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Influence of Anisotropic Soil Spatial Variability in the Horizontal Plane on 3D Slope Reliability

Divya Varkey[1], Michael A. Hicks[2] and Philip J. Vardon[3]

Section of Geo-Engineering, Faculty of Civil Engineering and Geosciences,
Delft University of Technology, Delft, The Netherlands.
[1]D.Varkey@tudelft.nl
[2]M.A.Hicks@tudelft.nl
[3]P.J.Vardon@tudelft.nl

Abstract: The influence of spatial variability in shear strength properties on the probabilistic stability of an idealised 3D slope has been investigated using the random finite element method. Considering the method of construction of long 3D slopes, such as dykes and embankments, different scales of fluctuation of shear strength properties in the two horizontal directions have been modelled using random fields. The results indicate that considering this anisotropy can have a significant influence on the probabilistic characteristics and potential failure volumes of a 3D embankment slope. It was observed that the spatial correlation of properties along the embankment length has a much greater influence than the spatial correlation of properties perpendicular to the length.

Keywords: random fields; slope stability; spatial variability; three dimensional.

1 Introduction

A complicating factor in the analyses of geotechnical structures is accounting for uncertainties, especially the uncertainty due to the inherent spatial variability of soil (Phoon and Kulhawy 1999) arising from a combination of various geological, environmental and physico-chemical processes, among others. The presence of this heterogeneity can significantly influence results and needs to be properly modelled, for example, by using the random finite element method (RFEM) (Fenton and Griffiths 2008). By combining random fields of soil properties with finite elements, this method does not make any prior assumptions regarding the location and shape of the failure mechanism. Much research has been done on using RFEM to investigate the influence of heterogeneity in soil shear strength on the stability of 2D slopes (for example, Hicks and Samy (2002, 2004), Griffiths and Fenton (2004), Hicks and Onisiphorou (2005), Griffiths et al. (2009a), Huang et al. (2010) and Vardon et al. (2016)). Hicks et al. (2019) and Varkey et al. (2020) illustrated the advantages of incorporating spatial variability of soil properties in the reliability-based stability assessment of a regional dyke in the Netherlands. Research has also been carried out to investigate the influence of 3D spatial variability of soil properties in the reliability-based assessment of slopes (Hicks et al. 2008; Griffiths et al. 2009b; Hicks and Spencer 2010; Huang et al. 2013; Hicks et al. 2014; Ji and Chan 2014; Hicks and Li 2018; Varkey et al. 2019), although this research has been limited due to computational requirements.

A challenging task in uncertainty modelling is estimating the scale of fluctuation of the material properties (i.e. the length over which the property values are significantly correlated), especially in the horizontal plane, as it often requires a large number of CPTs to be placed strategically (Lloret-Cabot et al. 2014; Ching et al. 2018; de Gast et al. 2021b). Therefore, an idealisation in 3D analyses in the literature has been to assume the same scale of fluctuation of the shear strength properties for all directions in the horizontal plane. However, a detailed investigation of horizontal scales of fluctuation below a Dutch regional dyke (de Gast et al. 2021b) revealed the scale of fluctuation along the dyke to be higher than across the dyke. Another typical idealisation in reliability-based analyses of 3D slopes has been to assume the embankment geometry and foundation layering to be deterministic for assumed 'uniform' sections in the longitudinal direction, although it is observed in practice that variations do occur in the third dimension.

Considering the above, Varkey et al. (2022) investigated the influence of three forms of uncertainty on the probabilistic stability of a 3D embankment slope: 1D spatial variability in the external geometry of the slope along its length, 2D spatial variability in the depth of the boundary between the embankment material and the foundation layer, and 3D spatial variability in the shear strength properties of the slope and foundation materials. Based on a detailed investigation using RFEM, they observed that the soil spatial variability had a much greater influence than uncertainties relating to embankment geometry and inter-layer boundary when using realistic variations. Furthermore, they demonstrated that the horizontal spatial correlation of properties along the embankment length had a greater influence on its probabilistic characteristics and potential failure lengths than the horizontal correlation length across a dyke cross-section and, based on the results, proposed a worst-case scenario. This paper provides a further RFEM demonstration of the influence of anisotropy in the horizontal spatial correlation of properties in predicting embankment response, particularly with respect to distributions of factors of safety and failure volumes.

2 Random finite element method

RFEM combines random fields with finite elements within a Monte Carlo framework. The random fields are used to model the spatial variability of soil properties and an ensemble of random fields are used to represent the uncertainty in the spatial distribution of soil properties. In this paper, the continuous random fields have been discretised as spatial averages using the Local Average Subdivision (LAS) method (Fenton and Vanmarcke 1990). The random fields are here generated using the Markov covariance function:

$$\beta_{\mathrm{M}} = \sigma^2 \exp\left(-\frac{2\tau_z}{\theta_z} - \sqrt{\left(\frac{2\tau_x}{\theta_x}\right)^2 + \left(\frac{2\tau_y}{\theta_y}\right)^2}\right) \quad (1)$$

where τ_x, τ_y and τ_z are the lag distances, and θ_x, θ_y and θ_z are the scales of fluctuation in the respective directions.

The method of random field generation follows Hicks and Spencer (2010), where the separation of the vertical (z) correlation structure from the two horizontal (x and y) directions was done to model the long-term depositional characteristic in the soil. It begins with generating isotropic fields, i.e. with $\theta_x = \theta_y = \theta_z$ in Eq. (1), followed by squashing and/or stretching the isotropic fields to generate the required degree of anisotropy. For example, to generate a field with θ_x = 10 m, θ_y = 100 m and θ_z = 1 m, an isotropic field may be initially generated with θ = 10 m and then squashed and stretched by a factor of 10 in the z and y directions, respectively.

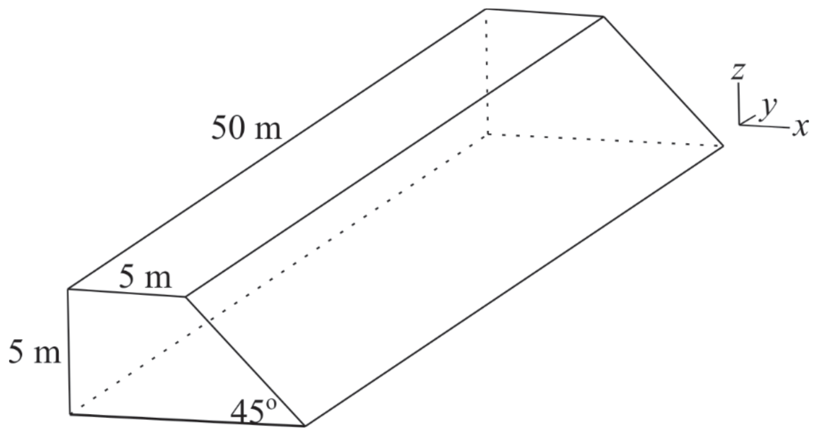


Figure 1. Geometry of the problem

Table 1. Soil parameter values

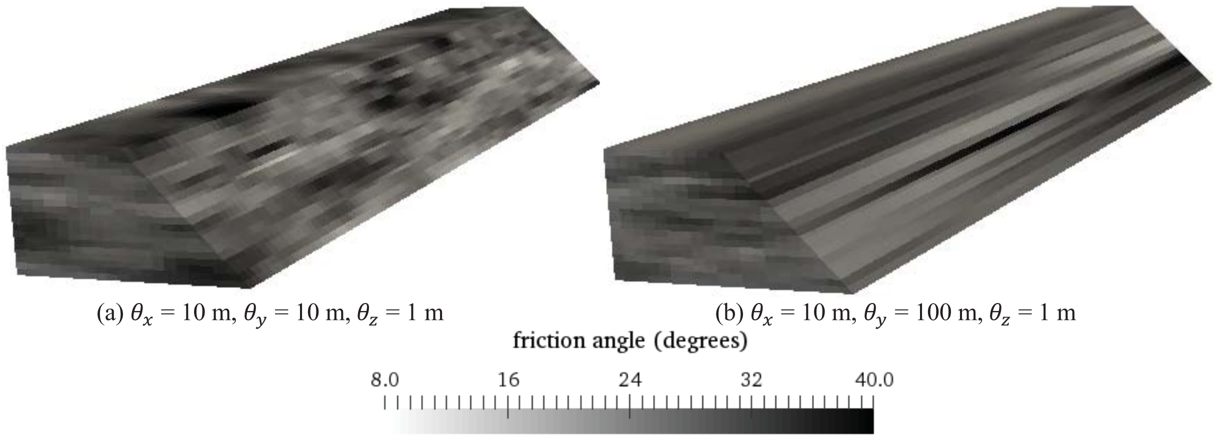
Parameter	Mean value	COV
Cohesion (c')	10 kPa	0.2
Friction angle (φ')	25°	0.2
Dilation angle	0°	-
Young's modulus	10^5 kPa	-
Poisson's ratio	0.3	-
Unit weight	20 kN/m^3	-

3 Problem description

An idealised 45° slope, as shown in Figure 1, has been analysed in this paper. The slope was meshed with a total of 4000, 20-node, regular hexahedral elements of approximate size 0.5 m in depth and 1 m×1 m in plan (except along the slope face), and using a 2×2×2 Gaussian integration scheme. This discretisation was chosen to ensure adequate characterisation of the spatial variability for the smallest scale of fluctuation considered (Spencer 2007). The mesh was fixed at the base, with 2D rollers on the y-z face preventing movement perpendicular to the face, and 1D rollers on the two x-z faces allowed movement only in the z direction. The end-faces were fixed against horizontal movement, as Spencer (2007) and Hicks and Li (2018) found that allowing horizontal movement on the end-faces appeared to result in a bias of failures congregating towards the ends of the slope; this was thought to be due to the implied symmetry of the random field about the mesh end boundaries. The used soil parameters are summarised in Table 1. A coefficient of variation (COV) of 0.2 was used for the shear strength parameters (cohesion c' and friction angle φ') and the other parameters were considered to be deterministic. Due to the relatively low values assumed for the COVs, normal distributions were considered adequate for modelling the uncertainty in the shear strength parameters. Note that uncorrelated random fields for c' and φ' were generated.

Hicks and Spencer (2010) and Hicks et al. (2014) analysed a slope similar to the slope in Figure 1 but with a different set of properties. Based on a detailed RFEM investigation using various horizontal correlation lengths (θ_{h}), they proposed the following three categories of failure mode. For a small θ_{h} relative to the slope height and for very large values of θ_{h} relative to the slope length, referred to as Mode 1 and Mode 3, respectively, the mean value of *F* using RFEM approached the deterministic *F* based on the mean property values. Since the failure mechanism is influenced by the geometry and size (as well as the relative distribution) of weak and strong zones, Mode 1 has a narrow range of solutions whereas Mode 3 has a large range of solutions. For intermediate values of θ_{h}, Mode 2 failures are characterised by discrete three-dimensional failures that tend to pass through semi-continuous weaker zones and thereby result in a relatively lower mean value of *F*. Mode 2 failures are therefore the most critical mode of failure. Varkey et al. (2019) analysed a slope with the same geometry and properties as in this paper and observed that the results obtained were also consistent with these three failure modes.

The above observations were based on the results of an RFEM investigation using isotropic random fields in the horizontal plane (i.e. with $\theta_{\mathrm{h}} = \theta_x = \theta_y$). However, different scales of fluctuation in the two horizontal directions have been modelled here, in line with the field observations of de Gast et al. (2021b). Figure 2 shows two typical realisations of 3D random fields of φ' analysed in this paper.



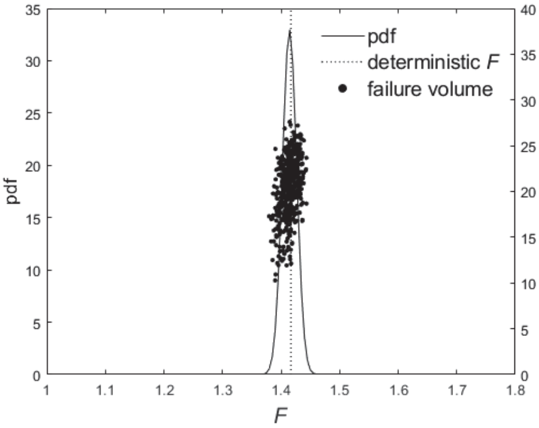
(a) θ_x = 10 m, θ_y = 10 m, θ_z = 1 m (b) θ_x = 10 m, θ_y = 100 m, θ_z = 1 m

Figure 2. Typical realisations of random fields of friction angle generated in the paper

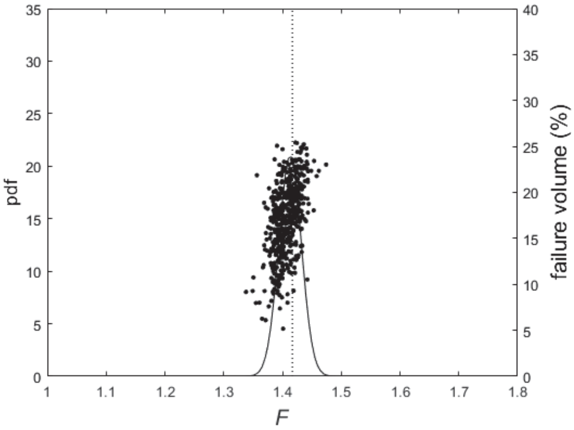
4 Results and discussion

The slope in Figure 1 was first analysed using the mean strength parameters in Table 1. This resulted in a deterministic factor of safety (*F*) of 1.417. Using a vertical scale of fluctuation (θ_z) of 1 m (de Gast et al. 2021a,b), 500 Monte Carlo realisations were then carried out for different combinations of θ_x and θ_y. Due to the general lack of measured field data for the horizontal correlation lengths, a wide range of values of θ_x and θ_y were considered. The probability density functions (pdfs) obtained by fitting normal distributions to the values of *F* obtained using RFEM are shown in Figure 3. In addition to computing *F* for each realisation, the slide volumes have also been computed based on the number of elements having an average out-of-face displacement more than a calibrated threshold value, as is described in Hicks et al. (2014). For the problem analysed in this paper, this was 37% of the maximum computed out-of-face displacement. The obtained failure volumes, expressed as a percentage of the total mesh volume, have also been plotted in Figure 3 against the corresponding values of *F*. Also shown in the figure is the deterministic value of *F* based on the mean strength parameters. The mean values of *F* and failure volumes averaged over all the realisations are listed in Table 2.

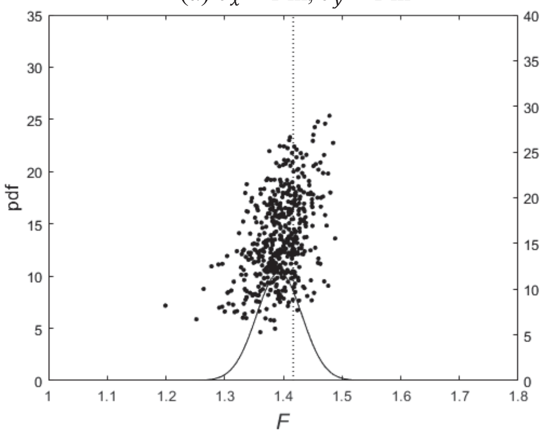
As can be seen from the figure, the pdfs of *F* and the failure volumes obtained with anisotropic horizontal soil variability are significantly different compared to those based on isotropic horizontal soil variability. The responses are more influenced by a change in θ_y than in θ_x. Of the six cases analysed in this paper, the ones with θ_y = 10 m resulted in a significantly lower mean value of *F* than the deterministic *F*, as well as a significantly lower mean failure volume. This intermediate value of θ_y compared to the slope length is therefore the critical one, as there is then more tendency for failure mechanisms to pass through local weaker zones. The results also demonstrate that a conservative solution is obtained by assuming isotropic variability in the horizontal plane based on this critical value of θ_y. With an increase in the value of θ_y, the range of possible solutions increases, as is reflected by a higher standard deviation of *F*.



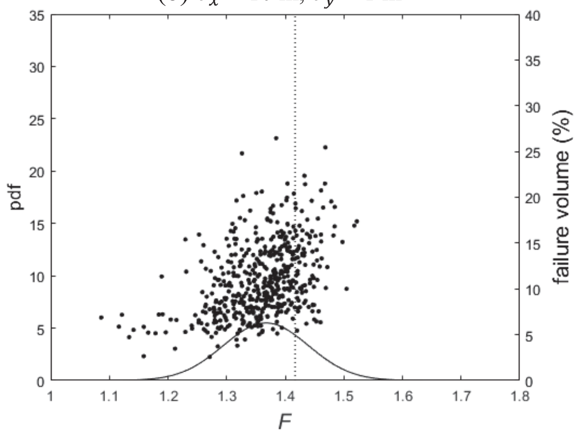
(a) θ_x = 1 m, θ_y = 1 m



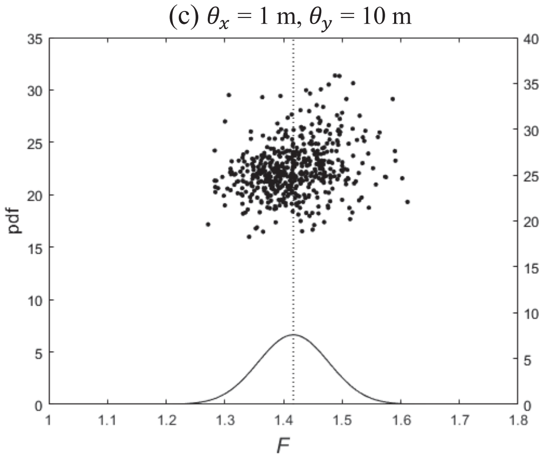
(b) θ_x = 10 m, θ_y = 1 m



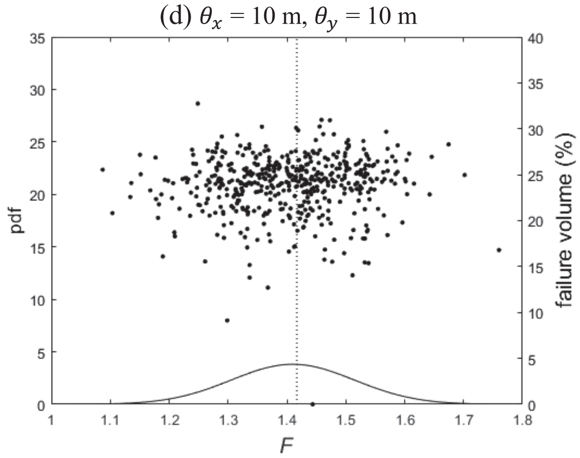
(c) θ_x = 1 m, θ_y = 10 m



(d) θ_x = 10 m, θ_y = 10 m



(e) θ_x = 1 m, θ_y = 1000 m



(f) θ_x = 10 m, θ_y = 1000 m

Figure 3. Pdfs of factor of safety and failure volumes obtained using RFEM with $\theta_z = 1$ m

The means and standard deviations of F, as well as mean discrete failure lengths, obtained from the RFEM analyses of slopes with different cross-sectional geometries and by considering a much wider range of θ_x and θ_y, are presented in Varkey et al. (2022). Based on a detailed investigation, they observed that a conservative solution is generally obtained by assuming isotropic variability in the horizontal plane based only on θ_y. For those cases in which an accurate knowledge of θ_y is not available, a worst case value of θ_y was also proposed.

Table 2. Mean values of F and failure volumes obtained using RFEM with θ_z = 1 m

	θ_x = 1 m, θ_y = 1 m	θ_x = 10 m, θ_y = 1 m	θ_x = 1 m, θ_y = 10 m	θ_x = 10 m, θ_y = 10 m	θ_x = 1 m, θ_y = 1000 m	θ_x = 10 m, θ_y = 1000 m
Mean F	1.415	1.413	1.393	1.367	1.417	1.410
Mean failure volume (%)	20.4	19.2	15.5	11.1	25.6	23.9

5 Conclusions

RFEM has been used in this paper to investigate the influence of anisotropy in the soil spatial variability in the horizontal plane on the probabilistic characteristics and potential failure volume of a 3D embankment slope. The results demonstrate that the spatial correlation of properties along the embankment length (θ_y) has a much greater influence than the spatial correlation of properties perpendicular to the length (θ_x). It was observed that intermediate values of θ_y relative to the embankment length were the critical ones, due to the tendency for failures to be attracted to weaker zones.

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