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A Probability-Based Upscaling Model of Grading Evolution of Calcareous Sand

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Abstract: Calcareous sand is vulnerable to particle breakage, which is problematic to the offshore engineering project, such as the pile installation. In this study, the grading evolution of calcareous sand was therefore investigated, and an upscaling model based the probabilistic analysis was proposed, which is capable to predict the grading evolution at the mesoscale (i.e., the change of the particle size distribution) by using the breakage characteristics of single particles at the microscale (e.g., the breakage strength and fragment size distribution). In order to examine this model, the uniaxial compression tests on 273 single calcareous sand particles were carried out to provide the microscopic breakage characteristics. Then, the grading evolutions of three calcareous sand samples under the isotopically consolidation test were predicted using the upscaling model, and in parallel, the experimental results of these samples were also acquired for the comparison. Differences between the experimental results and the predicted results existed but were moderate, demonstrating the effectiveness of the proposed upscaling model in predicting the grading evolution of calcareous sand.

Keywords: Calcareous sand; particle breakage; degradation; probabilistic upscaling model.

1 Introduction

Calcareous sand, a marine sediment comprising > 50% calcium carbonate ($CaCO_3$), is relatively vulnerable to particle breakage, compared with the quartz sand which is made up of silicon dioxide (SiO_2). Apparently, this kind of soil is problematic to the offshore engineering project. A typical example can be seen in 1982 the North Rankin gas/oil production platform of Australia, where the driven pile met unexpectedly low bearing capacity in calcareous sand stratum (Coop 1993).

Beside the engineering problem, the particle breakage complicates the constitutive modeling of calcareous sand, as the breakage-induced grading evolution is related to the change of the elastic stiffness and the energy dissipation for the plasticity (Wang et al. 2020). Yet, a universal model to predict the grading evolution is lacked in calcareous sand, and the existing results show that the evolution depends on the type of calcareous sand, loading condition and others (Airey et al. 1988). In fact, the evolution is fundamentally and only regulated by the particle breakage; that is, to propose a universe model, the characteristics of particle breakage must be involved. Cheng and Wang (2018) have made an attempt on such a model by using the probabilistic analysis, but their model was idealized for the particles in the numerical simulation of the discrete element method (DEM), not generalized to the calcareous sand.

Therefore, leveraging the idea from Cheng and Wang (2018), a probability-based upscaling model for calcareous sand was proposed here, which was designed to predict the grading evolution at the mesoscale by using the breakage characteristics of single particles at the microscale. To examine this model, the uniaxial compression tests of the single particles were then conducted to provide the microscopic breakage characteristics. Thereafter, the grading evolutions of the calcareous sand samples were predicted using the model and compared with the experimental results.

2 Upscaling Model of Grading Evolution of Calcareous Sand

Considering a sample is subjected to an external loading, the calcareous sand particles contact with each other and the forces emerge at the contact position. The particle stress σ_c is then generated from the interparticle contact force, and subsequently, the calcareous sand particle may break, as the loading σ_c exceeds the strength of its material σ_b. Fragments are then produced from the broken particles at the microscale, resulting in the evolution of particle size distribution at the mesoscale. According to the above causality, an upscaling model can be proposed. Yet all issues in the causality exist with their own probability density function, including the particle stress from the contact force, the strength of material, and the fragment size, and therefore the probabilistic analysis should be used for the model establishment as

$$f_d^{i+1}(D) = f_d^i(D) + \int_D^{\infty}\int_0^{\infty}\int_0^{\sigma_c} f_d^i(x) f_b(\sigma_b|x) f_c(\sigma_c|x) f_f(D|x)\cdot \mathrm{d}\sigma_b \mathrm{d}\sigma_c \mathrm{d}x \tag{1}$$

where f_d^i and f_d^{i+1} are the probabilistic density functions (PDFs) of particle size before and after the loading, respectively; D is the particle size; f_c is the PDF of the particle stress induced by the interparticle contact force; f_b is the PDF of the particle stress at breakage or the strength of material; and f_f is the PDF of the fragment size.

It has to be noted that the simplifications have been made in this model. For instance, under the external loading, the chain effect of particle breakage may occur. Therefore, an iteration may be executed on Eq. (1) until the convergence is met. Besides, f_c, f_b and f_f may not independent of each other. Nevertheless, in the following section, this probability-based upscaling model is examined by comparing the predicted results and the experimental results on the grading evolution, and the model improvement can be achieved upon the examination in the future study.

3 Examination on Upscaling Model of Calcareous sand

3.1 Uniaxial compression tests on single particles

To examine the proposed upscaling model, the uniaxial compression tests were first carried out on single calcareous sand particles to provide the PDFs of the particle stress at breakage and the fragment size (i.e., f_b and f_f). The calcareous sand particles were retrieved from Yongshu Island, a reclaimed coral island located within the coordinates of 9°32'-9°42'N, 112°52'-113°04'E in South China Sea. 273 particles were selected by manual pick-up and then taken for the X-ray computed tomography (CT) scan before testing. Thereby, the morphology (including the size and volume) of the testing particles were measured, and the particles were separated into three groups, i.e., Group 1 with the particle size ranging within 2.5-3.5 mm, Group 2 with the particle size ranging within 3.5-4.5mm and Group 3 with the particle size ranging within 4.5-5.5 mm.

A tailor-made equipment for the uniaxial compression test was created from the conventional triaxial system, as shown in Figure. 1a. In the equipment, the particle was compressed to break by two load plate. The linear variable differential transformer (LVDT) and load cell were mounted to measure the force-displacement curve during the compression, as exemplified in Figure 1b. In addition, the high-speed camera was set up to capture the particle behavior, aiming to perceive the moment of particle breakage visually (Wang and Coop 2016). In turn, based on the observation from the high-speed camera, the test could be terminated immediately and the breakage stress can be determined from the force-displacement curve as

$$\sigma_b = F_b \cdot d / V_s \tag{2}$$

where σ_b is the particle stress at breakage; F_b is the loading force at breakage, as indicated in Figure 1b; d is the distance between the two load plates at breakage; and V_s is the volume of the testing particle, which was measured by the CT scan. Finally, the fragments from the broken particles were collected cautiously and photographed. Sizes of the fragments were further analyzed using the open-source software Image J instead of CT scan, as the size extracted from the photo can effectively and efficiently represent that from CT scan result (Chow et al. 2019; Su and Yan 2020).

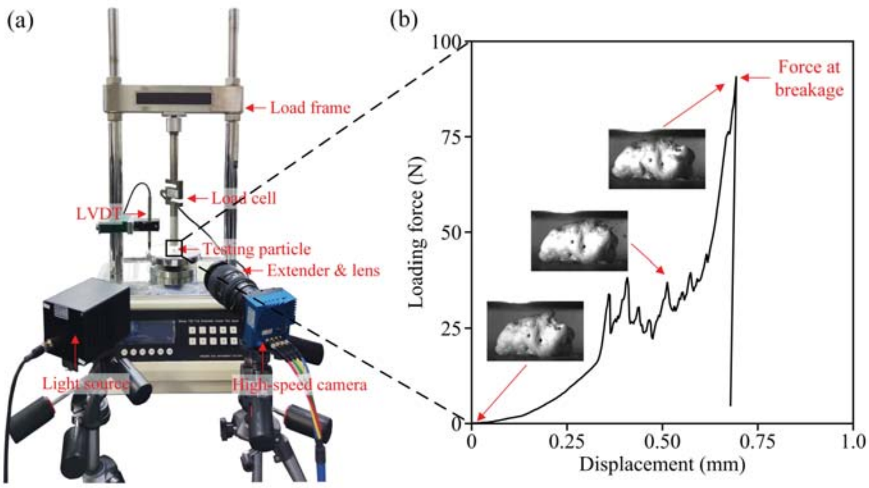


Figure 1. Experimental setup and determination of breakage strength.

Figure 2 shows the cumulative distribution functions (CDFs) and probability density functions (PDFs) of the breakage stress for Groups 1-3, and the gamma distribution was used to fit the data. Note that the stress has been normalized by the mean value of the 273 particles which is 12.0 MPa. As implied by the good agreement between the fitting curve and testing data for the CDFs, the breakage stress might inherently follow the gamma distribution in each group. Then, as indicated by the PDFs, both the mean value and fluctuation of the breakage stress increased with the increase of particle size. For instance, the mean and standard deviation of Group 1 are 14.2 MPa and 6.80 MPa, respectively, while those values of Group 3 are 10.2 and 4.55 MPa, respectively. Possibly, the effect of particle size is because the intra-particle flaw is less harmful to a larger particle. Therefore, towards the examination of the proposed upscaling model, the PDF of the particle stress at breakage f_b has been given in Figure 2 and can be further formulated by using the gamma distribution. Thereby, the effect of particle size can be represented by changing the parameters in the distribution.

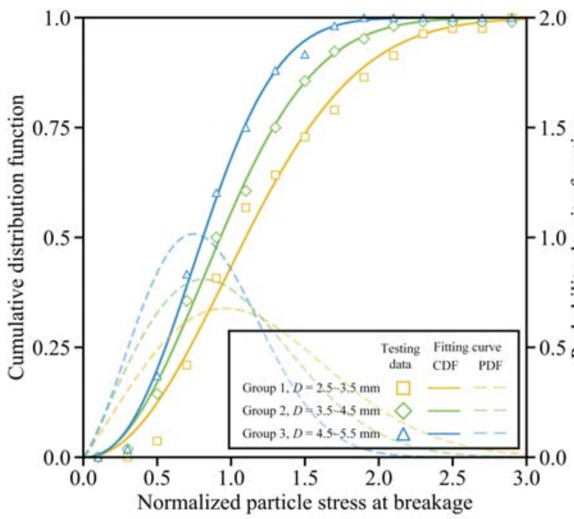


Figure 2. Distributions of breakage stress for Groups 1-3.

Figure 3 shows the histograms of fragment size for Groups 1-3. Obviously, two peaks emerged in each group; that is, the calcareous sand particle was broken into several small fragments (corresponding to the first peak) and large fragments (corresponding to the second peak) in the uniaxial compression test. The fragment size of the second peak is more or less proportional to the particle size, and subsequently, with the increase of particle size, the distance between the two peaks increased. In essence, the first peak of small fragment might originate from the scale smaller than particle breakage, e.g., contact fracturing and localized chipping, while the second peak of large fragment might originate mainly from the scale of particle breakage, e.g., particle splitting and explosion. Therefore, towards the examination of the upscaling model in this study, the PDF of the fragment size f_f has been also given by Figure 3, and can be further described by a mixture of two gamma distribution, as indicated by the dashed red lines in Figure 3. Again, the particle size effect can be characterized by changing the parameters of the gamma distribution for the second peak.

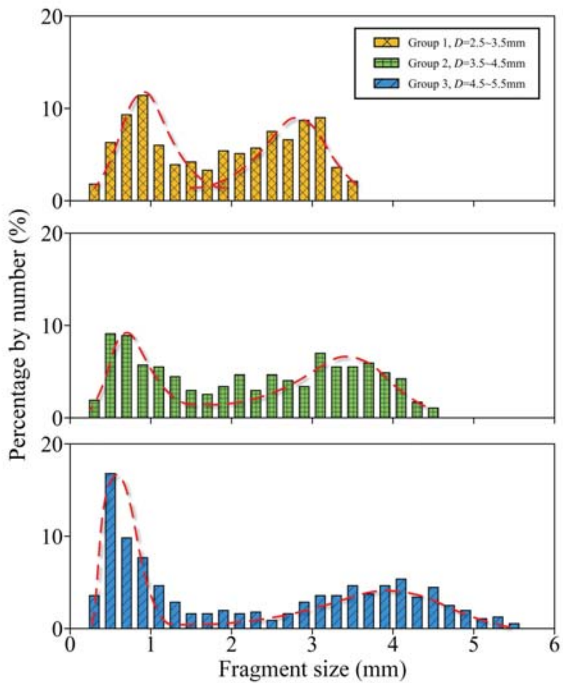


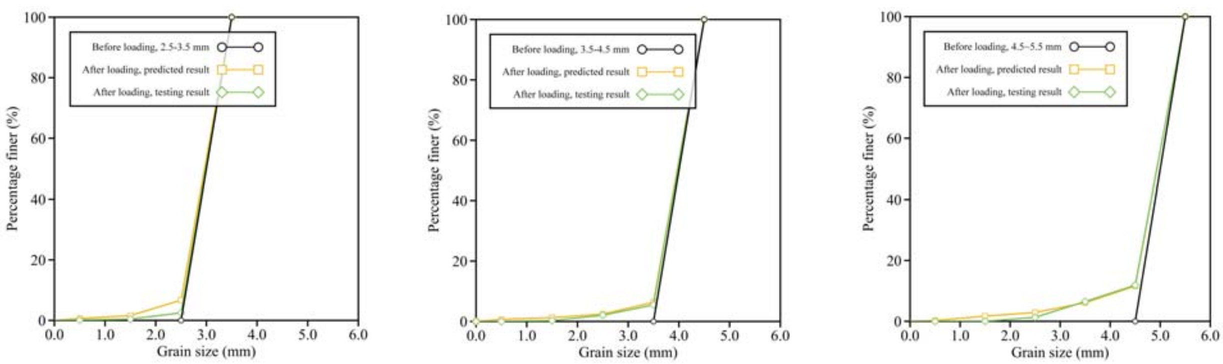
Figure 3. Histograms of fragment size after particle breakage for Groups 1-3.

According to Eq. (1), in order to examine the upscaling model, the PDF of particle stress from the inter-particle contact force f_c is also required. In fact, using the DEM simulation or the experimentation with the photoelastic technique, the distribution of f_c has been widely studied and well understood (Oda and Konishi 1974; Li et al 2017; Li et al. 2022). Specifically, the particle stress transmitted in packing follows a bimodal distribution, i.e., a power-law decay with a crossover to an exponential cut-off (Radjai et al. 1996). The changes of loading condition and packing density only alter the parameters in the distribution (O'Hern et al. 2001). Therefore, till now, f_c, f_b and f_f can be prepared for the examination of the upscaling model.

3.2 Result comparison on grading evolution

Based on the above testing results and the numerical findings from DEM simulation, f_c, f_b and f_f could be provided in a specific scenario, and a preliminarily examination on the upscaling model was enabled here. Three mono-sized sample were used in the examination, i.e., Sample A with the particle size ranging 2.5-3.5 mm, Sample B with the particle size ranging 3.5-4.5 mm, and Sample C with the particle size ranging 4.5-5.5 mm. All samples had a relative density as 90%. The loading scenario was chosen as the isotropic consolidation test, where the loading primarily compresses the particles along the contact normal; that is, the fragments are produced mainly by the particle breakage, instead of the wear of contact by interparticle shearing (Thornton 2000; Yuan et al. 2019). The confining pressure for consolidation was set as 1.5 MPa, which could produce a distinct mount of particle breakages on the sample after loading. Under these configurations, the grading evolutions for Samples A-C were predicted using the upscaling model in Eq. (1), and the predicted results after loading are shown in Figure. 4 (see the yellow lines). Note that to simplify the calculation in the model, the confining pressure was applied instantly as one step loading, instead of several step loadings, and the chain effect of particle breakage was not allowed; that is no iteration was executed on the model.

In parallel, the isotropic consolidation tests on Samples A-C with the relative density of 90% were carried out in the laboratory, and the particle size distributions after test were also measured and again plotted in Figure. 4 (see the green lines). As can be seen in Figure. 4, both the predicted and experimental results indicates that the particle breakages occurred largely in each sample, as a large number of fragments were created. In addition, the predicted results well matched with the experimental results, especially in Figure 4b and 4c for Samples B and C, demonstrating the effectiveness of the proposed upscaling modeling in predicting the grading evolution of calcareous sand. Still, the differences existed, and the fragments in the experiments were generally finer than those in the prediction. Probably, by considering the chain effect of the particle breakage and the real loading process of the confining pressure, the predicted results can be improved and become closer to the experimental results, which will be carried out in the future study.



(a) (b) (c)

Figure 4. Result comparison on the grading evolution in: (a) Sample A; (b) Sample B; and (c) Sample C.

4 Conclusion

In this study, a probability-based upscaling model of grading evolution of calcareous sand has been proposed and examined. The salient findings of this study are as follows.

- In the upscaling model, the grading evolution at the mesoscale is related to probability density functions of the particle stress induced by the interparticle contact force, the particle stress at breakage and the fragment size at the microscale.
- To examine the model, the uniaxial compression tests on 273 single calcareous sand particles were conducted to provide the microscopic breakage characteristics. The breakage strength followed the gamma distribution, and the fragment size distribution followed the mixture of two gamma distribution.
- The grading evolutions of three mono-sized calcareous sand samples under the isotropic consolidation test were obtained both by the upscaling model and the experimentation, respectively. Differences between the experimental results and those by the model were moderate, demonstrating the effectiveness of the upscaling

model, and they may be eliminated by considering the chain effect of particle breakage and real loading process of confining pressure.

Note that the examination here is limited to the samples with the mono-sized particles and the relative density of 90% under the isotropic confinement of 1.5 MPa. Future study will be extended to different initial particle size distribution, relative density and stress state, and then the effects of sampling conditions and loading conditions on grading evolution of calcareous sand can be further investigated.

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