at specific major $SOFx=1.0m$ and minor $SOFy=10m$ (b) for $\beta=0^\circ$; (c) for $\beta=45^\circ$

3. Results and Discussion

Monte Carlo simulations of 500 realizations assessed the mean and COV of settlement response across various major correlation lengths and random field orientations (Fig. 3), revealing that minor rotations (e.g., 5° to 20°) produce slightly similar settlement variations as non-rotated fields, spatially in case of mean vertical settlement (Fig. 3a). However, the convergence stability of COV along the major (horizontal) SOF was consistent from $SOFx=10m$ to infinity for both rotated and non-rotated anisotropic fields (Fig. 3b). Assuming a rough contact interface in the multi-layer soil model generally underestimates settlement.

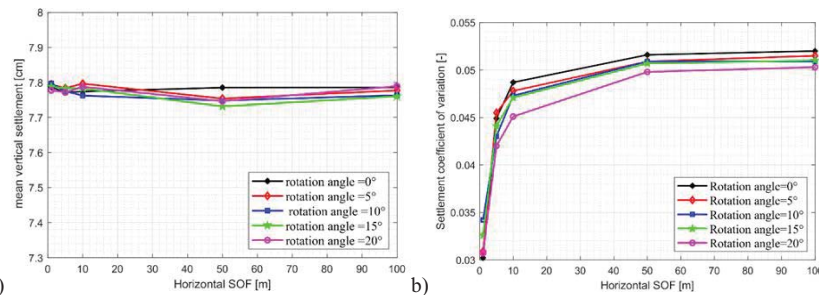


Fig.3.: Vertical settlement at rotated anisotropic random field for vertical (minor) $SOFy=1.0m$ and horizontal (major) SOF as a study parameter: a) Mean b) COV

Results in Fig. 4(a) indicate that mean vertical settlement is strongly affected by footing proximity. Across all spacing distances between footings, random field rotation angles (0° to 90°) for $SOF_x=10.0\text{m}$ and $SOF_y=1.0\text{m}$ have minimal impact on mean settlement. However, Fig. 4(b) shows that settlement COV is more sensitive to both footing spacing and random field rotation angles, suggesting that anisotropic orientations influence the variability of settlement response more than mean settlement. It is noteworthy that the largest settlements are observed for foundations located closest to each other (1B), as shown in Fig. 4a. Foundations with 3B-spacing behave in the range of the mean of settlements as single ones. The analysis of the COV shows that the single foundation demonstrates the greatest variability as well as widely spaced adjacent foundations (Fig. 4b). An interesting fact is that the smallest COV was observed for the closest located footings.

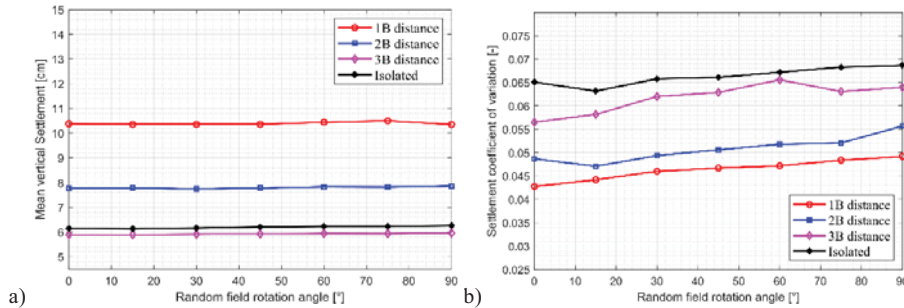


Fig.4.: Effect of random field rotation angle ($^\circ$) on (a) mean and (b) COV for various spacing distances of neighboring footings, with horizontal $SOF_x = 10\text{ m}$ and vertical $SOF_y = 1.0\text{ m}$.

The analysis of the 2B-spaced neighbors' footings was adopted for further analysis to represent adequate load distribution and interaction influence instead of too small or large spacing distance. Therefore, the study demonstrated in (Fig 5) that the random field configuration behaves differently with a change in the rotation angle, yielding a statistical response of settlement means and COV, as shown in Fig 6.

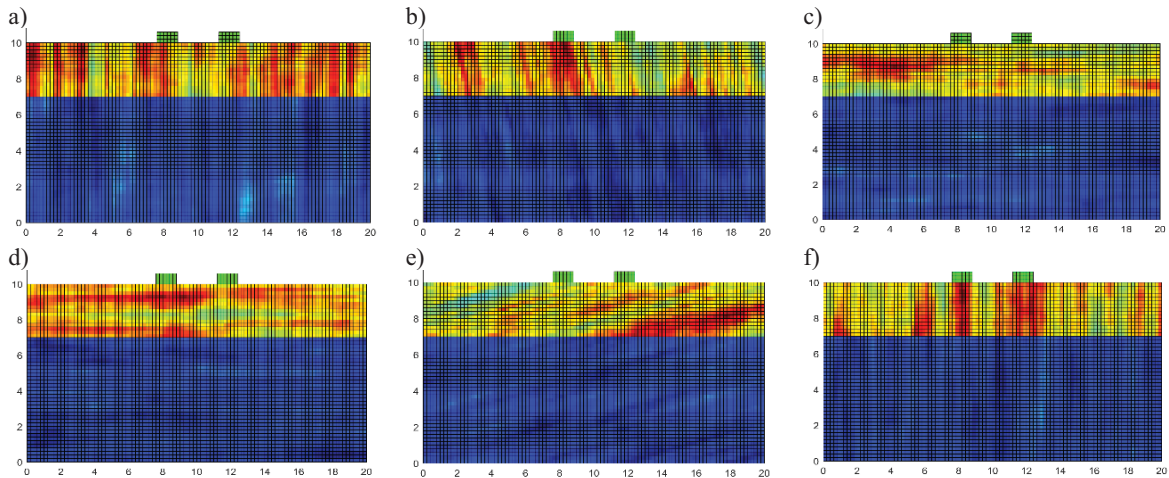


Fig. 5.: Effect of rotation on the random field configuration for neighboring footings spaced at 2B. Configurations shown are: with $SOF_x=1\text{ m}$, $SOF_y=10\text{ m}$ (a) 0° , (b) 15° , (c) 90° , with $SOF_x=10\text{ m}$, $SOF_y=1\text{ m}$ (d) 0° , (e) 15° , and (f) 90° .

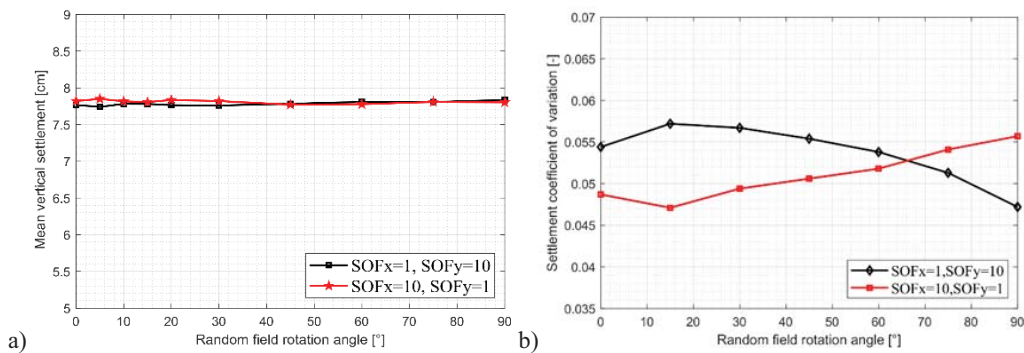


Fig.6.: Effect of random field rotation of vertical settlement response for neighboring spaced at 2B with two correlation length scenarios (a) mean (b) COV

The statistical response of settlement behavior under-rotated anisotropic random fields was analyzed by varying the fluctuation scale lengths from 0° (horizontal, major principal direction) to 90° (vertical, minor principal direction) with two configurations: $\text{SOF}_x = 10\text{m}$, $\text{SOF}_y = 1.0\text{m}$, and $\text{SOF}_x = 1.0\text{m}$, $\text{SOF}_y = 10\text{m}$ (Fig.5 and Fig.6). The mean vertical settlement demonstrates relative stability across various random field rotation angles for major and minor SOF orientation, as depicted in Fig. 6(a) for all Fig.5. However, Fig 6(b) illustrates two distinct patterns of discovered anisotropic random field configurations at 0° (Fig. 5(a) and (d)) and 90° rotations (Fig. 5(c) and (f)). The results reveal that for small major SOF_x (Fig. 5 (a-c)) and the black line in Fig.6(b) COV increases from perfectly horizontal (0°) to local maximum (15°) and then decreases till perfectly vertical (90°) random field configurations. Otherwise situation is observed for major $\text{SOF}_x=10\text{m}$ (Fig. 5(d-f) and the red line in Fig.6b). The COV values for the perfectly horizontal (0°) and vertical (90°) orientations establish the lower and upper bounds of the statistical variability for the rotated random fields. Fig.6(b) shows that the highest COV occurs at 90° for the $\text{SOF}_x=10\text{m}$, $\text{SOF}_y=1.0\text{m}$ configuration. On the contrary, a high COV is observed at 15° when the horizontal correlation length is reduced to $\text{SOF}_x=1.0\text{m}$, and the vertical length is increased to $\text{SOF}_y=10\text{m}$. These simulations demonstrate the significant impact of random field orientation on settlement variability, indicating that the vertical and horizontal spatial correlation lengths assumed in previous studies may represent the lower and upper bounds of settlement behavior in non-horizontal or non-vertical soil profiles.

4. Summary and Conclusions

This study investigated the impact of rotated anisotropic random fields of E and ϕ on settlement behavior under different scenarios of major and minor SOF in multi-layered soil. As presented in the study, the probabilistic analysis of the task significantly changes the understanding of the problem of settlement. The mean value is substantially constant and uniform regardless of the assumed fluctuation scale scenario (Fig.3a and Fig. 6a) or the rotation angle (Fig.4a and Fig.6a). The inclusion of a probabilistic approach is disclosed in the analysis of the coefficient of variation which exhibits sensitivity to the scale of fluctuations (Fig. 3b and Fig. 6b) and the angle of rotation (Fig. 4b and Fig. 6b). In addition, the work stated that footing spacing affected settlement variability, with closer spacing (1B) indicating higher mean settlement but minimum variability (lower COV). To this end, neighboring footings showed lower COV than isolated footings, indicating reduced variability yet increased mean settlement with closer spacing. An interesting case seems to be the rotation angle of 15° . Local maximum and minimum were observed here, depending on the adopted scale scenario. A similar conclusion appeared, among others, for example in the habilitation thesis by Kawa (2024), that the angle of 15° gave the most sensitive results in various geotechnical analyses.

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