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# Linear-System-Type Surrogate Model for Large-Scale Earth-Retaining Work Based on Dynamic Mode Decomposition

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**Abstract:** In this study, we developed a surrogate model that could efficiently calculate the displacement behavior of a wall to construct a robust design method and real-time control system for earth-retaining walls in large-scale underground spaces. Based on the construction of a dynamic mode decomposition approximation model with a focus on the cantilever state, we constructed a model that enables proxy calculations for various conditions using the stiffness matrix of the beam–spring model for conversion to various strut placement situations. Finally, the effectiveness of the surrogate model was verified from the viewpoints of the reproducibility of the elastoplasticity analysis results based on a simple problem setup and connectivity to the optimal design based on the optimization calculations of the strut placement.

Keywords: dynamic mode decomposition; reduced-order models; inverse analysis; time-series analysis; real-time control

## 1 Introduction

When constructing a large underground space, temporary structural walls, called earth retaining walls, are constructed, and struts and other support structures are placed to ensure the stability of the earth-retaining wall. In this case, it is necessary to achieve serviceability (to secure a large space for as long as possible) and safety (to prevent the collapse of the retaining wall) to improve the workability in the excavation space. This design and planning problem is complicated by the characteristics of geomaterials deposited around earth-retaining walls. Although the ground is multilayered and heterogeneously distributed, it cannot be fully understood at the design stage; therefore, the structural design must be robust (insensitive) to ground-derived uncertainties (Shih-Hsuan et al. 2012, Jianye et al. 2017, Otake et al. 2019). Therefore, it is crucial to treat the reliability assessment of earth-retaining walls as a site-specific problem, depending on the structure and environmental conditions of the target earth-retaining wall. In this design and planning problem, not only the structural properties, such as the stiffness of the wall and struts, but also 1) the location and time of the strut placement, 2) the magnitude and time of the forced load from the strut, and 3) the observation results during construction should be considered and reviewed sequentially. Therefore, it is necessary to develop a mathematical model formulated as a decision-making problem under uncertainty and contribute to robust design and real-time control.

Recent developments in data-driven science enable us to elucidate important principles from vast amounts of spatiotemporal information and quickly perform proxy calculations of complex physical phenomena. In previous studies (Otake et al. 2018, 2021), we showed that by applying modal decomposition (eigen orthogonal decomposition) to numerical results (spatiotemporal data of physical indicators), proxy calculations can be performed while maintaining the spatiotemporal characteristics of the ground. In this study, we investigated the use of dynamic mode decomposition (DMD, e.g., Kutz et al. 2016, Arai et al. 2021), a data-driven science method. DMD is a mode decomposition method proposed for fluid dynamics and is a relatively recent mathematical innovation. It is a dimensional contraction model in which dynamic and complex nonlinear physics are divided into time and space variables and represented by the mode superposition governed by eigenvalues. Because an exponential function approximates the time evolution, the future behavior of the target system is simplified, and optimal control reflecting the future evolution of the target physics may be efficiently implemented.

This study focuses onestablishing a robust design and real-time control model for the stiffness of earth-retaining walls and support arrangement planning. We utilized the displacement behavior of the retaining wall as the training data obtained from the numerical results, and a hybrid surrogate model was developed by integrating the DMD algorithm and beam theory on an elastic foundation. A characteristic approach reduces the surrogate model to a form of linear differential equation (a linear system). This principle is based on the ease of coupling with optimization theory and is intended to be extended to robust design methods and real-time control models in the future.

## 2 Configuration problem

### 2.1Target excavation section

Figure 1(a) shows a schematic of the retaining wall analyzed in this study. Table 1 lists the basic specifications of the earth-retaining wall. A hypothetical excavated ground with a wall length of L=20m and excavation depth of h=10m was assumed. The displacement behavior of the wall was evaluated when the overburden load, q, was increased from 0 to 30 $\mathrm{kN/m^2}$ at regular time intervals with $\Delta$q=1 $\mathrm{kN/m^2}$, assuming that the excavation was completed. The total number of steps, m, was 31.

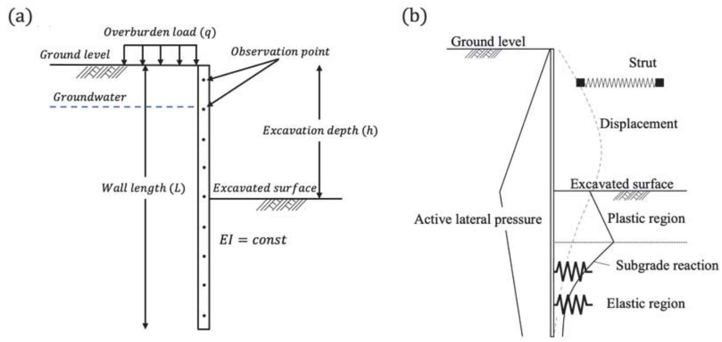


**Figure 1.** (a) Schematic of target excavation section and (b) conceptual diagram of elastoplastic method

**Table 1.** Structural and geotechnical parameters

| Parameter | Value | Unit |
|---|---|---|
| Wall length | 20 | m |
| Observation points | 41 | |
| Groundwater level | −1 | m |
| Young's modulus | $2.00 \times 10^8$ | $\mathrm{kN/m^2}$ |
| Second moment of area | $6.89 \times 10^{-4}$ | $\mathrm{m^2}$ |
| Excavation depth | 10 | m |

### 2.2Basic analysis methods and ground scenarios

The displacement calculation method of the retaining wall was based on the elastoplastic method used in the Japanese design standards (Otake et al. 2019). A conceptual diagram of the calculation model is shown in Figure 1(b). The retaining wall was modeled as a beam, and the ground was modeled as a spring. The effective lateral pressure, which is the earth pressure at rest subtracted from the active earth pressure, was applied to the back of the wall, and the ground reaction force on the excavation side (resistance side) was modeled as a fully elastoplastic spring with shear strength as the upper limit. Spatial variation was applied to the deformation coefficients (Figure 2). We established a linear system proxy model for the four spatial distributions generated by the stochastic process and discussed the effect of variation on the linear system surrogate model.

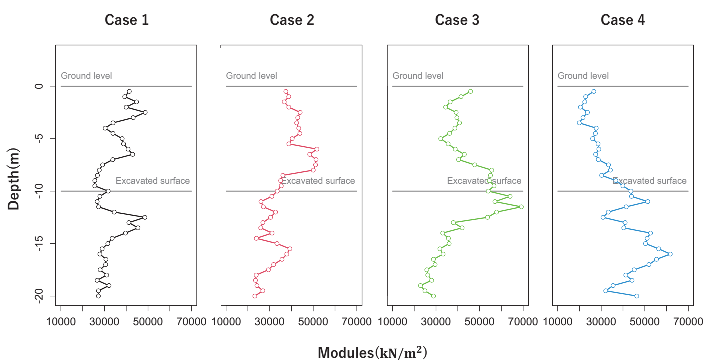


**Figure 2.** Four geotechnical scenarios (spatial distribution of deformation coefficients)

## 3 Research methods

### 3.1Proxy calculation model for each geological scenario

For a specific ground scenario $i$, the time evolution of the wall displacement distribution $\boldsymbol{u_i}$ is approximated by a linear system using the following equation. Additionally, this vector stores the displacement and rotation angles of each node.

$$\frac{d}{dt}\boldsymbol{u_i} = \mathbf{A_i^*}\boldsymbol{u} + \mathbf{B}\boldsymbol{f} \tag{1}$$

In Eq. (1),$\mathrm{A_i^*} \in \mathbb{R}^{2n\cdot 2n}$ is the time evolution matrix of the wall displacement vector,$u_i \in \mathbb{R}^{2n}$, and $f \in \mathbb{R}^{2n}$ is the forced load vector to control the wall deformation.$\mathrm{B} \in \mathbb{R}^{2n\cdot 2n}$ is a matrix of the column-wise array of retaining-wall deformation vectors for a unit forcing load from an arbitrary point, and it is called the forcing load operator. It is assumed that the forcing load operator is determined solely from the wall specifications.

$$\mathbf{B} = \mathbf{K_w}(E_w, I_w, L_w)^{-1} \tag{2}$$

In Eq. (2),$\mathrm{K_w}(E_w, I_w, L_w)$ is the stiffness matrix when the wall is modeled as an elastic beam, which can be calculated from the stiffness of the wall $(E_w, I_w)$ and length of the wall $(L_w)$. These parameters are fundamental indesigning earth-retaining walls and are determined (given as preconditions) at the beginning of the design. Therefore, the forcing load operator is treated as known parameter. In other words, the construction of a linear system proxy model results in the problem of determining the time evolution matrix $\mathrm{A_i^*}$ of the wall displacement vector.

It is assumed that $\mathrm{A_i^*}$ depends on the displacement level of the wall because it may be affected by the nonlinearity of the ground. In addition, because it depends on the strut placement plan (placement position and time), which can be assumed to have countless combinations, it is difficult to determine a general-purpose matrix. Therefore, in this study, we derived the matrix using the following process.

#### *3.1.1 Modeling deformation time evolution of wall in cantilevering state*

We focused on the time evolution matrix $\mathrm{A_i} \in \mathbb{R}^{2n\cdot 2n}$ of the displacement vector $u_{0,i}$ of the wall in the cantilever state (no struts in place). The temporal variation of the wall displacement distribution obtained through the elastoplastic method analysis was obtained using the DMD process. For details on the DMD algorithm, please refer to Kutz et al. (2016).

$$\frac{d}{dt}\boldsymbol{u_{0,i}} \approx \mathbf{A_i}\boldsymbol{u_{0,i}} = \boldsymbol{\Phi}_{\mathbf{r,i}}\boldsymbol{\Lambda}_{\mathbf{r,i}}\boldsymbol{\Phi}_{\mathbf{r,i}}^{\dagger}\boldsymbol{u_{0,i}} \tag{3}$$

$$\mathbf{A_i} = \underset{\mathbf{A_i}}{\mathbf{argmin}}\|\mathbf{U_i'} - \mathbf{A_i U_i}\|_F$$

In Eq. (3),$\phi_{r,i} \in \mathbb{C}^{2n\cdot r}$ is a time-independent DMD spatial mode function, and $\Lambda_i \in \mathbb{C}^{r\cdot r}$ is a matrix with eigenvalues arranged in diagonal terms. The DMD spatial mode function was approximated in a dimensionally compressed manner by extracting the $r$ largest eigenvalues. $\mathrm{U_i} \in \mathbb{R}^{2n\cdot m-1}$ and $\mathrm{U_i'} \in \mathbb{R}^{2n\cdot m-1}$ are data matrices defined as follows, which are columnar matrices of the wall displacement vectors obtained using the elastoplastic method:

$$\mathbf{U_i} = \begin{bmatrix} | & | & | \\ u_{0,i}(t_0) & u_{0,i}(t_1) & \cdots u_{0,i}(t_{m-1}) \\ | & | & | \end{bmatrix} \tag{4}$$

$$\mathbf{U_i'} = \begin{bmatrix} | & | & | \\ u_{0,i}(t_1) & u_{0,i}(t_2) & \cdots u_{0,i}(t_m) \\ | & | & | \end{bmatrix} \tag{5}$$

where $u_{0,i}(t)$ is the displacement vector of the wall at time $t$. Because we focused only on the deformation behavior in the cantilever state, it was assumed that r was extremely small. From the above, the wall displacement vector $u_0(t_k)$ at a certain time $k$ in the self-supporting state can be calculated usingEq. (6)with the initial wall displacement vector $u_0(t_0)$.

$$u_{0,i}(t_k) = \boldsymbol{\Phi}_{\mathbf{r,i}}\boldsymbol{\Lambda}_{\mathbf{r,i}}^{k}\boldsymbol{\Phi}_{\mathbf{r,i}}^{\dagger}u_{0,i}(t_0) \tag{6}$$

#### *3.1.2 Modeling of deformation time evolution of earth-retaining wall in cut-beam configuration*

In the elastoplastic method, the wall is modeled as a beam model, and the ground is modeled as a spring. The subgrade reaction force on the excavation side (resistance side) is modeled as a fully elastoplastic model with the shear strength as the upper limit. However, the nonlinearity of the ground is approximated using an equivalent linear model, as expressed by Eqs. (7) and (8):

$$\boldsymbol{P_{sa}} = \mathbf{K_{0,i}}\boldsymbol{u_{0,i}} \tag{7}$$

$$\mathbf{K_{0,i}} = \mathbf{K_w} + \mathbf{K_{c,i}^{EL}} \tag{8}$$

where$P_{sa} \in \mathbb{R}^{2n}$ is the external force vector (effective main active lateral pressure vector), and $K_{0,i} \in \mathbb{R}^{2n \cdot 2n}$ is the stiffness matrix, which is expressed as a linear sum of the stiffness matrix of the earth retaining wall,$K_W \in \mathbb{R}^{2n \cdot 2n}$, and the distributed van matrix of the ground,$K_{c,i}^{EL} \in \mathbb{R}^{2n \cdot 2n}$. The superscript,"EL," means equivalent linear, and $K_{c,i}^{EL}$ is the distribution spring matrix of the equivalent linearized ground, assuming that the ground is linear and has no spatial variability. The 2n·2n matrix has one unknown quantity because it assumes linearity and no spatial variability. $K_{c,i}^{EL}$ is identified by a particle filter using Eq. (9) as the objective function:

$$\mathbf{K}_{\mathbf{c,i}}^{\boldsymbol{EL}} = \underset{\mathbf{K}_{\mathbf{c,i}}^{\mathbf{EL}}}{\operatorname{argmin}} \left\| \mathbf{U}_{\mathbf{NL,i}} - \mathbf{U}_{\mathbf{EL,i}} \right\|_{\boldsymbol{F}} \tag{9}$$

where $U_{NL,i} \in \mathbb{R}^{2n \cdot m}$ is the data matrix of wall displacements based on the elastoplastic method, and $U_{EL,i} \in \mathbb{R}^{2n \cdot m}$ is the data matrix of wall displacements for linear analysis using $K_{c,i}^{EL}$. The assumption of linearity of the ground with no spatial variabilityis similar to the assumption based on the displacement method used in pile foundation design. The reason for introducing this assumption is that the displacement levels targeted in predicting the behavior of retaining walls are generally low, and the displacement of the wall is considered to depend on the local soil properties within a few meters from the excavation bottom, as shown by the beam theory on an elastic foundation.

Similarly, the displacement vector for a cut beam is expressed by Eqs. (10) and (11):

$$\boldsymbol{P_{sa}} = \mathbf{K_i} \boldsymbol{u_0} \tag{10}$$

$$\mathbf{K_i} = \mathbf{K_w} + \mathbf{K}_{\mathbf{c,i}}^{\mathbf{EL}} + \boldsymbol{K_{st}} \tag{11}$$

where $K_{st} \in \mathbb{R}^{2n \cdot 2n}$ is the stiffness matrix of the cut beam, where the stiffness of the strut ($E_{st}A_{st}$) is placed at the location where the strut are placed (diagonal term) and zero otherwise. Because $P_{sa}$ does not depend on the presence or absence of struts, the displacement vector when the struts are placed can be determinedusingEq. (12).

$$\boldsymbol{u_i} = \mathbf{K_i^{-1}} \mathbf{K_{0,i}} \boldsymbol{u_{0,i}} = \mathbf{T_{k,i}} \boldsymbol{u_{0,i}} \tag{12}$$

The linear system surrogate model can be rewritten as follows.

$$\frac{d}{dt} \boldsymbol{u_i} = \mathbf{A_i^*} \boldsymbol{u_i} + \mathbf{B} \boldsymbol{f} = \mathbf{T_{k_i}} \mathbf{A_i} \boldsymbol{u_{0,i}} + \mathbf{B} \boldsymbol{f} \tag{13}$$

Because the proposed model is based on DMD learning for the cantileverstate, it is expected to exhibit significant dimensionality compression. The wall displacements under various conditions can be calculated by simply converting the wall displacements in the cantilever state using a known stiffness matrix.

### 3.2 Extending the model with uncertainty

Assuming the soil spatial variability was modeled by stochastic processes, several soil scenarios were prepared. For each scenario, an elastoplastic analysis of the cantilever state was performed, and the DMD was approximated using Eq. (3). The analysis results of the four soil scenarios described below show that $\phi_{r,i}$ is insensitive to the spatial distribution of soil spatial variability and is approximately similar. Therefore, the DMD spatial mode function uses a certain reference spatial mode function $\phi_{r,ref}$, and the change in wall behavior for different ground scenarios is aggregated into eigenvalues, as expressed by Eq. (14):

$$u_{0,i}(t_k) = \boldsymbol{\phi}_{\mathbf{r,i}} \boldsymbol{\Lambda}_{\mathbf{r,i}}^{\mathbf{k}} \boldsymbol{\phi}_{\mathbf{r,i}}^{\dagger} u_{0,i}(t_0) \approx \boldsymbol{\phi}_{\mathbf{r,ref}} \boldsymbol{\Lambda'}_{\mathbf{r,i}}{}^{\mathbf{k}} \boldsymbol{\phi}_{\mathbf{r,ref}}^{\dagger} u_{0,i}(t_0) = \boldsymbol{\phi}_{\mathbf{r,ref}} \mathbf{x_{k,i}} \tag{14}$$

$$\mathbf{x_{k,i}} = \boldsymbol{\Lambda'}_{\mathbf{r,i}}{}^{k} \boldsymbol{\phi}_{\mathbf{r,ref}}^{\dagger} u_{0,i}(t_0) \tag{15}$$

where,$\boldsymbol{\Lambda'}_{\mathbf{r,i}}$ is converted by a particle filter using the following objective function.

$$\boldsymbol{\Lambda'}_{\mathbf{r,i}} = \underset{\Lambda'_{\mathbf{r,i}}}{\operatorname{argmin}} \left\| \mathbf{U'_i} - \boldsymbol{\phi}_{\mathbf{r,ref}} \boldsymbol{\Lambda'}_{\mathbf{r,i}} \boldsymbol{\phi}_{\mathbf{r,ref}}^{\dagger} \mathbf{U_i} \right\|_{\boldsymbol{F}} \tag{16}$$

Based on the above, a simple stochastic model was constructed by imposing the effect of soil spatial variability on the eigenvalue matrix $\Lambda'_{r,i}$. To adopt $u_0(t_k)$ and $x_k$as random variable vectors, we rewrite them as $\dot{u}_0(t_k)$ and $\dot{x}_k$. Because we cannot determine the statistics of the random variables in advance, we generated soil spatial variability scenarios based on the stochastic process theory, which simulates the soil characteristics at the target site. And we calculated its statistics based on numerical analysis results considering the soil spatial variability.Although the value of $\phi_{r,ref}$ is arbitrary, it is assumed to be the expected value of the soil scenario. If the expected value $E[\dot{x}_k]$ and covariance matrix$Cor[\dot{x}_k]$ of $\dot{x}_k$ can be obtained based on the numerical analysis as Eq.(17) and Eq.(18), the expected value and covariance matrix of $\dot{u}_{0,i}(t_k)$can be calculated using Eq. (19) and Eq. (20):

$$\mathrm{E}[\dot{\mathbf{x}}_{\mathbf{k}}] = \mathrm{E}\big[\mathrm{Re}[\dot{\mathbf{x}}_{\mathbf{k}}]\big] + \mathrm{E}\big[\mathrm{Im}[\dot{\mathbf{x}}_{\mathbf{k}}]\big]\boldsymbol{j} \tag{17}$$

where, $j$ means imaginary unit.

$$\mathrm{Cor}[\dot{\mathbf{x}}_{\mathbf{k}}] = \mathrm{E}[(\dot{\mathbf{x}}_{\mathbf{k}} - \mathrm{E}[\dot{\mathbf{x}}_{\mathbf{k}}])(\dot{\mathbf{x}}_{\mathbf{k}} - \mathrm{E}[\dot{\mathbf{x}}_{\mathbf{k}}])^{\mathrm{H}}] \tag{18}$$

$$\mathrm{E}[\dot{u}_0(t_k)] = \boldsymbol{\Phi}_{\mathbf{r,ref}}\mathrm{E}[\dot{\mathbf{x}}_{\mathbf{k}}] \tag{19}$$

$$\mathrm{Cor}[\dot{u}_0(t_k)] = \boldsymbol{\Phi}_{\mathbf{r,ref}}\mathrm{Cor}[\dot{\mathbf{x}}_{\mathbf{k}}]\boldsymbol{\Phi}_{\mathbf{r,ref}}{}^{\mathbf{H}} \tag{20}$$

where,the superscript "H" means conjugate complex.The time evolution of the strut arrangement is approximated using Eq. (17).

$$\mathrm{E}[\dot{u}(t_k)] \approx \mathbf{T}_{\mathbf{k,ref}}\mathrm{E}[\dot{u}_0(t_k)] + \mathbf{B}\boldsymbol{f} \tag{21}$$

$$\mathrm{Cor}[\dot{u}(t_k)] \approx \mathbf{T}_{\mathbf{k,ref}}\mathrm{Cor}[\dot{u}_0(t_k)]\mathbf{T}_{\mathbf{k,ref}}^{\mathrm{T}} \tag{22}$$

where$\mathrm{T}_{\mathrm{k,ref}}$ is the transformation matrix based on $\mathrm{K}_{\mathrm{c,ref}}^{\mathrm{EL}}$and is identified from the ground scenario of the reference case.$\mathrm{K}_{\mathrm{c,ref}}^{\mathrm{EL}}$ is associated with the displacement level at the bottom of the excavation during the learning process. The convergence $\mathrm{K}_{\mathrm{c,ref}}^{\mathrm{EL}}$ is obtained by a simple calculation used in the equivalent linear ground response analysis SHAKE (Schnable P.B. et al. 1972). This treatment of the nonlinearity of $\mathrm{K}_{\mathrm{c,ref}}^{\mathrm{EL}}$ is omitted for reasons of space limitation. The above equation is an approximate solution because it does not reflect the variation in $\mathrm{T}_{\mathrm{k,i}}$for each ground scenario. This assumption is made because $\mathrm{T}_{\mathrm{k,i}}$ is assumed to reflect the effect of $\mathrm{K}_{st}$more strongly than$\mathrm{K}_{\mathrm{c,i}}^{\mathrm{EL}}$ and on a convenience assumption to limit the particle filter to only one reference case. The accuracy of this approximation needs to be examined in future studies.

Therefore, if we can determine the mean vector and covariance matrix of the eigenvalues at arbitrary steps in the cantilever state during the learning process, the uncertainty in the displacement distribution under various design conditions can be computed immediately.

## 4 Research Results

Figures 3 shows the calculation results of the DMD spatial mode functions and time evolution (eigenvalues) from the elastoplastic analysis, assuming the cantilever state for the four ground scenarios. The number of significant modes was determined to be two because the cantilever state was used as the training target, and the unknowns (parameters) of the predictive model were the eigenvalues of the two modes. The number of mode functions was determined qualitatively by examining the magnitude of singular values. These results show that the effect of soil spatial variability is insensitive to the spatial mode function, and there is no significant difference in the spatial mode function. Figure 4 shows the superimposition of the elastoplastic method and proxy calculation values for the cut-beam configuration. The proposed method almost perfectly replicated the elastoplastic method results, and the effectiveness of the proposed method was verified. Figure 5 shows the range of expected values and a standard deviation of the predictions based on Eqs. (17) and (18) under the same conditions as those shown in Figure 3.

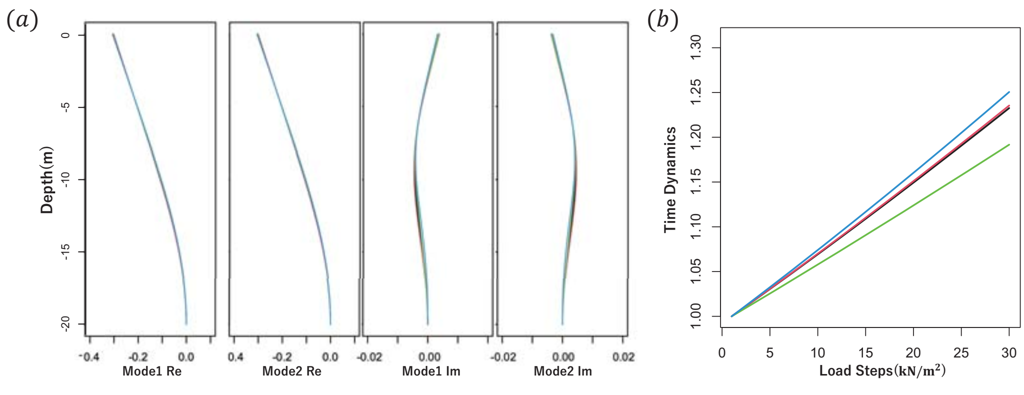


**Figure 3.** (a) DMD spatial mode functions and (b) time evolution(eigenvalues)

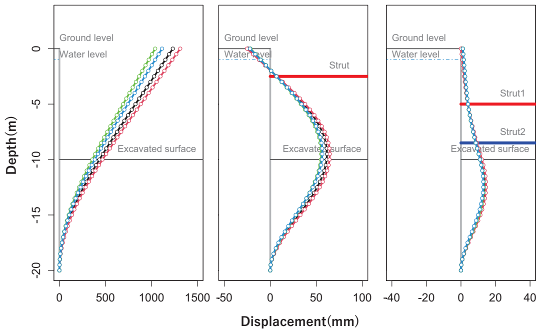


**Figure 4.** Comparison of elastoplastic method and proxy calculation values

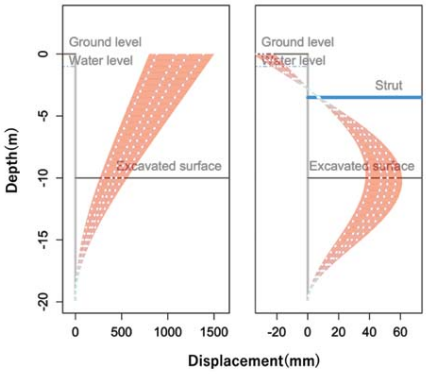


**Figure 5.** Range of one standard deviation of predictions

## 5 Conclusion

In this study, a linear-system-type surrogate model wasestablished to efficiently predict the displacement behavior of an earth-retaining wall while increasing the surface load after the excavation is completed. The proposed method wasdeveloped to construct an autonomous control model for large-scale earth retaining walls. This linear-system-type surrogate model is proposed to integrate DMD and the beam theory on an elastic foundation. Four simple soil heterogeneity scenarios were analyzed, and their effectiveness was verified numerically. Because the linear system model has a high affinity with the optimization and control theories, we believe that it will be an essential elemental technique for constructing autonomous control models. In the future, we plan to validate the model using actual observation records and extend it to a model for predicting the displacement behavior of earth-retaining walls during excavation. In addition, we plan to develop a robust design method for the initial design of struts and extend the model to a real-time control model during construction.

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