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Three-Dimensional Cross-Correlated Random Field Modelling Based on Hierarchical Archimedean Copulas

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Abstract: The spatial variability of geotechnical parameters is usually characterized by random field theory. However, there are few studies on three-dimensional (3D) random field modelling, especially considering the cross-correlation characteristics of geotechnical parameters. In this study, a 3D random field modelling method considering cross-correlation between different soil parameters using asymmetric hierarchical Archimedean Copulas (HACs) is presented. The proposed method considered the different cross-correlated structure of cohesion (c), friction angle (φ), and compressive modulus (E_s) of soils. The marginal distribution function of c, φ and E_s and the optimal HAC among these parameters are established firstly. The generated functions are then combined by Sklar's theorem to set up the joint distribution function. The joint distribution function between three different parameters is next sampled by inverse Laplace-Stieltjes transform. The autocorrelation of single parameter is finally generated using matrix decomposition method. A dataset of laboratory test results of soil strength and stiffness parameters is employed to verify the proposed approach. The proposed approach can not only describe the different asymmetric dependence structure by HACs among c, φ and E_s, but also be applied to the modelling of three-dimensional (3D) random field.

Keywords: Spatial variability; 3-D Random field; Hierarchical Archimedean Copulas; Cross-correlation structure; modified matrix decomposition method (M-MD).

1 Introduction

The physic-mechanical parameters of natural soils show a certain degree of spatial variability due to the effects of sedimentary conditions and stress history (Vanmarcke, 2010). Vanmarcke (2010) firstly proposed a random field model to describe the property of geotechnical parameters. Meanwhile, the concept of scale of fluctuation (SOF) is introduced in the model, and the autocorrelation function (ACF) is used to describe the autocorrelation of single geotechnical parameters between different discrete points in the space range. The autocorrelation random field of geotechnical strength and stiffness parameters has been studied extensively (Cheng et al., 2018, 2019a, 2019b; Hicks et al., 2014; Huang et al., 2017).

However, in addition to the autocorrelation of single geotechnical parameters, the representation of the cross-correlation among the strength and stiffness parameters based on random field model is also worthy of study. Previous studies focused on characterizing the cross-correlation by cross-correlated coefficient matrix (Cho, 2010; Fenton and Griffiths, 2003; Griffiths et al., 2009; Li et al., 2015a). In recent years, copula theory has also been introduced into geotechnical engineering (Li et al., 2014, 2015b; Tang et al., 2015). It can be used to characterize the non-Gaussian dependence structure among variables. Some scholars generated random field models based on copula theory and applied it to slope stability analysis (Ng et al., 2021; Tang et al., 2020; Wang et al., 2020; Zhu et al., 2017). However, most of the studies are focusing on the bivariate copula-based random field of geotechnical strength parameters. There is little research on the random field modelling of multivariate geotechnical parameters. At the same time, the above random field modelling methods are all carried out under two-dimensional conditions, and there are few studies on three-dimensional (3D) random field modelling considering the different dependence structures of parameters.

In this paper, a method for generating 3D cross-correlated random fields of cohesion (c), friction angle (φ), and compressive modulus (E_s) of soils using asymmetric hierarchical Archimedean Copulas (HACs) is proposed and presented. Firstly, the hierarchical Archimedean copulas is briefly introduced. Secondly, a procedure to construct joint cumulative distribution function (CDF) for two strength parameters and one stiffness parameter based on HACs is established, and a tri-HACs approach is put forward to describe the unique asymmetric dependence structure among the three parameters. Thirdly, the proposed tri-HACs method is extended to generate cross-correlation random fields of geotechnical strength and stiffness parameters based on Laplace-Stieltjes transform. Finally, a dataset of laboratory test results of soil strength and stiffness parameters is employed to verify the random fields generated using proposed approach.

2 Construction of CDF

2.1 Copulas

According to Sklar (1959), any n-variate CDF F can be regarded as a unique combination of n marginal distribution functions and a copula function. The copula function is used to describe the dependence structure among variables, and the marginal distribution function is used to describe the distribution characteristics of a single variable. From that, assuming the marginal distribution function of the n-dimensional continuous random vector $\mathbf{x}=(x_1, x_2,\ldots, x_n)$ is $F_1(x_1), F_2(x_2),\ldots, F_n(x_n)$, then the CDF $F(x_1, x_2,\ldots, x_n)$ can be expressed with an unique copula as follows:

$$F(x_1, x_2,\ldots, x_n) = C(F_1(x_1), F_2(x_2),\ldots, F_n(x_n)) \tag{1}$$

The copula C itself is a n-variate distribution function with standard uniform univariate margins. Let $u_i = F_i(x_i)$, the joint probability density functions (PDF) of $\mathbf{x}$ as follows:

$$f(x_1,x_2,\ldots,x_n) = c(u_1,u_2,\ldots,u_n)\prod_{i=1}^{n} f_i(x_i) \tag{2}$$

$$c(u_1,u_2,\ldots,u_n)=\frac{\partial^n C}{\partial u_1 \partial u_2 \ldots \partial u_n}(u_1,u_2,\ldots,u_n) \tag{3}$$

2.2 Hierarchical Archimedean Copulas

Archimedean copulas (ACs) are a specific class of copulas which can be constructed by one-dimensional generator function. An Archimedean generator is a continuous, decresing function $\psi(t)$: $[0,\infty]\rightarrow[0,1]$ which satisfies $\psi(0) = 1$, $\psi(\infty) := \lim_{t\rightarrow\infty}\psi(t) = 0$ and which is strictly decreasing on $[0,\inf\{t : \psi(t) = 0\}]$ (Nelsen, 2006). A bivariate AC can be expressed as follows:

$$C(u_1,u_2)=\psi(\psi^{-1}(u_1)+\psi^{-1}(u_2)) \tag{4}$$

A n-variate AC is constructed according to

$$C(u_1,u_2,\ldots,u_n)=\psi(\psi^{-1}(u_1)+\psi^{-1}(u_2)+\ldots+\psi^{-1}(u_n)) \tag{5}$$

$\psi(t)$ can generate a proper Archimedean copula in all dimensions $n\geqslant 2$ (Kimberling, 1974) if and only if ψ is completely monotone, so that it satisfies

$$(-1)^k\psi^{(k)}(t) \geq 0,\ k\in N_0,\ t\in[0,\infty) \tag{6}$$

The completely monotonic generators is called LT-Archimedean copula generators. We denote the set of all such generators by Ψ_∞. According to Bernstein's Theorem (Feller, 1971), if $\psi(t)\in\Psi_\infty$, it can be obtained by Laplace-Stieltjes transform of a distribution function. In what follows, we mainly assume that ψ is completely monotone.

To allow for the asymmetric dependences among the geotechnical strength and stiffness parameters, the class of HACs is considered. The basic characteristic of HACs is that its argument can be replaced by other HACs (Hofert, 2011). When n=3, the expression can be written as follows:

$$C(u_1,u_2,u_3)=\psi_1(\psi_1^{-1}(u_1)+\psi_1^{-1}\circ\psi_2(\psi_2^{-1}(u_2)+\psi_3^{-1}(u_3))) \tag{7}$$

Among them, $\psi_1(t)\in\Psi_\infty$ and $\psi_2(t)\in\Psi_\infty$.

Hofert (2010) proposed some forms of the tri-HACs with completely monotonic derivative of $\psi_1^{-1}\circ\psi_2$ and corresponding condition, as shown in Table 1. Among them, the first five family labels denote the popular families of Ali-Mikhail-Haq, Clayton, Frank, Gumbel and Joe respectively. The different dependence structures between geotechnical strength and stiffness parameters can be characterized at the same time by the tri-HACs shown in Table 1.

Table 1. Hierarchical Archimedean Copulas and sufficient nesting conditions

Family	Functional form	θ_1	θ_2	SNC
(A,A)	$C_2 = \dfrac{u_2 u_3}{1-\theta_2(1-u_2)(1-u_3)}$; $C = \dfrac{u_1 C_2}{1-\theta_1(1-u_1)(1-C_2)}$	[0, 1)	[0, 1)	$\theta_1 \leqslant \theta_2$
(C,C)	$C_2 = (u_2^{-\theta_2} + u_3^{-\theta_2} - 1)^{-1/\theta_2}$; $C = (u_1^{-\theta_1} + C_2^{-\theta_1} - 1)^{-1/\theta_1}$	(0, ∞)	(0, ∞)	$\theta_1 \leqslant \theta_2$
(F,F)	$C_2 = -\dfrac{1}{\theta_2}\ln[1+\dfrac{(e^{-\theta_2 u_2}-1)(e^{-\theta_2 u_3}-1)}{e^{-\theta_2}-1}]$; $C = -\dfrac{1}{\theta_1}\ln[1+\dfrac{(e^{-\theta_1 u_1}-1)(e^{-\theta_1 C_2}-1)}{e^{-\theta_1}-1}]$	(0, ∞)	(0, ∞)	$\theta_1 \leqslant \theta_2$
(G,G)	$C_2 = \exp(-((-\log u_2)^{\theta_2} + (-\log u_3)^{\theta_2})^{1/\theta_2})$; $C = \exp(-((-\log u_1)^{\theta_1} + (-\log C_2)^{\theta_1})^{1/\theta_1})$	[1, ∞)	[1, ∞)	$\theta_1 \leqslant \theta_2$
(J,J)	$C_2 = 1-[(1-u_2)^{\theta_2} + (1-u_3)^{\theta_2} - (1-u_2)^{\theta_2}(1-u_3)^{\theta_2}]^{1/\theta_2}$; $C = 1-[(1-u_1)^{\theta_1} + (1-C_2)^{\theta_1} - (1-u_1)^{\theta_1}(1-C_2)^{\theta_1}]^{1/\theta_1}$	[1, ∞)	[1, ∞)	$\theta_1 \leqslant \theta_2$
(12,12)	$C_2 = (1+[(u_2^{-1}-1)^{\theta_2} + (u_3^{-1}-1)^{\theta_2}]^{1/\theta_2})^{-1}$; $C = (1+[(u_1^{-1}-1)^{\theta_1} + (C_2^{-1}-1)^{\theta_1}]^{1/\theta_1})^{-1}$	[1, ∞)	[1, ∞)	$\theta_1 \leqslant \theta_2$
(19,19)	$C_2 = \dfrac{\theta_2}{\log(e^{\theta_2/u_2} + e^{\theta_2/u_3} - e^{\theta_2})}$; $C = \dfrac{\theta_1}{\log(e^{\theta_1/u_1} + e^{\theta_1/C_2} - e^{\theta_1})}$	[1, ∞)	[1, ∞)	$\theta_1 \leqslant \theta_2$
(20,20)	$C_2 = (\log(\exp(u_2^{-\theta_2}) + \exp(u_3^{-\theta_2}) - e))^{-1/\theta_2}$; $C = (\log(\exp(u_1^{-\theta_1}) + \exp(C_2^{-\theta_1}) - e))^{-1/\theta_1}$	(0, ∞)	(0, ∞)	$\theta_1 \leqslant \theta_2$
(A,C)	$C_2 = (u_2^{-\theta_2} + u_3^{-\theta_2} - 1)^{-1/\theta_2}$; $C = \dfrac{u_1 C_2}{1-\theta_1(1-u_1)(1-C_2)}$	[0, 1)	(0, ∞)	$\theta_2 \in [1, \infty)$
(A,20)	$C_2 = (\log(\exp(u_2^{-\theta_2}) + \exp(u_3^{-\theta_2}) - e))^{-1/\theta_2}$; $C = \dfrac{u_1 C_2}{1-\theta_1(1-u_1)(1-C_2)}$	[0, 1)	(0, ∞)	$\theta_2 \in [1, \infty)$
(C,12)	$C_2 = (1+[(u_2^{-1}-1)^{\theta_2} + (u_3^{-1}-1)^{\theta_2}]^{1/\theta_2})^{-1}$; $C = (u_1^{-\theta_1} + C_2^{-\theta_1} - 1)^{-1/\theta_1}$	(0, ∞)	[1, ∞)	$\theta_1 \in (0,1]$
(C,20)	$C_2 = (\log(\exp(u_2^{-\theta_2}) + \exp(u_3^{-\theta_2}) - e))^{-1/\theta_2}$; $C = (u_1^{-\theta_1} + C_2^{-\theta_1} - 1)^{-1/\theta_1}$	(0, ∞)	(0, ∞)	$\theta_1 \leqslant \theta_2$

2.3 Construction of CDF

The CDF can be identified by determining the best-fit marginal distribution and the best-fit HACs. In this study, we mainly consider the CDF of c, φ, and E_s. The HACs presented in Table 1 are selected herein. Given a dataset of three parameters, $\{[c_i; \varphi_i; E_{si}], i=1, 2,\ldots, N\}$, the CDF can be identified step-by-step as follows:

(1) Determination of marginal distribution for single parameter

Considering the non-negative property of soil parameters and the range of friction angle φ, the best marginal distribution of c, φ, and E_s is selected from the normal distribution, lognormal distribution, truncated lognormal distribution, Weibull distribution. The AIC (Akaike, 1974) and BIC (Schwarz, 1978) criteria are used to determine the best-fit marginal distribution functions of c, φ, and E_s.

$$AIC = -2\sum_{i=1}^{N}\ln f\left(x_i; p,q\right) + 2k \tag{8}$$

$$BIC = -2\sum_{i=1}^{N}\ln f\left(x_i; p,q\right) + k\ln N \tag{9}$$

where k is the number of parameters in the distribution function; N is the number of data of c, φ, and E_s, x_i represents the c_i, φ_i and E_{si}, respectively. The function that can minimize the AIC and BIC values is the best-fit marginal distribution function.

(2) Identification of the best-fit HACs

As shown in Table 1, we note that $\theta_2 \geqslant \theta_1$ provides a constraint on the dependence structure that the HACs can represent. The condition means that the cross-correlation between $\boldsymbol{U_2}$ and $\boldsymbol{U_3}$ is greater than that between $\boldsymbol{U_1}$, $\boldsymbol{U_3}$ and $\boldsymbol{U_1}$, $\boldsymbol{U_2}$. It can be seen from Table 1 that the value range of θ_1 and θ_2 are constrained. As mentioned above, the dependences among all pairs of c, φ, and E_s show positive and negative cross-correlation at the same time in most cases. In view of the above situation, in order to calculate conveniently, we can always find a parameter of c and φ and transform its symbol (e.g. c to $-c$), so that the cross-correlations among all pairs of c, φ, and E_s are positive at the same time.

The maximum likelihood estimation method (Lumb, 1970) is used to obtain the unknown parameters θ_1 and θ_2. Construct the log-likelihood function of the copula function as following:

$$L(\theta_1,\theta_2) = \sum_{i=1}^{N} \ln c\left(u_1,(u_2,u_3;\theta_2);\theta_1\right) \tag{10}$$

$$(\theta_1^{'},\theta_2^{'}) = (\theta_1,\theta_2)\Big|_{L=L_{\max}} \tag{11}$$

where N is the sample size; θ_1 and θ_2 are the parameters of the HACs to be estimated.

Similarly, the AIC (Akaike, 1974) and BIC (Schwarz, 1978) criteria are used to determine the best-fit HACs, as follows

$$AIC = -2\sum_{i=1}^{N} \ln c\left(u_1,(u_2,u_3;\theta_2);\theta_1\right) + 2k \tag{12}$$

$$BIC = -2\sum_{i=1}^{N} \ln c\left(u_1,(u_2,u_3;\theta_2);\theta_1\right) + k\ln N \tag{13}$$

where k is the number of unknown parameters of selected HACs and k=2; u_1, u_2 and u_3 can use empirical probability value calculated as follows

$$u_{1i} = \frac{rank(x_{1i})}{N+1} \tag{14}$$

$$u_{2i} = \frac{rank(x_{2i})}{N+1} \tag{15}$$

$$u_{3i} = \frac{rank(x_{3i})}{N+1} \tag{16}$$

The HAC is identified as the best-fit HAC when it produces the lowest AIC and BIC value. It can be seen from Eq.12 and Eq.13 that AIC and BIC always reach the minimum at the same time.

3 Generation of random field

3.1 Generation of 3D autocorrelation random fields

In random field theory, the autocorrelation of a variable is often characterized by autocorrelation coefficient $\rho_{x,y,z}$, which is only related to the relative position of elements in space. In order to simplify the calculation, the theoretical ACF is usually used to describe the autocorrelation coefficient $\rho_{x,y,z}$. The most commonly used ACF is exponential function (Ji et al., 2012). In this paper, the exponential correlation function is selected to characterize the autocorrelation as follows:

$$\rho\left(\tau_x,\tau_y\right) = \exp\left[-2\left(\frac{|\tau_x|}{\theta_x}+\frac{|\tau_y|}{\theta_y}+\frac{|\tau_z|}{\theta_z}\right)\right] \tag{17}$$

where θ_x, θ_y and θ_z represent the SOF along with the horizontal and vertical directions, respectively; τ_x, τ_y and τ_z are the relative distances of horizontal and vertical directions between any two elements.

Since the covariance matrix decomposition method is easy to implement, with high calculation accuracy and fast calculation efficiency, it is used to model the autocorrelation random field in this paper. $\boldsymbol{D}_{n\times n}$ is the n-order covariance matrix composed of the covariance between n points where $D_{ij}=\rho_{ij}(\tau_{xij}, \tau_{yij}, \tau_{zij})$, as follows:

$$D = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & \rho_{nn} \end{bmatrix} \tag{18}$$

Perform Cholesky decomposition on the covariance matrix D to obtain upper and lower triangular matrices, as follows

$$D = LU = LL^T \tag{19}$$

where $\boldsymbol{L}$ is the lower triangular matrix and $\boldsymbol{L}^T$ is the transpose of the matrix $\boldsymbol{L}$.

Then the n-order autocorrelation random field matrix $\boldsymbol{Z}$ can be expressed as follows:

$$Z = LY \tag{20}$$

where $\boldsymbol{Y}$ is an n×3 matrix of independent standard normal random variables. The matrix $\boldsymbol{Z}$ can be used as a realization of independent standard normal autocorrelation random field.

3.2 Sampling HACs

Hofert (2010) listed some single-parameter LT-Archimedean generators with explicit inverse Laplace-Stieltjes transforms. Assuming that $\psi_1(t)$ can be obtained by Laplace-Stieltjes transform of distribution function $F_1(v_1)$ on $[0,\infty]$. The generator of $\psi_1(t)$ can be shown as follows:

$$\psi_1(t) = \int_0^\infty e^{-tv_1} dF_1(v_1) \tag{21}$$

Eq. 8 can be converted to:

$$C(u_1,u_2,u_3) = \int_0^\infty e^{-v_1\psi_1^{-1}(u_1)} e^{-v_1\psi_1^{-1}\circ\psi_2(\psi_2^{-1}(u_2)+\psi_2^{-1}(u_3))} dF_1(v_1) \tag{22}$$

$$\psi_{12}(t;v) = \exp(-v_1\psi_1^{-1}\circ\psi_2(\bullet)) \tag{23}$$

The HACs explicit sampling algorithm based on inverse Laplace transform is as follows:

(1) Generate a variate $\boldsymbol{V_1}$ with distribution function $F_1$with LT ψ_1;

(2) Generate a variate $\boldsymbol{V_{12}}$ with distribution function $\boldsymbol{F_{12}}$ with LT ψ_{12};

(3) Generate independent uniform variates $\boldsymbol{X_1}$, $\boldsymbol{X_2}$, $\boldsymbol{X_3}$;

(4) Return ($\boldsymbol{U_1}$, $\boldsymbol{U_2}$, $\boldsymbol{U_3}$) = (ψ_1(-ln($\boldsymbol{X_1}$)/ $\boldsymbol{V_1}$), ψ_2(-ln($\boldsymbol{X_2}$)/ $\boldsymbol{V_{12}}$), ψ_2(-ln($\boldsymbol{X_3}$)/ $\boldsymbol{V_{12}}$)).

3.3 Generation of the trivariate cross-correlated random field

Now, we can generate the trivariate cross-correlated random field of c, φ, and E_s to combine the matrix $\boldsymbol{Z}$ with the HACs sampling results. The process of generating the random field is as follows:

(1) Generate standard normal random fields $\boldsymbol{Z}$ = ($\boldsymbol{Z_1}$, $\boldsymbol{Z_2}$, $\boldsymbol{Z_3}$), where $\boldsymbol{Z_1}$, $\boldsymbol{Z_2}$ and $\boldsymbol{Z_3}$ are independent of each other;

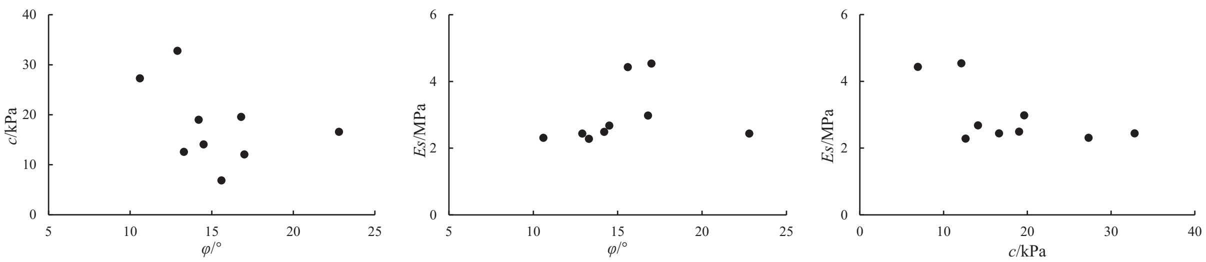
(2) Transform standard normal random fields $\boldsymbol{Z}$ = ($\boldsymbol{Z_1}$, $\boldsymbol{Z_2}$, $\boldsymbol{Z_3}$) into standard uniform distributed random fields $\boldsymbol{X}$ = ($\boldsymbol{X_1}$, $\boldsymbol{X_2}$, $\boldsymbol{X_3}$);

(3) As described above, sample HACs to get $\boldsymbol{U}$ = ($\boldsymbol{U_1}$, $\boldsymbol{U_2}$, $\boldsymbol{U_3}$) using $\boldsymbol{X}$;

(4) The inverse function of the marginal distribution function of c, φ, and E_s is solved, and the random field model satisfies the HACs' structure.

4 Implementation

Undrained shear tests and consolidation tests are conducted on the undisturbed marine soft soil located on Pearl River Estuary. The test results pertaining to c, φ, and E_s are listed in Figure 1 and the dataset are used as an example to describe the process of generation of the trivariate cross-correlated random field



(a) (b) (c)

Figure 1. Test data of strength and stiffness parameters (a) c-φ; (b) φ-E_s; (c) c-E_s

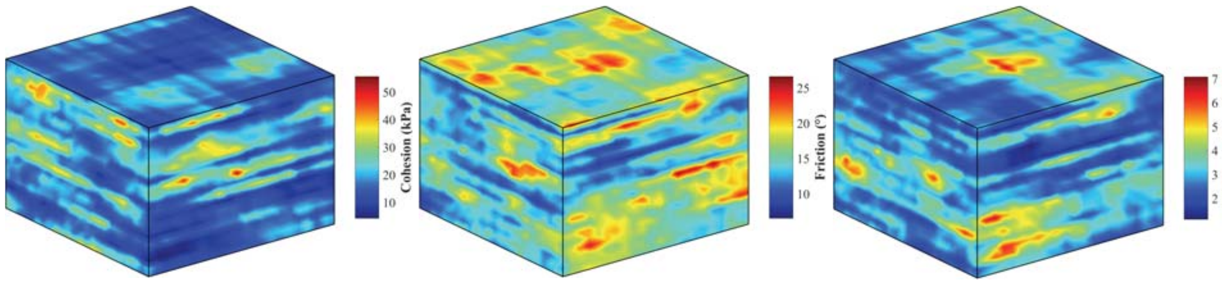
Based on the dataset and Eq.8 and Eq.9, the best-fit marginal distribution functions of c, φ, and E_s are determined to be the lognormal, truncated lognormal, and lognormal distributions, as shown in Table 2.

Table 2. AIC and BIC values of c, φ and E_s under different marginal distribution

Marginal distribution	AIC			BIC		
	c	φ	E_s	c	φ	E_s
Normal distribution	65.99	50.83	26.51	66.39	51.22	26.90
Lognormal distribution	**64.96**	49.64	**24.10**	**65.36**	50.03	**24.49**
Truncated lognormal distribution	64.96	**-33.61**	24.10	65.36	**-33.22**	24.49
Weibull distribution	-	102.20	-	-	102.59	-

The Kendall correlation coefficients of (c, φ), (c, E_s) and (φ, E_s) are -0.333, -0.310, and 0.535, respectively. According to Section 2.2, just converting c to $-c$ and the pair of (φ, E_s) as an inner pair with generator $\psi_{2}(:,\theta_2)$, the joint distribution function can be characterized by HACs. The θ_1 and θ_2 of the selected HACs are calculated by Eq. 10 and 11 based on the measured data. The AIC and BIC criteria are used to identify the best-fit HACs. Based on the calculation results, (12, 12) is identified the best-fit HAC according to section 4.2. The θ_1 and θ_2 values of (12, 12) are 1.218 and 1.396.

The length, width and height of the 3D random field simulation space are 10m, 11m and 12m respectively, the grid size is 0.5m, and $\theta_x=\theta_y=10$m, $\theta_z=1$m. The ACF takes the form of exponential function. Combining with the above marginal distribution types and (12,12) HAC function, the 3D cross-correlation random field model of c, φ, and E_s is generated, as shown in Figure 2. It can be seen that c, φ and c, E_s show negative cross-correlation, while φ, E_s show obvious positive cross-correlation. It is consistent with the cross-correlation characteristics shown by the tested dataset.



(a) (b) (c)

Figure 2. 3D cross-correlation random field model (a) c; (b) φ; (c) E_s

5 Conclusion

This study proposed an approach for generating cross-correlation geotechnical random field of c, φ, and E_s based on the asymmetric HACs, the inverse Laplace-Stieltjes transform and Cholesky decomposition of the covariance matrix. The main conclusions are as follows:

(1) Compared with the conventional method, it can characterize the different dependence structures of geotechnical strength and stiffness parameters rather than just describe the correlation coefficient. Moreover, it can describe the non-Gaussian dependence structures among the parameters. Compared with the traditional multivariate copula-based method, it can describe the asymmetric dependence structures among c, φ, and E_s at the same time.

(2) This study provides a more flexible selection on the characterization of cross-correlation among multivariate geotechnical random fields and three-dimensional (3D) random field. The proposed approach is considered rational, because the CDF of geotechnical parameters can be obtained based on the measured data and can be explicit representation.

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