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Simulations Used in Geotechnical Practice: Part 2 – Comparing Normal and Non-normal PDF in Sampling

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Abstract: Monte Carlo (MC) and Latin Hypercube (LH) sampling were compared in the Part 1 companion paper. The simulation results are compared to show that the LH better represents low probability outcomes by forcing the sampling of the simulation to include events in the tails. At a high number of simulation iterations both provide similar outputs, but at low simulation iterations the LH is more reliable. However, both the MC and LH sampling suffer from impractical values at low or high probability events, when using the normal probability density function (PDF). The normal PDF is commonly used in statistical modelling. Non-normal PDFs often represent the best fit PDF when a goodness of fit test is carried out. The errors associated with using the common normal PDF are shown with the above-mentioned simulation models. This best fit PDF applies whether simulation models are used or even with simple "what if" sensitivity models in traditional spreadsheet analysis.

Keywords: Simulation; Probability density functions; characteristic value: normal PDF; non normal PDF.

1 Introduction

Geotechnical engineering practice often relies on characteristic values which are moderately conservative. The most likely value is unlikely to be the mean, yet data parameters are also assumed to have a normally distributed probability density function (PDF). In practice, visually removing outliers is used in non-statistical approaches to determine a characteristic value for design.When statistical approaches are used these "outliers" create unrealistic statistical models if all data is used, yet selectively removing data is acceptable by the less reliable visual method, but considered cherry picking when statistical approaches are used. An approach using the best fit PDF, or bounded values, can be used to overcome such anomalies.

Monte Carlo (MC) sampling is the traditional technique for generating random numbers to sample from a probability distribution. This simulation has been used to assess the impacts of parameter uncertainty on an analytical model. A companion paper (Part 1) examined the effect of MC vs Latin Hypercube (LH) sampling. The unrealistic "failure" in a bearing capacity simulation was shown using a normal PDF. Similarly negative settlements for a simulation of embankment settlements were shown. A designer should input a more realistic PDF. This paper examines the Normal PDF vs a non-normal PDF. The former is often used as a simplified statistical model but geotechnical parameters are typically non normally distributed.

1.1 Characteristic value and use of appropriate distributions

A key task in any analysis is the selection of the design value. This selection is often based on experience and personal judgment. The moderately conservative parameter is not a precisely defined value, and needs to be considered in the context of data, reliability and sufficiency as to what partial factors or factors of safety (FS) are applied in the analytical process. A high (or low) FS may result from the choice of parameters rather than providing clarity on a "safe" vs. "unsafe" product.

The use of statistics provides both transparency and accountability as judgment often varies between practitioners. Assuming sufficient tests are available for a defined "homogenous" layer then a statistical approach may be used. This approach assumes other ground assessment factors (such as time dependent properties, ground water, stress dependency or strain softening) have been given due consideration. Using sensitivity analysis to assess critical input parameters is not the same as risk assessment and does not have the full benefit of simulation modelling.

Geotechnical material parameters typically have a large coefficient of variation (COV) as compared with man-made materials such as steel or concrete. A COV of 20% for concrete is considered poor (Phoon, 2008) while in soil and rock materials such a strength result would be considered unusual and values of 40% typical. The seminal paper by Phoon and Kulhawy (1999) evaluates geotechnical property variability and is considered a practical initial input for reliability-based analysis in the absence of sufficient project specific data. COVs of 90% are common for strength index tests such as CBR (Look, 2009) or Point Load index tests (Look and Wijeyakulasuriya, 2009). Such index tests and large COVs were shown to be not well modelled by the Normal distribution. The assumed distribution can affect the moderately conservative or simulation results considerably.

Goodness of fit tests are required to determine the appropriate probability density function (PDF). Case studies from 5 bridge sites, 24 deep rock socketed piles at one site, 4 roadway segments and a major pile test site with almost 1,000 driven piles were described in Look (2015). These examples showed how characterisation of the strength properties requires best fitting distribution through goodness-of-fit tests and concluded a normal PDF should be used in geotechnical practice, only when there is a COV below 25%.

Unrealistically low and negative values at the low fractiles, based on a normal distribution, are shown in Figure 1. The normal distribution model is not appropriate at the 10% fractile or below, when high COV occur. The normal distribution was comparable with the lognormal and best fit PDF at the 20% to 30% fractile (Look, 2015).

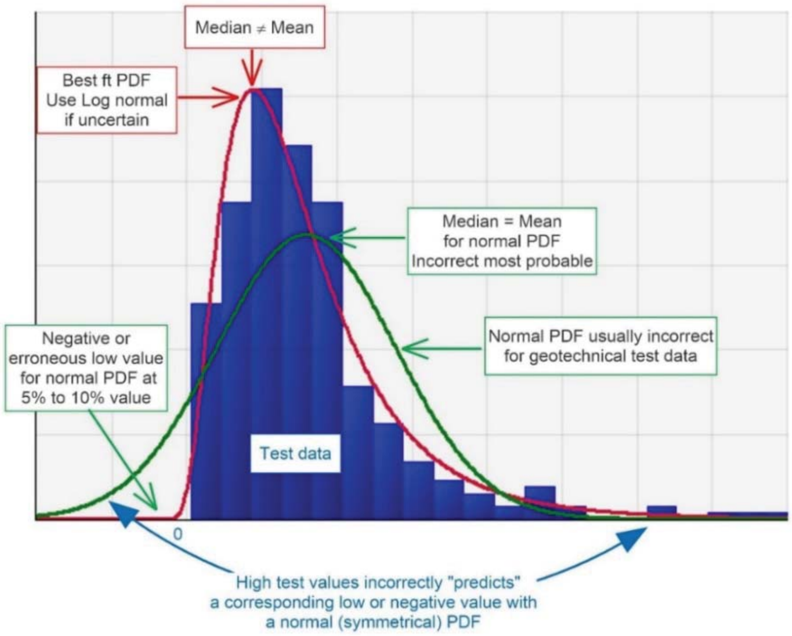


Figure 1. Avoid incorrect PDF in design input (Look, 2022)

2 Bearing Capacity Simulation

This example is for a working platform design described in the Part 1 companion paper Look and He (2022, Part 1). A medium dense granular platform without a geogrid is placed on a stiff clay subgrade. The eccentric applied pressures are 188kPa, and 306 kPa for the travelling and drilling operations, respectively,

The issue with any analysis based on a mean and standard deviation is that unrealistic tails become self-evident. For example, using the typical cohesion values in Table 1 (Look and He, 2022, Part 1) shows that for a firm clay with a mean C_u = 35 kPa, there is a 18.6% probability of soft clay (Cu < 25kPa). This is unrealistic and immediately infers even the material strength classification is incorrect. Thus, the strength should be bounded (or truncated) at 25 and 50 kPa to meet its strength classification, or reclassified if 18.6% likelihood of being "soft". Given this conundrum of the mismatch between practical classifications and theoretical variability, the application of a PERT (Project Evaluation and Review Techniques) PDF provides maximum and minimum bounded values. Similarly, for a working platform material with a 19.7% probability of the material being either denser or looser than the assumption of medium dense, is clearly unrealistic to advance the analysis and a truncation is also required.

2.1 Analysis output with a non-normal PDF

The corresponding tornado graph using a non-normal PERT distribution which uses the expected minimum, likely and maximum (Table 1) is shown in Figure 2 for a simulation model using 10,000 iterations. The PERT distribution is a special case of the Beta General PDF, with the benefit of a simplified input. The false model of data symmetry and kurtosis which occurs in a normal PDF is then accounted for by skewing to the likely (median /mode) value. Further details on best fit and simulation models are provided in the @Risk user's manual, which states "it is often possible to fit normal-like data to a Beta General distribution with very large shape parameters and artificially wide minimum and maximum parameters. Although this can lead to a good fit from a mathematical viewpoint, the fit is questionable from a practical standpoint."

This simulation modelling in Part 1 is repeated using the PERT PDF which is a more practical representation of both variability and skewness in parameters. The LH simulation shows little change, whether 100 or 10,000 iterations are used (Table 1). This applies for both travelling and drilling operations

Table 1. Summary statistics for bearing resistance using a PERT PDF (Unacceptable values highlighted)

Percentile	Bearing Resistance (kPa) using a PERT PDF									
	Monte Carlo simulation				Latin Hypercube simulation					
Iterations →	100		10,000		100		1,000		10,000	
	Trav	Drill	Trav	Drill	Trav	Drill	Trav	Drill	Trav	Drill
Minimum	258	298	232	268	233	295	236	276	233	270
5%	270	312	269	311	269	313	269	308	269	311
10%	280	322	280	323	279	320	279	322	279	323
15%	289	336	288	332	287	333	288	330	286	331
25%	308	355	299	346	301	348	300	348	299	346
50%	329	383	323	374	323	376	322	375	323	374
85%	371	431	364	425	359	416	364	422	364	424
90%	381	443	374	437	376	437	373	436	374	436
95%	397	474	390	456	385	457	388	460	390	456
Maximum	422	498	478	552	437	514	456	539	500	587

Figure 2 shows the tornado graph for the 5 variables in this analysis for travelling and drilling. The variation of the working platform angle of friction governs for this case study with the subgrade cohesion the next governing parameter. The effect is greater for the drilling operation as compared to travelling and standing.

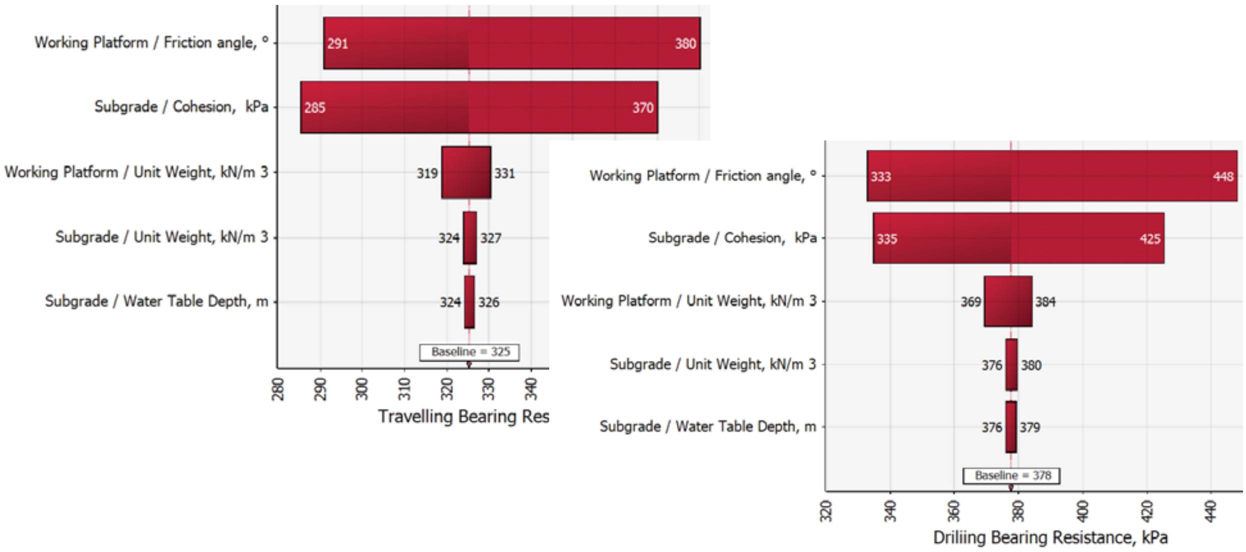


Figure 2. Tornado graph showing effect on output mean for 10,000 Latin Hypercube iterations.

The 270 kPa minimum calculated output value provides confidence that the required 188 kPa (unfactored) bearing resistance for the travelling tracked plant is not exceeded – a low risk. Arguably the applied load factor is conservative for this standing / traveling operation example. For the drilling operations for 10,000 iterations the Latin Hypercube simulation shows there is a 3.2% probability of the 306 kPa load not having sufficient bearing resistance. A risk of 3.2% for this temporary condition suggests BRE 470 is not over conservative for this case. Note for this simulation analysis the most likely soil condition was used and not a conservative value.

There was little difference between 100 and 10,000 iterations in LH simulation with the upper and lower quartile values within 1.2%. The MC at 100 iterations provided the smallest prediction range of 200kPa. The LH simulations consistently provided a higher range than the MC, although both increase proportionately.

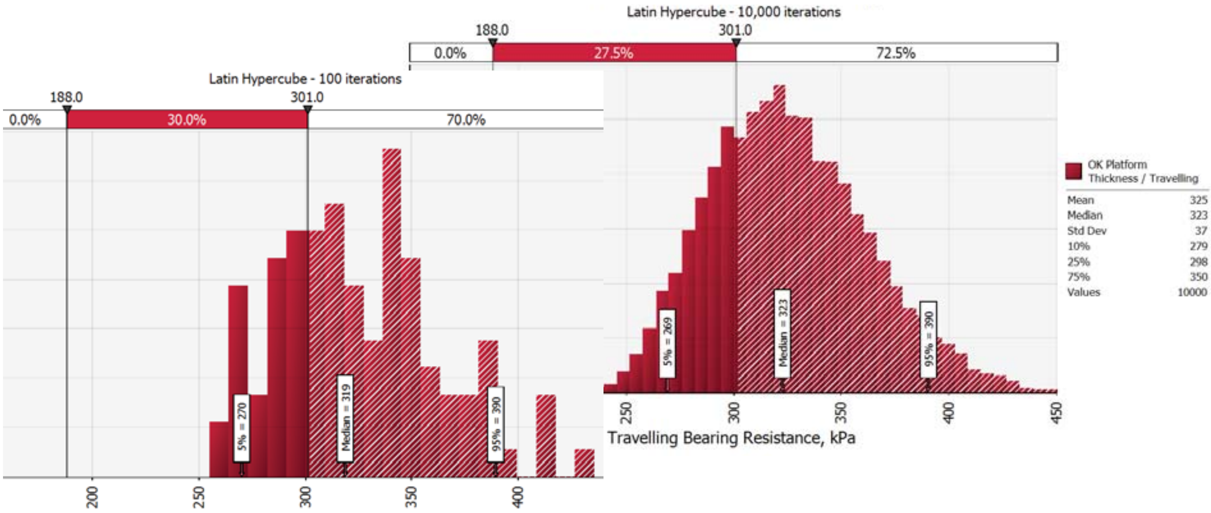


Figure 3. Distribution of travelling bearing resistance for 100 and 10,000 iterations. 188kPa is unfactored load.

The various analyses consistently show that the upper and lower 5 to 10 percentile, would provide an unrealistic output (Figure 4). In summary:

- A normal PDF provides an excessively wide output range with 25% of simulation results not achieving the unfactored load, this must clearly be rejected as incorrect. Changing iterations or using LH or MC models also show dubious results. When a PERT distribution is used, realistic output values are evident.
- 100 iterations are dissimilar to 1,000 and 10,000 iterations which are approximately similar. The MC simulation is most dissimilar from the other outputs at 100 iterations and would be the least accurate.
- This case study demonstrates simulation models should use a PERT PDF as most reliable, and a LH simulation at 1,000 iterations is optimal. Normal PDFs are unreliable, and the MC simulation is unreliable at low simulations (100 No.).
- Similar conclusions are evident for the travelling operations

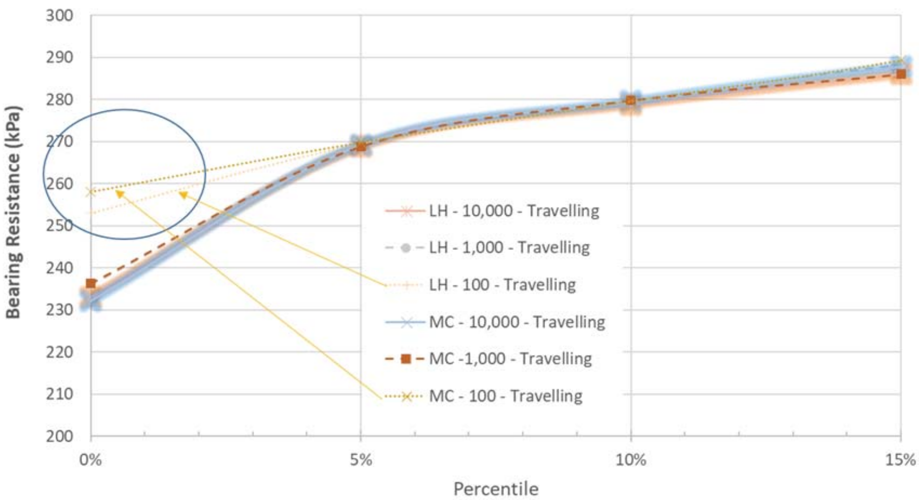


Figure 4. Comparison of simulation results at lower 5% bearing resistance

3 Settlement Simulation

Another example is a settlement simulation for a 10m thick layer of soft clay under an embankment loading of up to 6m, illustrated in Look and He (2022, Part 1). Six parameters were considered in this analysis, including unit weight, compression index Cc, initial void ratio e_0, thickness of clay, groundwater, and embankment load.

One-dimensional consolidation tests had been conducted on eleven soft soil samples across the site to obtain key settlement parameters – compression index, C_c, and initial void ratio, e_0. These test results were statistically analyzed using three probability distribution functions – Normal, Log-normal, and Pert, as illustrated in Figure 5. The comparison of three PDFs indicates that Normal distributions lead to unreasonable results as 8.1% of the C_c samples fall below the minimum measured value and also into a negative zone, and the similar issue is also observed on the initial void ratio, e_0. Density with a low COV has a normal PDF as the best fit.

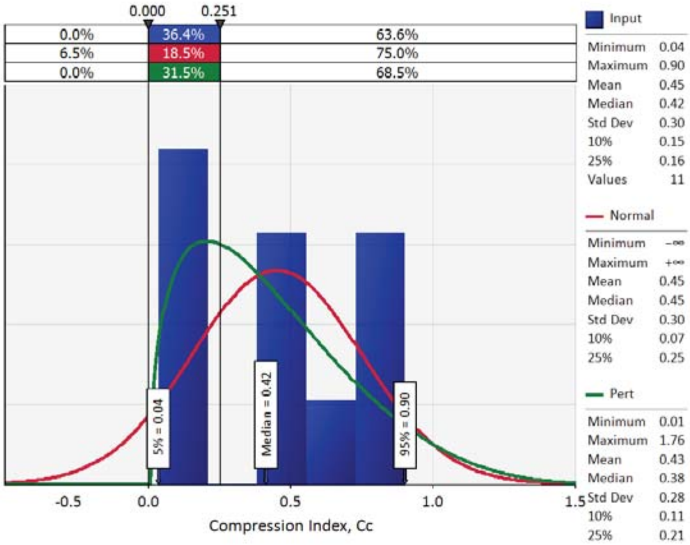


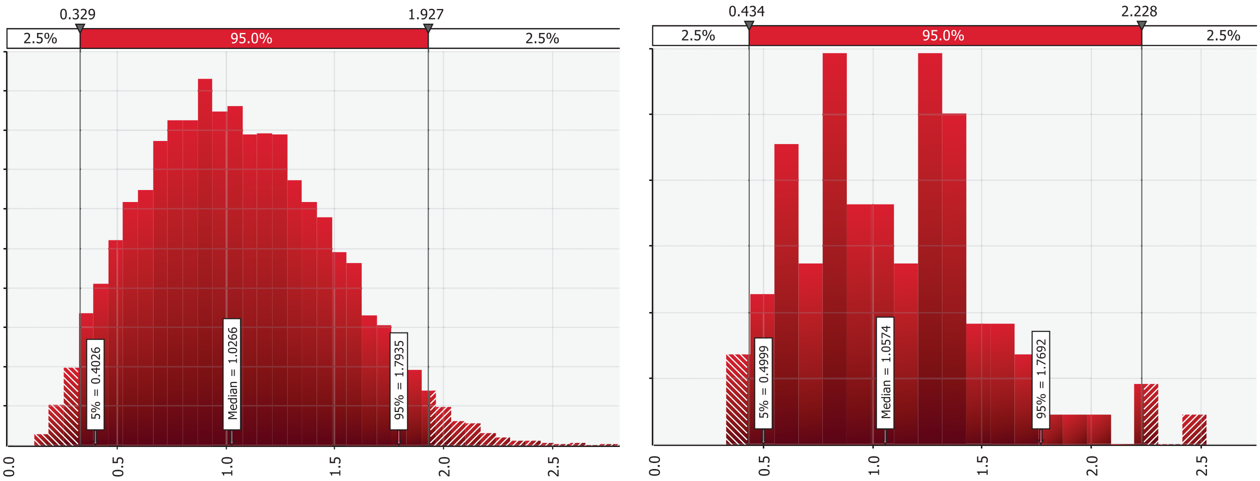
Figure 5. Statistical analysis of compression index, Cc

3.1 Analysis output with a non-normal PDF

Compared to the normal PDFs, a PERT distribution can obtain positive values across the whole range, as indicated in Figure 5. This result is consistent with the example of bearing capacity. The statistical analysis results using the PERT PDF are tabulated in Table 2. The statistical analysis results of 10,000 and 100 simulations using LH sampling are shown in Figure 6. Part 1 showed negative values in all simulation models at the 5% percentile.

Table 2. Summary statistics for settlement analysis using a non-normal PDF

Percentile	Settlement (m) using PERT PDF				
	Monte Carlo simulation		Latin Hypercube simulation		
Iterations →	100	10,000	100	1,000	10,000
Min	0.251	0.087	0.332	0.097	0.113
5%	0.408	0.423	0.500	0.419	0.413
10%	0.503	0.529	0.559	0.529	0.519
15%	0.527	0.609	0.631	0.605	0.605
25%	0.586	0.739	0.770	0.728	0.749
90%	1.498	1.619	1.601	1.642	1.617
95%	1.597	1.793	1.769	1.819	1.781
Max	1.905	2.588	2.525	2.527	2.693



(a) 10,000 simulation using LH sampling (b) 100 simulation using LH sampling

Figure 6. Statistical analysis settlement result of 10,000 and 100 simulation using a non-normal PERT PDF

Using 10,000 LH result as the example, the analysis resulted in a settlement between 113mm and 2,693mm, and the range with 95% confidence is from 413mm to 1,781mm. This settlement range is more reasonable and reliable than that using a normal PDF, as all parameters, especially for compression index and initial void ratio, have been constrained within reasonable limits. In addition, using the NORMAL PDF, 100 MC simulations obtained a result which significantly deviated from other analysis, while LH simulation shows more consistent results (Figure 7).

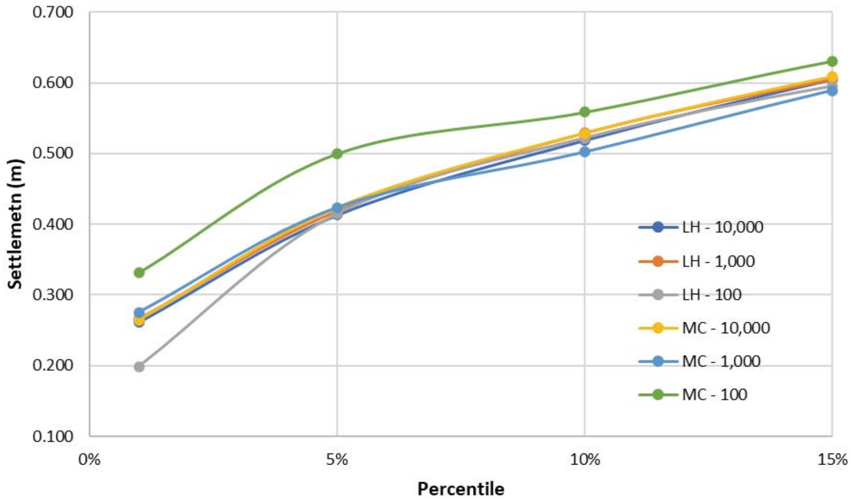


Figure 7. Simulation effect at lower 10% settlement for simulation iterations

4 Conclusion

In reliability analysis, probability of occurrence is combined with simulation models. The following conclusions can be drawn based on the two examples:

1. The normal PDF should be avoided for most geotechnical parameters. Unreasonable negative or low results may be calculated when adopted. The PERT PDF provides a practical simulation tool, although the Log normal PDF (not discussed herein) is also known to provide realistic output results
2. Latin Hypercube sampling better represents low or high probability outcomes by forcing the sampling of the simulation to include data from the tails. Low numbers of Monte Carlo simulations would produce unrealistic results. This is less likely to occur with Latin Hypercube simulations
3. Industry is often hesitant to use statistical approaches even though they are superior to visual selection of parameters. This is mainly due to the inconsistent results obtained when selecting a normal PDF as shown herein, as Goodness of fit tests on the data typically show a better fit PDF.

The simulation sampling using the LH and MC sampling methods was discussed in a companion paper.

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