Proc. of the 8th International Symposium on Geotechnical Safety and Risk (ISGSR)
Edited by Jinsong Huang, D.V. Griffiths, Shui-Hua Jiang, Anna Giacomini and Richard Kelly
 Published by Research Publishing, Singapore.
doi: 10.3850/978-981-18-5182-7_21-00501-cd

ISGSR 2022

Simulations Used in Geotechnical Practice: Part 1 – Comparing Monte Carlo and Latin Hypercube Sampling

Burt Look[1] and Wade He[2]

[1 2] FSG – Geotechnics + Foundations, Level 2, 527 Gregory Tce
Fortitude Valley, QLD, 4006, Australia.
E-mail: blook@fsg-geotechnics.com.au
E-mail: whe@fsg-geotechnics.com.au

Abstract: Monte Carlo (MC) sampling is the traditional technique for generating random numbers to sample from a probability distribution. When low probability events occur, a small number of MC iterations might not sample sufficient quantities of these outcomes for inclusion in the simulation model. Latin Hypercube (LH) sampling uses stratification of the input probability distributions, to overcome the limitations of Monte Carlo sampling. Both the LH and MC sampling are compared to demonstrate this effect in common geotechnical engineering simulation applications. These simulation results show the LH better represents low probability outcomes by forcing the sampling of the simulation to include outlying events. At a high number of simulation iterations both provide similar outputs, but at low simulation iterations the LH is more reliable. A companion paper (Part 2) factors in the probability density function (PDF) used. Non-normal PDFs often represent the best fit PDF when a goodness of fit test is carried out.

Keywords: Simulation; Monte Carlo; Latin Hypercube; Probability density functions

1 Introduction

Duncan (2000) discusses factors of safety (FS) and reliability in Geotechnical Engineering to show the same value of FS is often applied to conditions that involve widely varying degrees of uncertainty. A high (or low) factor of safety, may result from the choice of parameters rather than providing clarity on a "safe" vs. "unsafe" product. Lacasse (2013) illustrates the example of two slopes; 1) The first slope has a central FS of 1.4 and a probability of failure (P_f) of 10^{-4} per year; 2) The second slope has a higher FS of 1.8, but a wider variation, and therefore is more likely to fail due to a higher P_f of 10^{-3} per year. Reliability analysis can capture variation of material parameters in a quantitative way, which is not considered in the traditional FS methodology.

Monte Carlo sampling is the traditional technique for generating random numbers to sample from a probability distribution. This simulation has been typically used to assess the impacts of parameter uncertainty on an analytical model. Latin Hypercube (LH) is a type of stratified Monte Carlo (MC). This paper compares MC and LH sampling. MC is often used in simulation models but, because it relies on pure randomness, has a closer clustering around the mean than LH, which spreads the sample points more evenly across all possible values. Stratified sampling allows a weighting according to its conditional probability. Low (and high) probability events are then better captured using the LH sampling approach.

A key failing in using the normal probability density function (PDF) is exposed with low or high probability events. A companion paper (Part 2) combines the 2 sampling approaches with Normal PDF vs a non-normal PDF. The former is used as a simplified statistical model but geotechnical parameters are often non normally distributed.

1.1 Simulation modelling

The traditional MC sampling may lead to incorrect answers for a low probability event. When low probability events occur, a small number of MC iterations might not sample sufficient quantities of these outcomes for inclusion in the simulation model. A 5% probability does not mean that there is a 5% chance of being sampled in a MC simulation for a small number of simulations.

LH sampling uses stratification of the input probability distributions, to overcome the limitations of MC sampling. Stratification divides the cumulative curve vertically into equal probability intervals. A sample is then randomly taken from each stratification of the input distribution, thus forcing sampling to represent values in each interval. Therefore, all values in the input distribution have an equal chance at being sampled, whereas low probability events are not as well "represented" in MC sampling. Both the LH and MC sampling are compared in common geotechnical simulation applications: Bearing capacity for working platforms and embankment settlement.

2 Bearing Capacity Simulation

This example is for a working platform design using the BRE 470 method for an 800mm wide tracked plant for installation of a 1800mm diameter bored pier. A medium dense granular platform without a geogrid is placed on a firm clay subgrade with the water table 2.0m below the top of subgrade. The eccentric applied pressures are 188kPa, and 306 kPa for the travelling and drilling operations, respectively. A platform thickness of 1.150 m is calculated based on the mean parameters below. A designer would use more conservative parameters and the mean is used only to show the simulation model using MC sampling for the standard deviation shown in Table 1.

Given the mode and median are not necessarily equal to the mean value, the likely value is also shown in Table 1. That effect will be covered in the part 2 companion paper. Specifically, although the clay is firm (25 – 50kPa), the normal PDF suggests that 18.6% (Figure 1) will be soft (< 25kPa) and 9% will be stiff (> 50 kPa) using "typical" coefficient of variation (COV). The 27.6% of values as outliers are unreasonable and should not happen in any real project. Similarly, for the angle of friction with 2 x 19.7% = 39.4% being "misclassified" when a normal PDF is used.

Table 1. Ground data input for the working platform design.

Design element	Parameter	COV (%)	Mean / Standard Deviation	Minimum / Likely / Maximum
Granular platform	Angle of Friction (°)	13%[1]	36 / 4.7	33 / 36 / 42
	Unit Weight (kN/ m^3)	9%[1]	20 / 1.8	19 / 20 / 21.5
Clay subgrade	Cohesion (kPa)	32%[1]	35 / 11.2	25 / 35 / 50
	Unit Weight (kN/ m^3)	9%[1]	18 /1.6	17 / 18 / 19.5
Water Table	Depth (m)	25%	2.0 / 0.5	1.5 / 2.0 / 2.5

[1] Phoon and Kulhawy, 1999

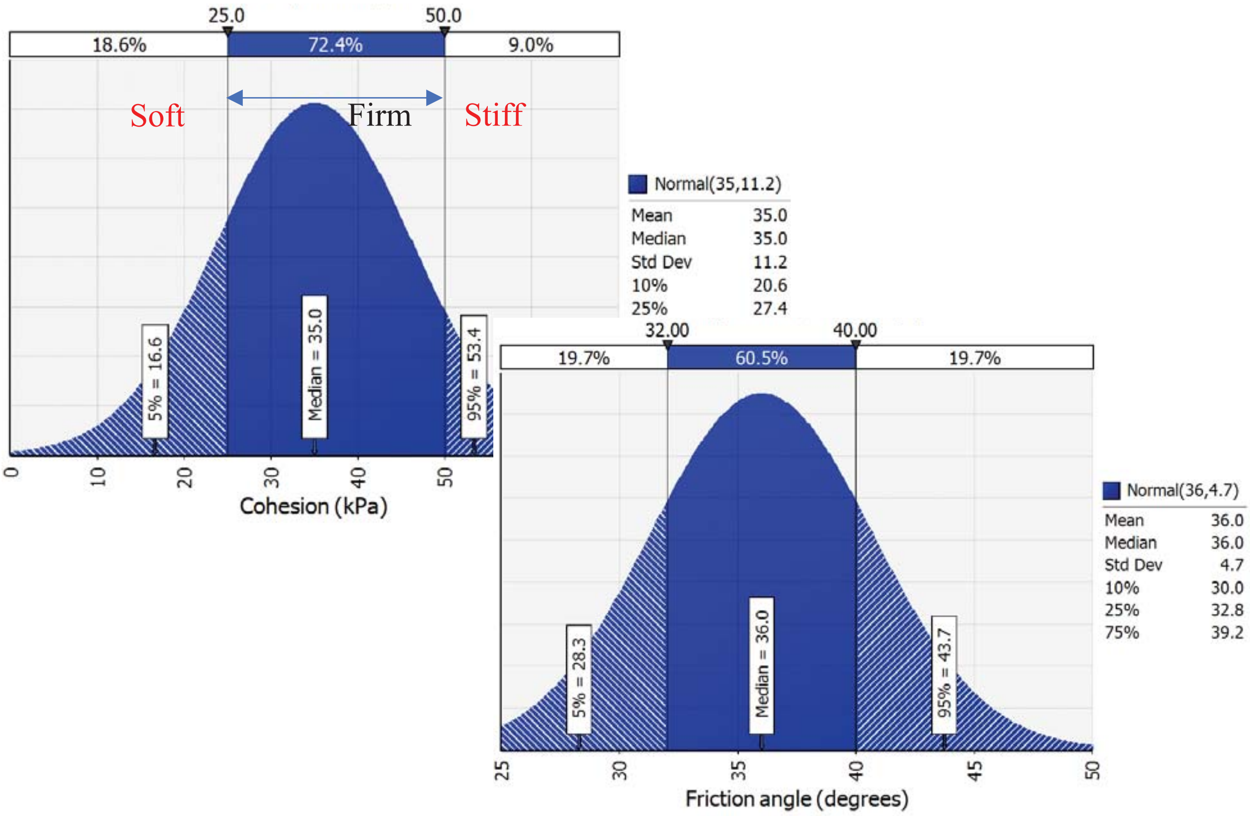

Figure 1. Variability based on normal distribution of results with mean and standard deviation

2.1 Analysis output comparing MC and LH simulations with a normal PDF

The tornado graph (Figure 2) shows the relative effect on the output mean bearing resistance for 10,000 Monte Carlo simulations with a normal PDF. The working platform friction angle governs for the travelling case, with the subgrade unit weight having the least effect in this example. The corresponding statistical output is compared in Table 2 using MC and LH simulation models, for varying numbers of iterations for a normal PDF. This shows:

- Output varies as simulations increase. The 100 MC simulations is considered inaccurate (
- Figure **3**), while the 100 LH simulation is approximately compatible with the higher 10,000 simulations

● The MC simulation with 10,000 iterations has the greatest variation (Figure 4), and questionable values. An output which provides 22.1% of values less than 306kPa (the unfactored load for drilling) is a high risk while showing 54% of results greater than 367 kPa (the factored load from drilling)

● The 10,000 LH provides less variability than the MC simulation, but still has unrealistically low and high bearing resistance values for the upper and lower 10% values.

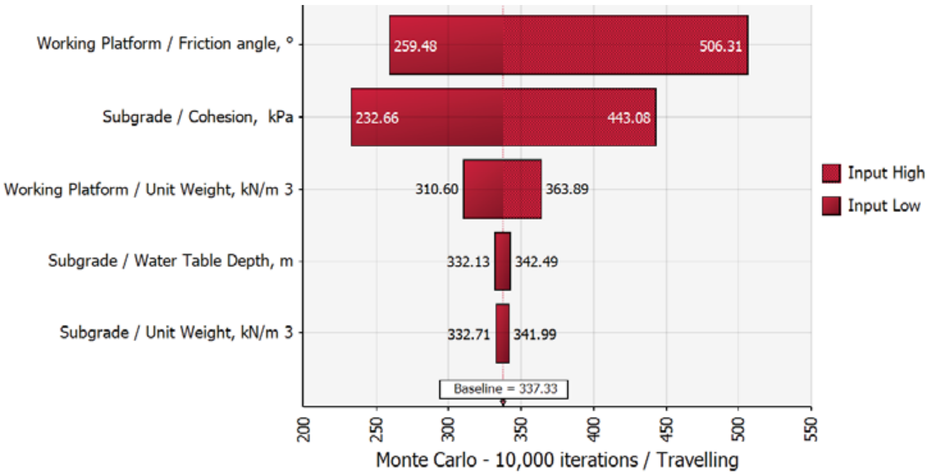

Figure 2. Tornado graph showing effect of input variability (standard deviation) on the mean

Table 2. Summary statistics for ultimate bearing capacity using a normal PDF (Unacceptable values highlighted)

Percentile	Bearing Resistance (kPa) using a Normal PDF									
	Monte Carlo simulation				Latin Hypercube simulation					
Iterations →	100		10,000		100		1,000		10,000	
	Trav	Drill	Trav	Drill	Trav	Drill	Trav	Drill	Trav	Drill
Min	108	132	38	57	102	124	121	127	52	70
5%	204	242	199	234	199	233	194	234	201	236
10%	228	264	226	262	205	245	223	258	228	265
15%	239	272	244	284	229	266	244	280	245	284
25%	268	305	271	314	277	317	273	313	272	315
95%	478	572	517	619	508	617	531	616	520	623
Max	610	725	819	970	643	768	698	814	747	895

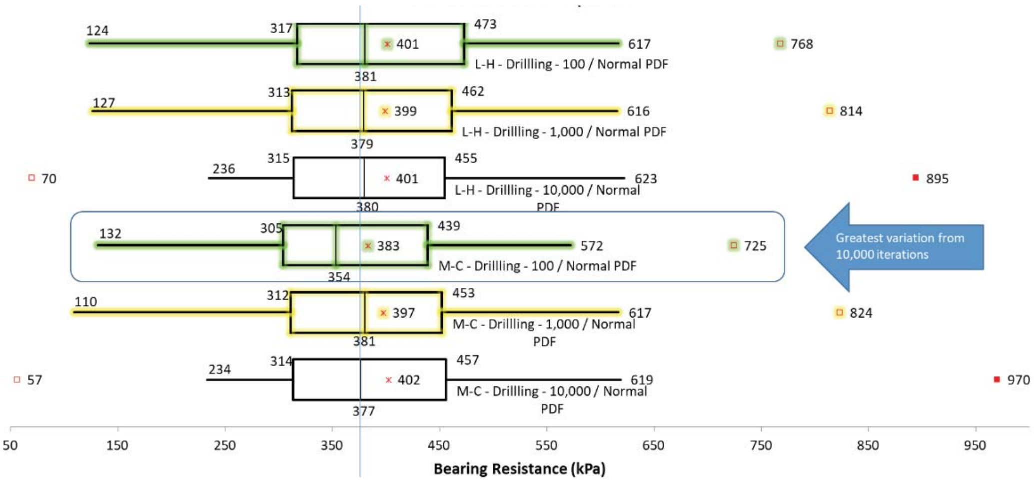

Figure 3. Box and Whisker plot shows the 100 iterations MC simulation varies from other simulations

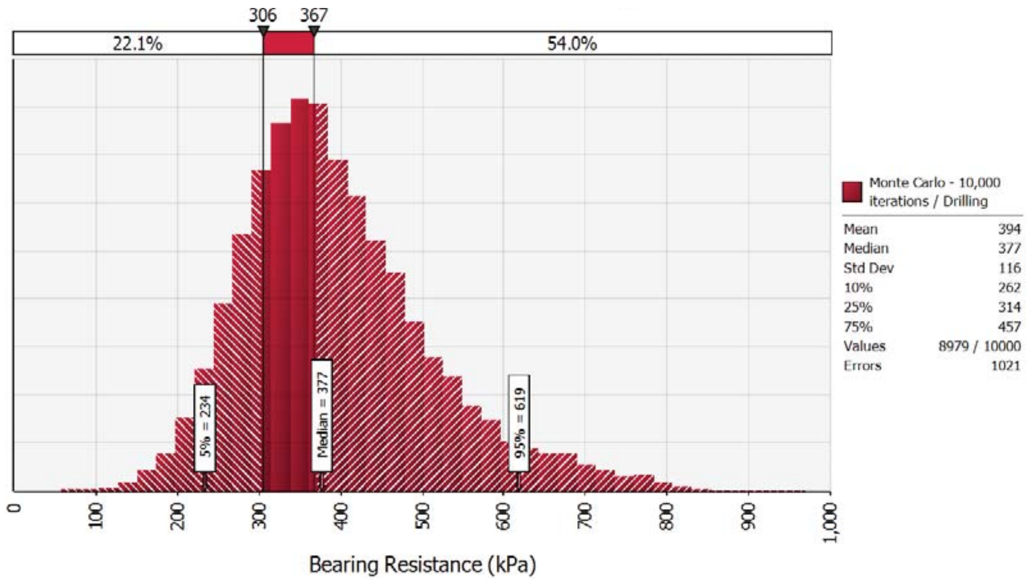

Figure 4. Bearing resistance for 10,000 MC. 306kPa is unfactored load and 367kPa is factored

Overall, the issues previously discussed associated with the normal PDF are now being transferred to the simulation output. Using these simulation model outputs would lead a practicing engineer to a poor rationale for risk decision making. The input of the normal PDF is examined further in the companion paper by using an alternate non normal PDF with "similar" input.

3 Settlement simulation

Another simulation example is for a settlement for a 10m thick soft clay under an embankment loading of up to 6m, as illustrated in Figure 5. The ground comprises 2m of sand, and 10m of clay, underlain by extremely weathered rock. It is reasonable to neglect compression within the sand and rock layers after the construction phase, hence only the 10m of clay is considered in the analysis.

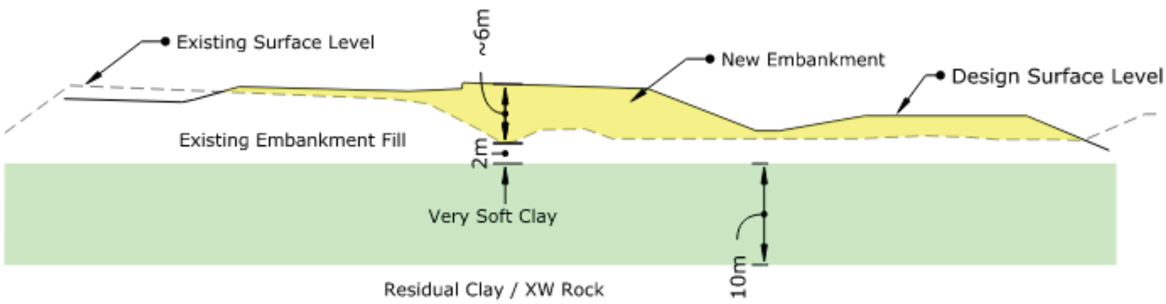

Figure 5. Typical cross section of a Motorway Upgrade Project

Six parameters were considered in this analysis, including unit weight, compression index C_c, initial void ratio e_0, thickness of clay, groundwater table, and embankment load. To simplify the problem, the soft clay was assumed to be normally consolidated and a fixed pre-consolidation value used in the analysis. Similar to the bearing capacity case, the adopted COV, mean value, standard deviation of each design parameter, and the likely values are tabulated in Table 3.

Table 3. Ground data input for the settlement simulation.

Design element	Parameter	COV (%)	Mean / Standard Deviation	Minimum / Likely / Maximum
Clay	Unit Weight (kN/ m^3)	9%[1]	17.0 / 1.53	16 / 17 / 19
	Compression Index, C_c	66%	0.45 / 0.30	0.04 / 0.42 / 0.90
	Initial Void Ratio, e_0	39%	0.97 / 0.38	0.50 / 0.97 / 1.73
	Thickness (m)	20%	10.0 / 2.0	8.0 / 10.0 / 12.0
Water Table	Depth (m)	/	1.5 / 0.5	1.0 / 1.5 / 2.0
Embankment Load	Load (kN)	10%[1]	136 / 13.6	130 / 136 / 142

[1] Phoon and Kulhawy, 1999

One-dimensional consolidation tests have been conducted from an accredited laboratory on eleven soft soil samples across the site to obtain key settlement parameters – compression index, C_c, and initial void ratio, e_0. These testing results have been statistically analyzed using three probability distribution functions – Normal (This paper), and Log-normal /PERT (Part 2 paper).

3.1 Analysis output comparing MC and LH simulations with a normal PDF

The tornado graph (Figure 6) shows the effect on the output mean for settlement analysis with a normal PDF, which indicate that the compression index governs the settlement analysis, with the groundwater level having the least effect. It should be noted that the high contribution of compression index, Cc, to settlement is due to not only its direct correlation, but also its high COV.

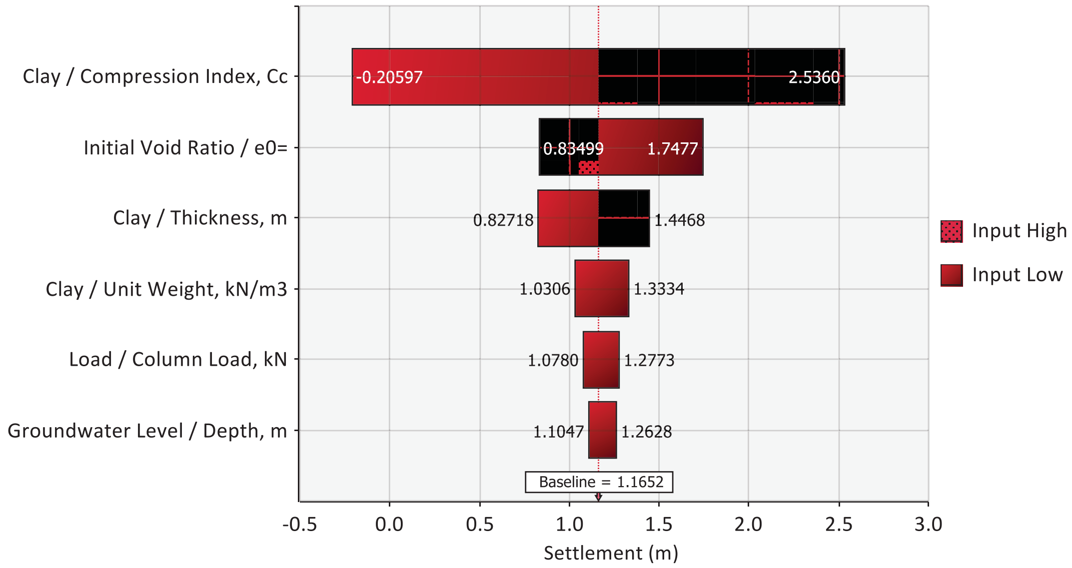

Figure 6. Tornado graph (10,00 iterations using MC) showing effect of input variability on settlement

The corresponding statistical output is compared in Figure 7 using MC and LH simulation models, for varying number of iterations with a normal PDF. This shows common similar results to those for bearing capacity:

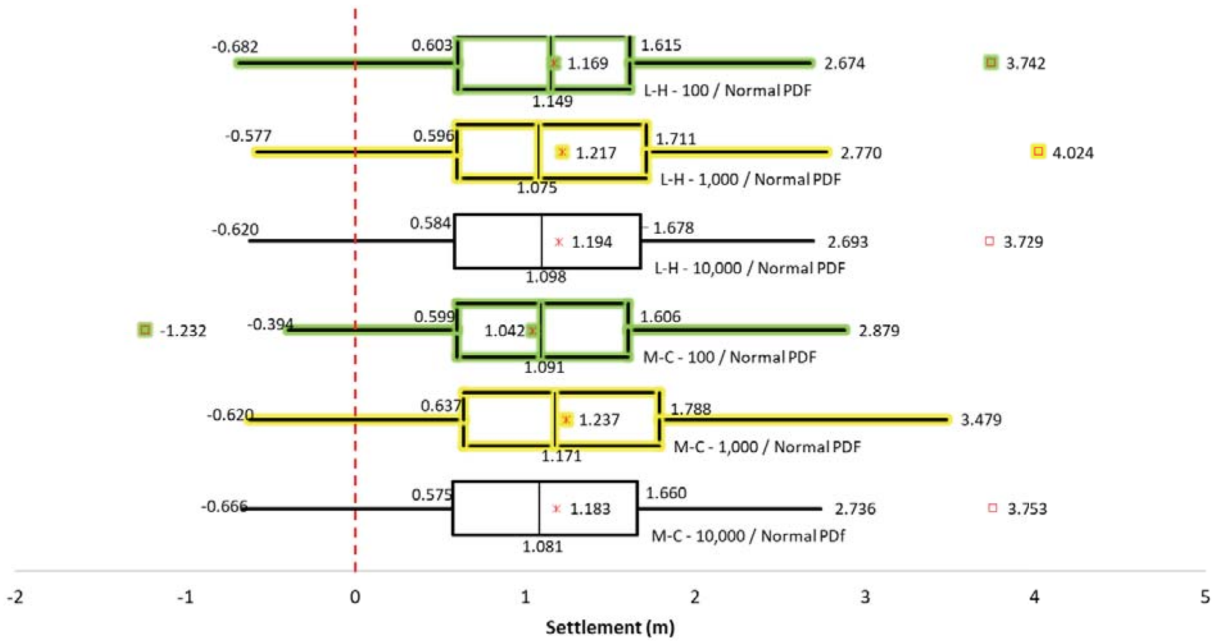

Figure 7. Box and Whisker plot shows the 100 iterations MC simulation varies from other simulations

- Using a normal PDF obtained questionable values, as all statistical analyses calculated negative settlement

● Output varies as simulations increase. Similar to the bearing capacity, the settlement of 100 MC simulations significantly deviates from other simulations (settlement) thus it is considered inaccurate, while the 100 LH simulation is broadly compatible to others.

● The 10,000 LH provides less variability than the MC simulation, but still has unrealistically low and high settlement values for the upper and lower 10% values

The probability distribution of settlement using 10,000 MC and a normal PDF is illustrated in Figure 8, which indicated 6.9% of settlement is negative (i.e., heave). This is obviously contradicting any field observation.

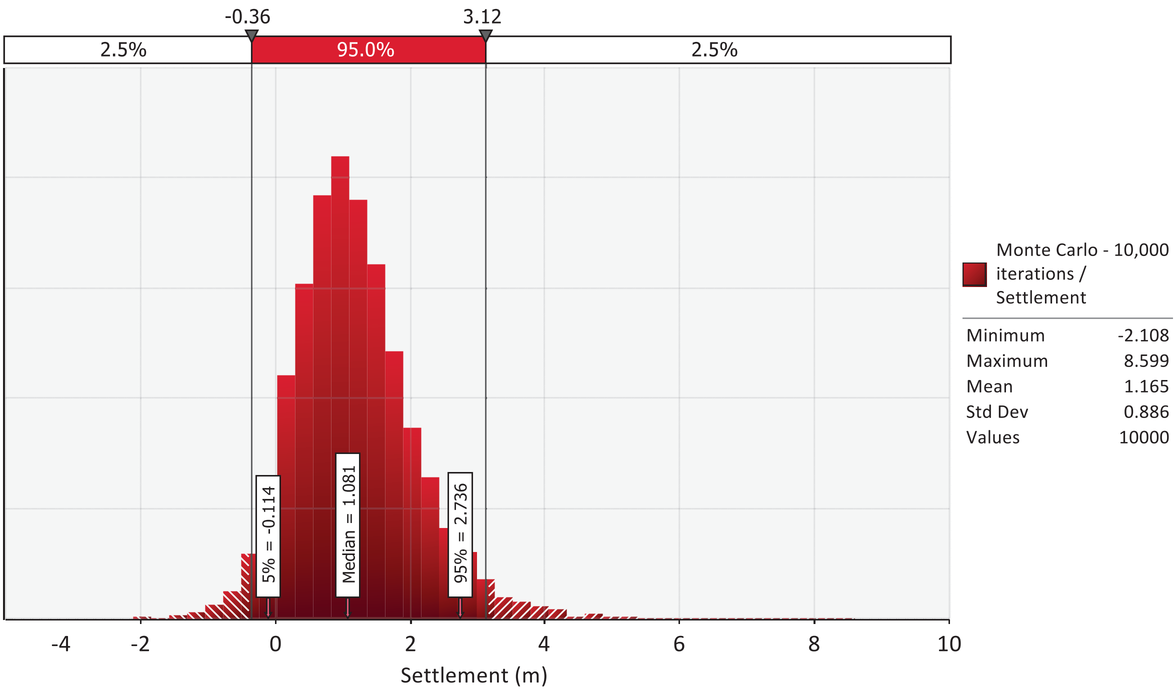

Figure 8. Settlement analysis with 10,000 simulations using MC sampling

4 Conclusions

This paper compares simulation results to show that the Latin Hypercube better represents low or high probability outcomes by forcing the sampling of the simulation to include low probability events. If a sufficiently high number of iterations is carried out then this issue is not apparent. The unrealistic "failure" in bearing capacity was shown using a normal PDF. Similarly negative settlements for a simulation of embankment settlements were shown. A designer can either truncate such unrealistic values or use a more realistic PDF. This will be examined in the companion paper (Part 2) using non normal PDFs.

References

BRE 470 (2004). Working Platforms for tracked plant. *Building Research Establishment, UK*

Duncan, J.M. (2000). Factors of safety and reliability in Geotechnical Engineering. *Journal of Geotechnical and Geoenvironmental Engineering*, 126(4), pp 307-316.

Lacasse, S. (2013). Protecting society from landslides – the role of the geotechnical engineer. *Proceedings of the 18th International Conference on Soil Mechanics and Geotechnical Engineering, Paris, France,* September 2-6, 2013.

Palisade Corporation 2015. @RISK User's Guide - Risk Analysis and Simulation. *Add-In for Microsoft Excel*, Version 7

Phoon K. and Kulhawy, F.H. (1999). Characterization of geotechnical variability. *Canadian Geotechnical Journal*, Volume 36, pp 612 – 624.

Phoon K-K and Kulhawy F.K. (2008). Serviceability limit state reliability-based design. *Reliability based design in geotechnical engineering* Ed Kok-Kwang Phoon, Taylor and Francis Publishers, pp 344 – 384.