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Calibration of Resistance Factor Based on Pile Load Test Conducted to Failure

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Abstract: In Load and Resistance Factor Design (LRFD), the resistance factor for pile foundation is used to account for uncertainties in pile capacities.With pile load tests conducted, the uncertainties associated with pile capacity prediction can be greatly reduced, thus, the resistance factor can be updated based on theload test results. In this paper, a probabilistic approach based on Bayes' theorem and the Monte Carlo Simulation (MCS) is proposed to calibrate the resistance factor based on pile load tests conducted to failure. To illustrate the proposed approach, the pile capacity predicted by the standard penetration test (SPT) is adopted. The measured pile capacity is normalized by the predicted pile capacity to obtain the resistance bias factor. Parametric studies are performed to evaluate the effect of the number of load tests, mean and minimum resistance bias factors on the resistance factor. Results show that the resistance factor increases when the number of load tests increases if all resistance bias factors are larger than 0.75. The resistance factor increases when the mean resistance bias factor increases. For the same mean resistance bias factor, the resistance factor decreases as the minimum resistance bias factor decreases. The results may explain why the specification in Eurocode 7 recommends the equivalent resistance factors depending on the mean and minimum measured pile capacities in load tests.

Keywords: LRFD; resistance factor; pile load test; Bayes' theorem; MCS.

1 Introduction

In recent decades, geotechnical design codes have beengradually migrating towards reliability-based design (RBD)(Fenton et al., 2016; Tang et al., 2019).A number of geotechnical RBD codeswere developed worldwide, such as the Canadian Highway Bridge Design Code (CSA, 2014), the Load and Resistance Factor Design (LRFD) Bridge Design Specifications (AASHTO, 2014) in the United States, Eurocode 7 (CEN, 2004) in Europe, and the Australian Standard for Piling-Design and Installation (AS, 2009).

The LRFD is a simpler variant of the RBD, in which the uncertainties in resistance are quantified by the resistance factor.It is generally accepted that higher resistance factors should be used when pile load tests are performed, and lower resistance factors should be used when pile load tests are not prescribed.For example, the Australian Standard for Piling-Design and Installation (AS, 2009)suggestsresistance factors range from 0.4 to 0.9, depending onthe percentage of piles tested by static load tests.However, these values are mainly determined by engineering judgement(Rausche et al., 2012; Huang et al., 2016). Currently, research efforts focus on updating pile capacities based on load tests(Zhang, 2004; Najjar & Gilbert, 2009; Abdallah et al., 2015; Huang et al., 2016), butvery limited studies concerning the calibration of resistance factorsbased on load tests within the LRFD framework.Zhang and Tang (2002)adopted an analytical solution to update pile capacities based on the mean measured capacity and used the First Order Second Moment (FOSM) to calculate resistance factors. However, the effect of minimum test results was not considered. Besides, Kwak et al. (2010) indicated that the FOSMmight not be reliable,and the resistance factors calculatedbythe FOSM were about 4-19% less than that obtained by the First Order Reliability Method (FORM) and the Monte Carlo Simulation (MCS).

In this paper, a probabilistic approach based on Bayes' theorem and the MCS is proposed to calibrate the resistance factor based on load tests conducted to failure. The first part of this paper illustrates the proposed approach for calibrating resistance factors based on load test results. The second part of this paper investigates the effect of the number of load tests, mean and minimum resistance bias factors on the resistance factors.

2 Methodology

In LRFD, the resistance factor is calibrated based on the statistics of load and resistance bias factors, which are defined as the ratio of measured to predicted values (Zhang et al., 2006; Yoon et al., 2008).Generally, the load bias factors are adopted from superstructure analysis(Paikowsky, 2004), and the empirical distribution of resistance bias factors is constructed based on existing load test databases(Zhang, 2004). When additional load tests are conducted, the distribution of resistance bias factorsis updated by the load test results, and the resistance factor is calibrated based on the updated resistance bias factors.Therefore,the proposed approach contains two main parts:one is the calibration of resistance factors in LRFD based on the statistics of resistance bias factor, and theother is updating the resistance bias factor with load test results.

2.1Calibration of resistance factors in LRFD

In LRFD, if only dead and live loads are considered, the design equation is as follows:

$$\phi R_n \geq \gamma_D Q_{Dn} + \gamma_L Q_{Ln} \tag{1}$$

where ϕ, γ_D and γ_L are the resistance factor, dead load factor and live load factor, respectively. R_n, Q_{Dn} and Q_{Ln} are the nominal values for resistance, dead load and live load, respectively.

The limit state function used to calibrate resistance factors is derived asfollows(Kwak et al., 2010; Tang et al., 2019):

$$g = \left[\frac{\lambda_R}{\phi} \times \left(\gamma_D \times \kappa + \gamma_L \right) \right] - \left(\lambda_D \times \kappa + \lambda_L \right) = 0 \tag{2}$$

where $\kappa = Q_{Dn} / Q_{Ln}$. λ_R, λ_D and λ_L are bias factors for resistance, dead load and live load, respectively.

AMATLAB program is developed to calibrate the resistance factors based on the MCS method.The steps are summarized as follows:

Step 1: Define the limit state function.

Step 2: Determine the target reliability index β_T.

Step3: Select a trial resistance factorand generate samples for the random variables(λ_R, λ_D and λ_L).

Step 4: Find the number of cases where $g < 0$, N_f ,the probability of failure is given by $P_f = N_f / N$ (N is the total number of Monte Carlo simulations, which is 1000000here).Calculate the reliability index $\beta = -\Phi^{-1}(P_f)$, where Φ^{-1} is the inverse cumulative distribution function of the standard normal distribution.

Step 5: Compare the reliability index β with the target reliability index β_T, and repeat steps 3-4 until $|\beta\text{-}\beta_T| \leq 0.01$.

2.2 Updating the resistance bias factor with load tests

Suppose n piles are tested to failure under a specific failure criterion (e.g., Davisson's criterion).The measured pile capacityis normalized by the predicted pile capacity (e.g., SPT method) to obtain the corresponding resistance bias factor. It should be noted that the pile capacities defined by different failure criteria,and the pile capacities predicted by different design methods are considerably different (Zhang et al., 2005; Tang & Phoon, 2018), which would further lead to significant differences in resistance factors.Whitman (1984) indicated that the within-site distribution of resistance bias factor λ_R can be assumed to be lognormally distributedwith the mean μ_{λ_R} and standard deviation σ_{λ_R} .Therefore, the likelihood that the measured resistance bias factor is λ :

$$L\left(\lambda_R = \lambda\right) = L\left(\ln \lambda_R = \ln \lambda\right) = \exp\left(-\frac{\left(\ln \lambda - \mu_{\ln \lambda_R}\right)^2}{2\sigma_{\ln \lambda_R}^2} \right) \tag{3}$$

where $\sigma_{\ln \lambda_R} = \sqrt{\ln\left(1 + \left(\sigma_{\lambda_R} / \mu_{\lambda_R}\right)^2\right)}$ and $\mu_{\ln \lambda_R} = \ln \mu_{\lambda_R} - 1/2\, \sigma_{\ln \lambda_R}^2$.

The within-site variability ($COV_{\lambda_R} = \sigma_{\lambda_R} / \mu_{\lambda_R}$) describes the variation of pile capacities among the same site, which is resultedfrom the variation of material properties, dimensional errors and construction quality(Evangelista et al., 1977).In this paper, COV_{λ_R} is assumed to beknown and μ_{λ_R} is treated as a lognormal random variable. If themeasuredresistance bias factoris denoted as λ_i, $i = 1, 2, ..., n$, the likelihood distribution of $\mu_{\ln \lambda_R}$ is:

$$L\left(\mu_{\ln \lambda_R}\right) = \exp\left(-\frac{\sum_{i=1}^{n} \left(\ln \lambda_i - \mu_{\ln \lambda_R}\right)^2}{2\sigma_{\ln \lambda_R}^2} \right) \tag{4}$$

Theprior distribution of $\mu_{\ln\lambda_R}$ withthe mean $\mu'_{\ln\mu}$ and standard deviation $\sigma'_{\ln\mu}$ can be derivedfromthe prior distribution of λ_R and within-site variability(Zhang & Tang, 2002; Huang et al., 2016). The prior distribution of $\mu_{\ln\lambda_R}$ is as follows:

$$f'\left(\mu_{\ln\lambda_R}\right)=\exp\left(-\frac{\left(\mu_{\ln\lambda_R}-\mu'_{\ln\mu}\right)^2}{2\sigma'^{2}_{\ln\mu}}\right) \tag{5}$$

The posteriordistributionof $\mu_{\ln\lambda_R}$ iscalculated based onBayes' theorem(Ang & Tang, 2007):

$$f''\left(\mu_{\ln\lambda_R}\right)\propto\exp\left(-\frac{\sum_{i=1}^{n}\left(\ln\lambda_i-\mu_{\ln\lambda_R}\right)^2}{2\sigma^2_{\ln\lambda_R}}\right)\times\exp\left(-\frac{\left(\mu_{\ln\lambda_R}-\mu'_{\ln\mu}\right)^2}{2\sigma'^{2}_{\ln\mu}}\right) \tag{6}$$

The Delayed RejectionAdaptive Metropolis (DRAM) (Haario et al., 2006)is adoptedto sample the mean and standard deviation of $f''\left(\mu_{\ln\lambda_R}\right)$. The mean $\mu''_{\ln\lambda_R}$ and standard deviation $\sigma''_{\ln\lambda_R}$ ofthe posterior distribution of $\ln\lambda_R$ is obtained by the statistics of $f''\left(\mu_{\ln\lambda_R}\right)$ and within-site variability. Finally, the mean μ''_{λ_R} and standard deviation σ''_{λ_R} for the posterior distribution of the resistance bias factor is obtained by Eq. (7). These two values are used to calibrate resistance factors.

$$\begin{aligned}\mu''_{\lambda_R}&=\exp\left(\mu''_{\ln\lambda_R}+\frac{1}{2}\sigma''^{2}_{\ln\lambda_R}\right)\\ \sigma''_{\lambda_R}&=\mu''_{\lambda_R}\sqrt{\exp\left(\sigma''^{2}_{\ln\lambda_R}\right)-1}\end{aligned} \tag{7}$$

3 Example

The summary of parameters used inthe exampleis listed in Table1. The dead load factor γ_D and live load factor γ_L are different for various codes, generally, γ_D ranges from 1.00 to 1.40 while γ_L ranges from 1.30 to 1.75 (Goble, 1999). The load bias factors λ_D and λ_L are treated as lognormal random variables with mean (μ_{λ_D} and μ_{λ_L}) and coefficient of variation (COV_{λ_D} and COV_{λ_L});these values are adopted from Paikowsky (2004). The pile capacity predicted by the standard penetration test (SPT) is adopted, for which the prior mean μ'_{λ_R} and coefficient of variation COV'_{λ_R} of λ_R were investigated by Orchant et al. (1988). The ratio of dead and live load κ has a wide range for various constructions, but previous studies (McVay et al., 2000; AbdelSalam et al., 2011)showed that κ has a small effect on the resistance factors. The target reliability index β_T is critical for the calibration of resistance factors and $\beta_T=2.33$ is adopted in this paper, which is used for the design of pile groupsin AASHTO (2014).The within-site variability COV_{λ_R} is adopted from Zhang (2004).

Table1. Summary of parameters used in the example

Parameter		Value
Dead load factor γ_D		1.25
Live load factor γ_L		1.75
Dead load bias factor λ_D	μ_{λ_D}	1.05
	COV_{λ_D}	0.1
Live load bias factor λ_L	μ_{λ_L}	1.15
	COV_{λ_L}	0.2
Resistance bias factor λ_R	μ'_{λ_R}	1.30
	COV'_{λ_R}	0.5
Parameter		Value
Ratio of dead and live load κ		3
Target reliability index β_T		2.33
Within-site variability COV_{λ_R}		0.2

3.1Effect of the number of tests

When the measured resistance bias factors are the same for all the tested piles, the effect of the number of tests on the resistance factors for different measured resistance bias factorsis shown in Figure 1. Whenthe measuredresistance bias factor λ = 0.75, 1.0, 1.25 and 1.5, the resistance factors increase as the number of tests increases. However, the change of resistance factors is insignificant when the number of tests is larger than three. For example, when λ = 1.25, the resistance factor increases from 0.49 to 0.91 when the number of tests increases from zero to three, and only slightly increases to 0.96 when the number of tests continually increases to ten.In contrast, when λ = 0.5 and 0.2, the resistance factors decrease as the number of tests increases.However, the resistance factors become almost constant when the number of tests is larger than one. For instance, when λ = 0.5, the resistance factor decreases from 0.49 to 0.39with one load test conducted, and the resistance factor keeps unchangedwith more load tests conducted.

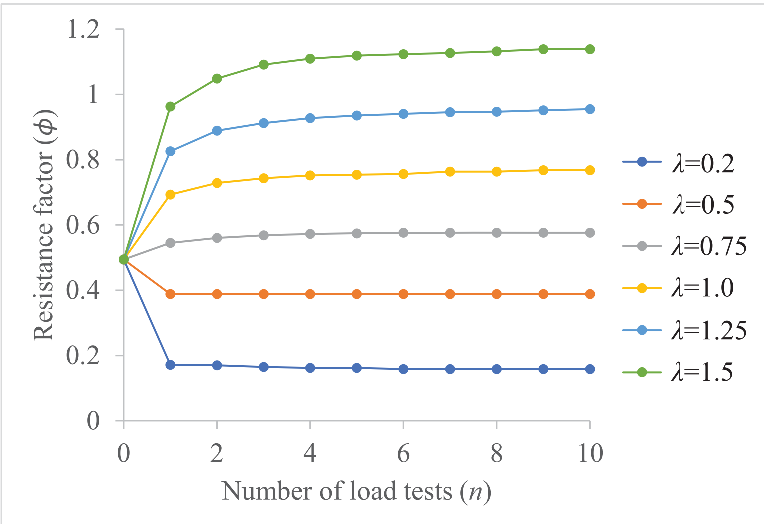

Figure1. Resistance factors as a function of the number of tests and the measured resistance bias factors.

3.2 Effect of the mean resistance bias factor

When the measured resistance bias factors are the same for all the tested piles, the effect of the mean resistance bias factor on the resistance factors for different numbers of testsis shown in Figure 2.The initial resistance factor ($n=0$), which is calibrated based on the prior distribution of the resistance bias factor, is also shown in Figure 2 for comparison. Figure 2 shows that the resistance factors increasealmost linearly with the mean resistance bias factor.When $n=1$, the resistance factor increasesfrom0.17 to 0.96 when the measured resistance bias factor increases from 0.2 to 1.50. It can be seen from Figure 2 that the resistance factor is increased compared to the initial resistance factor when the mean resistance bias factor is larger than a thresholdvalue, while the resistance factor is decreasedcompared to the initial resistance factor when the mean resistance bias factor is less than the threshold value.For example, when $n=1$, the resistance factor is larger than the initial resistance factor, if the measured resistance bias factor is larger than 0.67.It is also noted that the threshold valuesare insensitive to the number of tests. As shown in Figure 2, the threshold valuesare 0.67 and 0.64 for $n=1$ and $n=10$, respectively. In engineering practice, the threshold value can be used as the minimum acceptable results ofload tests for design verification.

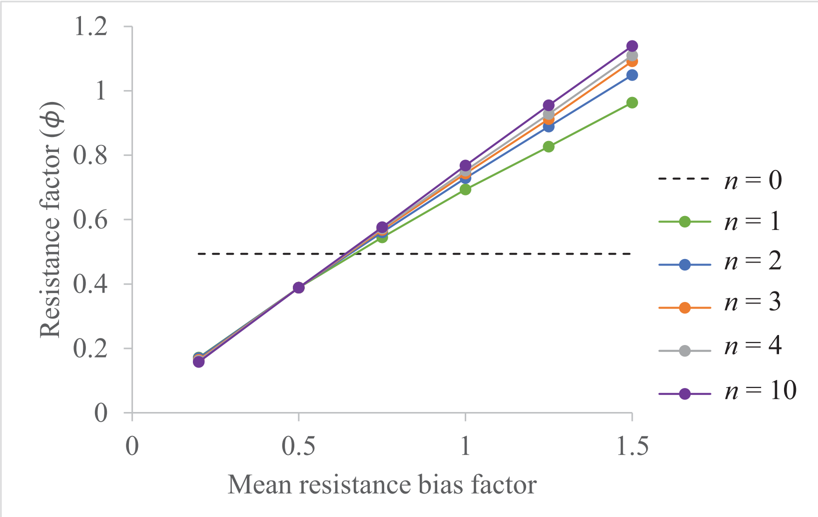

Figure2. Resistance factors with differentmean resistance bias factors.

3.3 Effect of the minimum resistance bias factor

Table 2 shows how the resistance factors vary with the minimum resistance bias factors while the mean resistance bias factor is1.0.It can be seen from Table 2 that the resistance factors decrease as the minimum resistance bias factors decrease. For example, when $n=2$, the resistance factor decreases from 0.73to 0.46 when the minimum resistance bias factor decreases from 1.0 to 0.2.This may explain why Eurocode 7 recommends the equivalent resistance factors depending on the mean and minimum measured pile capacities.For each row in Table2, the minimum and maximum resistance bias factors are the same while the number of tests with $\lambda_i=1.0$ increases, resulting in the resistance factor increasing.This is consistent with section 3.2 that the resistance factor increases if the measured resistance bias factor islarger than thethreshold value (i.e., about 0.67).

Table2. Summary of resistance factors for variousmeasured resistance bias factors

n=2		n=3		n=4	
λ_i	ϕ	λ_i	ϕ	λ_i	ϕ
1.0, 1.0	0.73	1.0, 1.0, 1.0	0.74	1.0, 1.0, 1.0, 1.0	0.75
0.9, 1.1	0.72	0.9, 1.0, 1.1	0.74	0.9, 1.0, 1.0, 1.1	0.74
0.8, 1.2	0.72	0.8, 1.0, 1.2	0.73	0.8, 1.0, 1.0, 1.2	0.74
0.7, 1.3	0.70	0.7, 1.0, 1.3	0.72	0.7, 1.0, 1.0, 1.3	0.73
0.6, 1.4	0.67	0.6, 1.0, 1.4	0.70	0.6, 1.0, 1.0, 1.4	0.72
0.5, 1.5	0.63	0.5, 1.0, 1.5	0.68	0.5, 1.0, 1.0, 1.5	0.70
0.4, 1.6	0.59	0.4, 1.0, 1.6	0.64	0.4, 1.0, 1.0, 1.6	0.67
0.3, 1.7	0.53	0.3, 1.0, 1.7	0.60	0.3, 1.0, 1.0, 1.7	0.64
0.2, 1.8	0.46	0.2, 1.0, 1.8	0.54	0.2, 1.0, 1.0, 1.8	0.59

4 Conclusions

This paper proposes a probabilistic approach based on Bayes' theorem and the Monte Carlo Simulation (MCS) to calibrate the resistance factor based on pile load tests conducted to failure.The effect of the number of load tests, mean and minimum resistance bias factors on the resistance factorsis investigated. The following conclusionscan be made:

1. Most of the change in resistance factors is achieved with a small number of tests.For the measured resistance bias factors are 0.75, 1.0, 1.25 and 1.50,the change of resistance factors is insignificant with the number of tests when the number of tests is larger than three. For the measured resistance bias factors are 0.5 and 0.2, the resistance factors are almost constant when the number of tests is larger than one.

2. Thereis a threshold value of the measured resistance bias factor, for which the resistance factor increases if the measured resistance bias factor is larger than the threshold value. In engineering practice, the threshold value can be used as the minimum acceptable results of load tests for design verification.

3. The mean and minimum measured pile capacities have a significant effect on resistance factors.Both of them need to be considered in pile designs based on load test results.

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