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Uncertainty in the Critical Secant Gradient Function for Prediction of Backward Erosion Piping

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Abstract: Backward erosion piping is a type of internal erosion that occurs when small erosion channels progress through erodible foundation materials beneath an embankment in the direction opposite of flow. Recent research has proposed the concept of a critical secant gradient function (CSGF) as a means of quantifying the critical hydraulic conditions for erosion progression. In this study, uncertainty in the prediction of the CSGF was evaluated through a series of 24 laboratory measurement on different soils. An analytical solution for the CSGF was fitted to each experiment, and a model for predicting the analytical CSGF was developed through a non-linear multivariate analysis of the results. Results indicate that the uncertainty in the CSGF is relatively small as indicated by the narrow confidence intervals on the predicted CSGF values. Additionally, the model for predicting the CSGF was found to reliably bound the measured CSGF values with 95% confidence for 20 of the 24 experiments conducted. This suggests that the CSGF can be reliably predicted based on soil gradation characteristics for clean sands.

Keywords: internal erosion, backward erosion piping, levees, dams, prediction

1 Introduction

Backward erosion piping (BEP) is a type of internal erosion that occurs when shallow erosion channels progress upstream through the foundation sand beneath an embankment as illustrated in Figure 1a. If the erosion channels are allowed to progress to the upstream water source, the channels will rapidly enlarge leading to collapse and breach of the overlying embankment (Van Beek et al., 2011). Because BEP is a significant contributor to failure risks for dams and levees (Foster et al. , 2000), it is essential to have methods for reliable estimation of BEP failure probabilities in order to assess flood risk associated with existing infrastructure. Robbins (2022) developed a physics-based approach for predicting BEP progression using the concept of the critical secant gradient function (CSGF). In this study, uncertainty in the CSGF is evaluated through an analysis of 24 laboratory experiments in order to inform probabilistic, physics-based modeling of BEP. The sections that follow first describe the concept of the CSGF in detail before then describing the experiments, experiment results, and analysis conducted to assess the uncertainty in predictions of the CSGF.

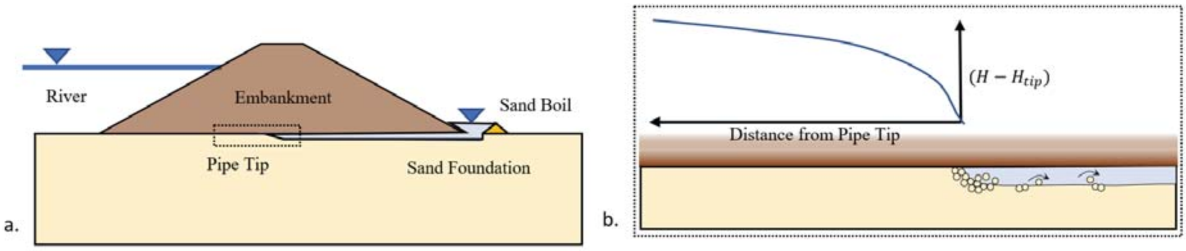

Figure 1. Illustration of (a) backward erosion piping progression beneath an embankment and (b) the non-linear head profile near the upstream pipe tip due to the concentration of flow towards the pipe.

2 The CSGF

The CSGF is a spatial function that describes the critical hydraulic conditions near the pipe tip that cause the pipe to progress further. The erosion channels that form are open voids that have significantly higher hydraulic conductivity than the surrounding soil. Because of this, groundwater flow concentrates severely towards the erosion channels resulting in a highly nonlinear profile of the head (H) upstream of the erosion pipe (Figure 1b). As a result, the hydraulic gradient ($i = \nabla H$) near the pipe tip becomes quite large. It is the high seepage forces associated with the large hydraulic gradient near the pipe tip that are responsible for the progression of the pipe.

Until recently, a consistent definition of the hydraulic conditions near the pipe tip has been difficult to obtain. This is because the head profile in front of the erosion pipe is highly nonlinear. As a result, hydraulic gradients calculated over different distances will be of different magnitudes. To remedy this issue, Robbins (2022) developed the concept of the CSGF to describe the full spatial function of the hydraulic gradient in front of the erosion pipe. The CSGF describes the critical, secant gradient (i_{cs}) between the pipe tip and all points in front of the pipe tip. The term "secant" is used to be explicitly clear that the values are calculated as linear averages between two points as given by the slope of a secant chord reaching from the pipe tip to any point along the head profile in front of the erosion pipe. As one moves further and further away from the pipe tip, the secant gradient decreases in magnitude as illustrated in Figure 2. The critical value of i_{cs} is defined as the value of i_{cs} associated with the last stable head profile prior to BEP progression. The pipe will progress further if the secant gradient exceeds i_{cs}, and the pipe will stop progressing if the secant gradient is less than i_{cs}. In this manner, the CSGF provides a physics-based criterion for BEP progression.

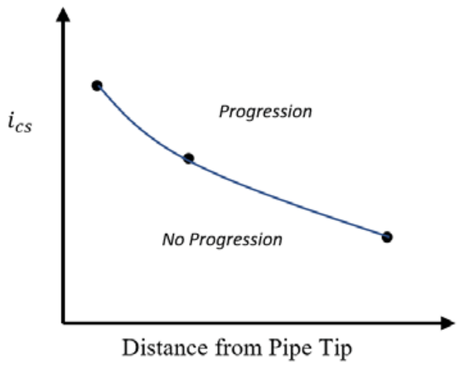

Figure 2. Illustration of the CSGF

The analytical form of the CSGF was derived by Robbins (2022) as shown in Eq. 1. The function is defined in terms of the head, H_o, measured at some arbitrary distance, x_o, in front of the erosion pipe. The values of H_o and x_o are constant values that define the head profile for a sand of a given gradation and density. As such, the CSGF can be defined entirely by the constant $C = H_o/\sqrt{x_o}$.

$$i_{cs} = \frac{H_o}{\sqrt{x_o x}} = \frac{C}{\sqrt{x}} \qquad (1)$$

To define the constant C, it is necessary to measure H_o at some point located a distance x_o in front of the erosion pipe. The following section describes experiments for obtaining these measurements on various sands.

3 Experiments

Robbins et al. (2018) developed a cylindrical laboratory apparatus for measuring the hydraulic conditions near a progressing erosion channel as illustrated in Figure 3. Because of the cylindrical shape, the erosion channels that form must pass directly beneath the pore pressure transducers in the top of the cylinder. The device in Robbins et al. (2018) was modified to include sensors spaced at 2-cm intervals as illustrated in Figure 3 to measure the shape and magnitude of the head profile and corresponding CSGF in front of the erosion pipe.

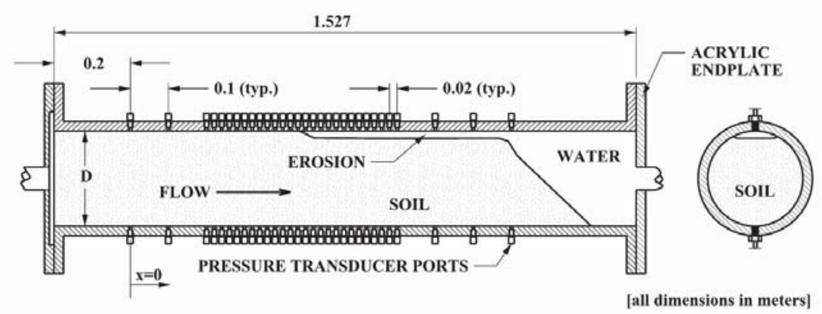

Figure 3. Schematic of laboratory equipment for measuring the CSGF.

The experiments were conducted in a cylinder with a diameter of D=15.24 cm. The sample preparation and testing procedure consisted of the following steps:

1. The cylinder was rotated to a vertical position with the downstream end cap oriented vertically in order to prepare the sample. The cylinder was filled with de-aired water, and the sample was prepared by water

pluviating sand into the container until the desired sample length was obtained. To obtain dense samples, the container was vibrated by tapping with a rubber mallet during water pluviation.

2. Once the sample was prepared, the device was slowly rotated to the horizontal position for testing. As the sample was rotated, the downstream slope would slide to the angle of repose of the sand as illustrated in Figure 3.
3. With the upstream inlet valve still closed, the pore pressure transducers were all zeroed to the downstream water level.
4. The upstream water level was then gradually increased in small increments until erosion initiated and the pipe began progressing.
5. As the pipe reached the zone of dense, 2-cm spaced pressure transducers, the upstream head was decreased to cause the erosion pipe to stop near the downstream end of the sensing interval.
6. After the pipe was stopped, the upstream head was then gradually increased again in small increments. After each increase, the head was held constant to allow sand to wash out of the erosion pipe in order to ensure the pipe was truly at equilibrium. This process was repeated until the pipe began to progress upstream once again. The last stable head profile measured before upstream progression re-initiated was taken as the critical head profile, and the data from this step was used to determine the CSGF.

For this study, a series of 24 experiments was conducted on 7 different sands. The grain size (d_{50}) and coefficient of uniformity (C_u) for the 7 sands are provided in Table 1. Additionally, Table 1 indicates the test numbers for which each sand was used.

Table 1. Properties of sands tested and associated test numbers.

Sand	d_{50} (mm)	C_u	Test No.'s (Figure 5)
40/70	0.304	1.42	3, 4, 5, 6, 16, 17, 18
B15	0.150	1.60	9
B25	0.230	1.60	10, 11,22
GZB1	1.400	3.70	14, 28
GZB2	0.870	2.50	12, 15, 19, 21, 27, 30
GZB3	0.910	1.30	7, 25
MZ	0.380	2.40	23, 24, 26

4 Results

For each test, an attempt was made to stop the erosion pipe immediately at a pressure transducer. In this manner, the pressure head could be measured at the pipe tip and every 2 cm in front of the erosion pipe. From these measurements, i_{cs} was able to be calculated directly from the pressure transducers as shown in Figure 4. For cases where the pipe tip did not stop exactly at a pressure transducer, an estimate of the head at the pipe tip was obtained through a linear fit of the pressure transducers in the erosion pipe, and the value of i_{cs} at each pressure transducer was determined on the basis of the estimated head at the pipe tip. This approach was taken as it has been well established that the head loss in the erosion channel varies linearly along the channel length (Van Beek et al. 2019). In addition to calculating i_{cs} directly from the pressure transducers, Equation 1 was also fitted to the measurements using the measured head and distance from the pipe tip associated with each pressure transducer. In this manner, an estimate of the constant C for the CSGF was able to be obtained for each pressure transducer measurement. A value of C was calculated for the first 5 pressure transducers in front of the pipe tip at distances of 2 cm to 10 cm away from the tip. An example of the 5 estimates of the CSGF obtained from the 5 measurement points is provided for Test 30C in Figure 4. As shown, estimating C at multiple points provides a means of estimating the uncertainty in the CSGF measurement for one test. For this example, the uncertainty is relatively small based on the limited variability between the various curves. Additionally, each estimate of the CSGF provides a reasonable fit to the measured data indicating that the analytical form of the CSGF provided by Equation 1 adequately describes the actual CSGF shape measured in the experiments.

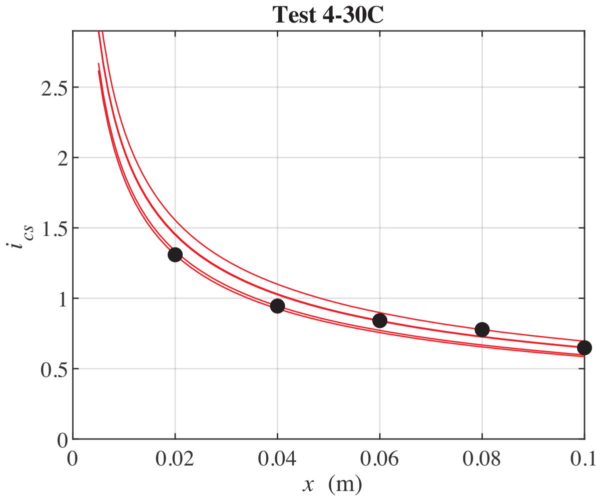

Figure 4. Measured CSGF values and range of analytical fits for Test 30C.

While Figure 4 demonstrates the uncertainty in measuring the CSGF for a single test, it does not address the uncertainty in our ability to predict the CSGF on the basis of soil properties. In order to assess the uncertainty in predicting the CSGF, a statistical regression was performed to predict the constant C in Equation 1 on the basis of the sand's grain size (d_{50}), coefficient of uniformity (C_u), and void ratio (e). For each test, the average of the 5 values of C calculated was taken as the best estimate of C. A regression on the resulting 24 combinations of soil properties and C values yielded equation 2 as the best fit equation for predicting the CSGF. In Equation 2, the coefficients β_i are given by Equations 3 through 6 where t is the value of the Student t-distribution for a desired confidence level and 20 degrees of freedom. In this manner, the CSGF can readily be predicted with confidence intervals in order to assess the uncertainty in estimates of the CSGF for use in practice. This will be explored further in the following section through a comparison of the predicted CSGF to the measured values for the 24 experiments.

$$C = \beta_1 \left(\frac{d_{50}}{0.581}\right) + \beta_2 \left(\frac{C_u}{2.02}\right) + \beta_3 \left(\frac{e}{0.581}\right)^{\beta_4} \tag{2}$$

$$\beta_1 = 0.010 \pm 0.017\, t \tag{3}$$

$$\beta_2 = 0.012 \pm 0.021\, t \tag{4}$$

$$\beta_3 = 0.005 \pm 0.006\, t \tag{5}$$

$$\beta_4 = -20.751 \pm 6.682t \tag{6}$$

5 Predictive Model Evaluation

The best estimate and 95% confidence intervals for the CSGF were predicted using Equations 2 through 6 for all 24 experiments. The predictions are compared to the measured values in for all 24 tests in Figure 5. From these results, a few observations can be made regarding (1) the uncertainty in the CSGF predictions and (2) the ability of the prediction equations to describe the variation in the CSGF with soil properties. First, with regards to the uncertainty in the CSGF prediction, it is observed that the confidence intervals are closely spaced for most of the experiments suggesting that the uncertainty in the prediction given the data in this study is quite small. Secondly, it is also observed from the results that the predicted CSGF adequately describes the test data for the majority of the experiments. Upon close inspection, it is seen that the predicted CSGF range bounds the measured points for 20 of the 24 experiments. This indicates that the proposed Equation 5 does an excellent job of explaining the variation in the CSGF with soil properties, and this type of approach can be taken to reliably predict the CSGF

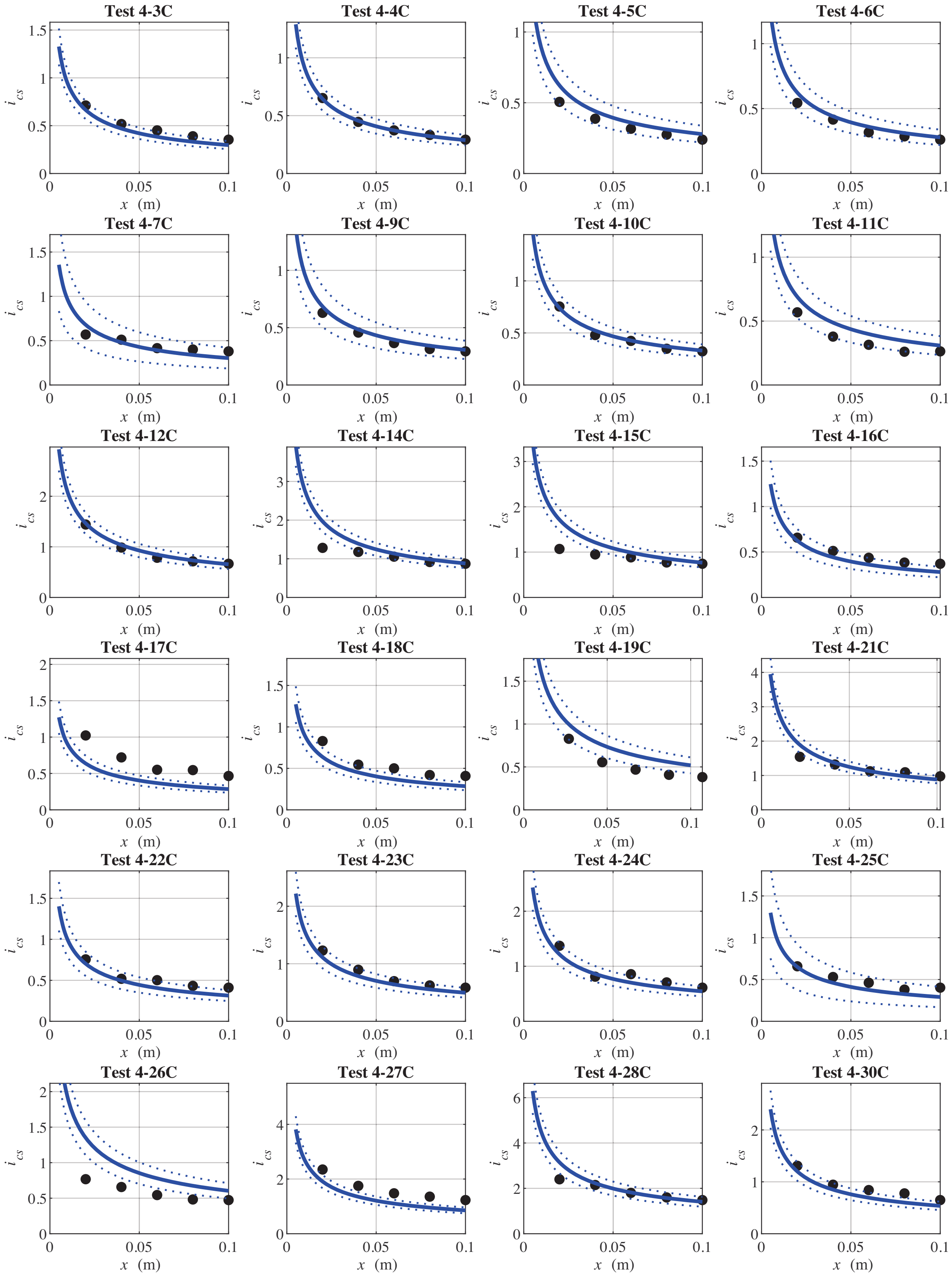

Figure 5. Comparison of predicted CSGF and 95% confidence bounds obtained from Eq. 2 to measured results.

for use in practice. More work is needed to further verify these concepts through comparison of additional, independent laboratory measurements to predicted values obtained using the equations proposed in this study.

6 Conclusions

Backward erosion piping is a type of internal erosion that is responsible for the failure of many dams and levees, and this failure mechanism has been especially difficult to predict due to the lack of a clear definition of the hydraulic conditions causing BEP channels to progress upstream. The authors have recently proposed the critical secant gradient function (CSGF) as a spatial definition of the hydraulic conditions at the pipe tip that cause erosion progression. In this paper, a preliminary assessment of the uncertainty in CSGF predictions was made by developing a statistical model for predicting the CSGF and comparing the predictions to the measured CSGF for the 24 tests used to develop the model. The results demonstrate that the predictive model adequately captures the variation in the CSGF with soil properties and the uncertainty in the model predictions is relatively small. The ability to reliably predict the CSGF from a statistical model indicates this may be a very promising means of predicting BEP progression in the future.

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