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Design of a Probabilistic FEM Analysis Using the Sellmeijer Model for Backward Erosion Piping

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Abstract: Backward erosion piping (BEP) is an important failure mechanism for clay dikes founded on sandy soil or dikes resting on an impermeable blanket above a sandy layer. To compute a failure probability for this mechanism, probabilistic calculations are required. Until now these are computed in the Netherlands using the analyticalSellmeijer model. This model is based on a strongly simplified subsurface. Therefore, it is desirable to perform probabilistic calculations with models that can more accurately depict the subsurface features such as with a Finite Element Methods (FEM). In this paper, the FEM BEP model DgFlow is used, which couples the Sellmeijer erosion model to a FEM model for groundwater flow, allowing to take complex features, such as heterogeneity and anisotropy, into account when assessing dikes. This paper shows the first attempt to perform probabilistic BEP calculations using DgFlow. Different limit state functions and probabilistic methods are tested to find a method that provides accurate, fast, and reliable results. It is found that FORM computations result in sufficiently quick (time) and accurate (compared to MC) results.This method is applied to more realistic conditions to test its robustness,and hence, allows for a wider application of probabilistic BEP FEM calculations.

Keywords: Dike; Backward erosion piping; DgFlow; D-Seepage, Probabilistic models

1 General Aspects

Dikes are used around the world to protect the land from flooding and are becoming increasingly important due to sea-level rise, land subsidence, population growth, and more severe weather events. In the Netherlands, dikes are preferably designed and assessed based on their failure probability since this allows for less conservatism in dealing with uncertainties as compared to a traditional load and resistance factor based approach.

Backward erosion piping (BEP) is an important failure mode for clay dikes founded on sandy soil or dikes resting on an impermeable blanket above a sandy layer. During high water periods, seepage flow through the aquifer can erode grains, which are transported to an unfiltered exit (such as a crack in the cover layer) forming sand boils on the land side of the dike. Erosion leads to the formation of hollow pipes at the top of the aquifer just below the blanket. These pipes lengthen and, when a critical head drop is exceeded, a pipe can progress upstream leading to failure.

Analytical assessment models are available to assess this mechanism such as fpr example Bligh (1910), Lane (1935), Sellmeijer (1988), Schmertmann, J. H. (2000), or Hoffmans en Van Rijn (2018)these require a strong simplification of the subsurface. The Sellmeijer model of BEP describes the erosion of grains in a pipe due to groundwater flow. This modelisthe basis of the assessment model of Sellmeijer (often used in the Netherlands) andhas also been implemented in a prototype Finite Element Method (FEM) groundwater flow model called DgFlow. This FEM model can, therefore, compute the pipe growth beneath a dike given a certain head drop, as well as the critical head. The critical head is the head drop above which the pipe can progress upstream, resulting in failure. The pipe length found at the critical head drop is called the critical pipe length. DgFlow allows taking more complex features, such as heterogeneity and anisotropy of the permeability into account in the assessment of BEP (Van Esch, Sellmeijer and Stolle 2013). Heterogeneity here refers to an aquifer containing two or more distinct homogenous layers instead of one, it does not refer to the spatial variability in a layer.

Whereas for the analytical assessment model of Sellmeijer it is relatively straightforward to make probabilistic computations and derive a semi-probabilistic safety format based on partial factors (Teixeira, Wojciechowska and ter Horst 2016), a suitable probabilistic implementation is not yet available for DgFlow. This paper presents the results of the research performed to find a suitable method to perform probabilistic computations in DgFlow. Several limit state functions are considered in combination with different probabilistic methods to derive a fast, robust, and precise method for determining the failure probability. The computations are made using the Probabilistic Toolkit (PTK) of Deltares (2021). In the PTK a script, an executable (such as DgFlow), or a combination of both can be implemented and used for probabilistic calculations.

In a simplified schematization, the probability of failure is computed using Monte Carlo (MC) analysis based on a critical head limit state function. This method is slow but accurate and thereby provides a benchmark to test other methods. The effects of defining the limit state function as a function of the pipe length are compared to defining the limit state function as a function of the critical head. This definition with the pipe length ideally allows for increased speed, possibly at the expense of accuracy. Alternative probabilistic methods that are considered are Monte Carlo Importance Sampling (MCIS) and First Order Reliability Method (FORM). The combination of limit state function and probabilistic method that is the most accurate and fastest is then tested using a more complex schematization.

These and other details of the study are presented such that insights regarding the influence of relevant parameters and fundamentals of physical and/or numerical behavior are better understood. The study enables the implementation of a probabilistic framework in DgFlow and its successor D-Geo Flow 2022. For more information about DgFlow,please refer toVan Esch, Sellmeijer and Stolle (2013).

2 Method

2.1Limit state functions

A limit state function defines whether failure of a dike occurs (Z<0) or not (Z≥0).In this study, the limit state function for BEP is provided by the Dutch guidelines (RWS 2021). Traditionally, these guidelines use the critical head drop found with the Sellmeijer assessment model. However, in this study the critical head found in DgFlow is used(Section 2.1.1). A second limit state function is proposed based on the pipe length found with DgFlow (Section 2.1.2.).

2.1.1 Based on the hydraulic head

According to the Dutch guidelines, failure occurs when the occurring hydraulic head exceeds the critical hydraulic head, see Equation 1. In the occurring hydraulic head, a correction is made for a loss of hydraulic head in the channel through the blanket. The critical hydraulic head contains a model factor.

$$Z_p = m_p H_c - (h - h_{exit} - r_c d) \tag{1}$$

where:

- Z_p (m): limit state function based on the hydraulic head
- m_p (-): model factor to consider uncertainties in the model
- H_c (m): critical hydraulic head, until now computed with the analytical Sellmeijer assessment model, in this paper with DgFlow
- h (m): river waterlevel
- h_{exit} (m): polder groundwater level
- r_c (-):correction factor for the resistance in the channel through the blanket layer, currently taken to be 0.3
- d (m): thickness of the blanket layer

It should be noted that the computation of $H_c$with DgFlow is not straightforward. DgFlow only computes the pipe length given a certain head drop and hence only whether H_c is exceeded or not. Therefore, an iterative or stepwise procedure, in which the head drop is incrementally increased to find the maximum head drop where the pipe reaches an equilibrium, is needed to find H_c.

2.1.2 Based on the pipe length

For this study, an alternative limit state function is defined based on the pipe length to reduce the computational time by avoiding the procedure of computing $H_c$by incrementally increasing the head drop mentioned above. In this limit state function, failure is defined as the moment the pipe, given an occurring hydraulic head, reaches the outer toe. When the exit point is located at the inner toe of the dike,this means that failure occurs when the pipe length exceeds the width of the dike, see Equation 2.

$$Z_{pl} = DB - L_{pipe} \tag{2}$$

where:

- Z_{pl} (m): limit state function based on the pipe length
- DB (m): length of the base of the dike
- L_{pipe} (m): pipe length as computed in DgFlow at a given time and hydraulic headdrop noted as Δh (see below).

The occurring head drop (Δh) used to calculate the pipe length is based on Equation 1. It is the minimum critical head where failure is about to occur ($Z_p = 0$), see below.

$$\Delta h = \frac{h - h_{exit} - r_c\, d}{m_p} \tag{3}$$

2.2 Limit state functions

Once failure due to BEP is defined, its failure probability can be computed using probabilistic methods. The three probabilistic methods used in this paper are commonly used and explained shortly below, more detailed information can be found in Deltares (2021). Less common probabilistic methods become necessary if these first three candidates don't comply to the requirements.

2.2.1 Monte Carlo

Monte Carlo (MC) simulation is a probabilistic method known for its robustness and high accuracy. During an MC analysis, several simulations are performed: each time with a different realization of the stochastic parameters. After each simulation, it is checked whether a failure has occurred. The sum of all failed simulations divided by the number of simulations is the failure probability, see Equation 4.

$$P_f = \frac{N_f}{N} \tag{4}$$

where:

- P_f (-): Failure probability
- N_f (-): Number of failed simulations
- N (-): Total number of simulations

However, to get accurate results,a MC simulation needs a minimal number of simulations. For models with a low failure probability, or for large time-consuming models, this can lead to an undesirably long computational time. To get accurate resultsa minimum of $N = {}^{400}/_{P_f}$ simulations are required. This is based on an acceptable error of 0,1 and a 95% confidence interval(Vrouwenvelder and Vrijling, 1987). Due to the low allowed failure probabilities of dikes, in the order of magnitude of 10^{-5} or lower, this results in millions of computations. When one realization takes more than a second, computational time runs into several days and thus other probabilistic methods are preferable.

2.2.2 First Order reliability method

The First Order Reliability Method (FORM) is another frequently used method to determine the failure probability of dikes. This method determines the joint density functions of all strength and load stochastic variables. This n-dimensional probability space is divided into safe and non-safe regions using the limit state function. To determine the probability, the stochastic variables are transformed to a standard normal distribution. In this new space, the point on the limit state function that is the closest to the origin, called the design point, is determined. The smallest distance from the origin to design point is called the reliability index, which can be translated into a failure probability using the following formula:

$$P_f = \Phi(-\beta) \tag{4}$$

where:

- β (-): reliability index;
- $\Phi()$: cumulative density function of the standard normal distribution.

To find the design point, a first approximation is made of its location. The first approximation is commonly found using the mean value of all stochastic variables, however, other technics exist such as the 'direction' or 'sensitivity' starting methods Deltares (2021). Thereafter, using gradients, points closer to the design point are determined until no other closer location is found. The advantage of FORM is the limited computation time needed to determine the failure probability. A disadvantage is its sensitivity to local minima and maxima, therefore, a relatively linearand continuous solution space is needed.

2.2.3 Importance sampling

Monte Carlo - Importance Sampling (MCIS) is a smart MC analysis that moves the mean values of the stochastic variables towards the direction of the design point.If the design point is unknown, the standard deviation of the stochastic variables can be increased to cover more ground when looking for the design point. By displacing and widening the distribution of stochastic variables towards the design point,more failures are simulated. This reduces the number of simulations needed for accurate results. The failure probability is then computed using a weighted average, see Equation 6, in order to account for the displaced/widened distributions. The weight of each simulation is equal to the actual density function of the limit state function divided by the density function of the shifted limit state function, see Equation 7. Simulations where failure occurred have a lower weight than simulations without failure.

$$P_f = \frac{\sum_{i=1}^{N} I[g(\underline{x})<0]w_i}{N} \tag{6}$$

$$w_i = \frac{f_{\underline{X}}(\underline{x})}{f_S(\underline{x})} \tag{7}$$

where:

- $g(\underline{x})$: limit state function;
- w_i (-): weight of simulation i;
- $f_{\underline{X}}(\underline{x})$: density function of limit state function;
- $f_S(\underline{x})$: density function of the shifted limit state function.

The advantage of a MCIS is the lower computational time required to obtain accurate results. However, the stochastic parameter must be shifted as close as possible to the design point, which can be a difficult task.

3 Model set-up

In this study, two approaches are analyzed based on the two limit state functions mentioned above. For the PTK to calculate probabilistically, the stochastic input variables need to be defined. To reduce the computational time, in this study, four variables are chosen to be stochastic.In practice, it is common to find more stochastic variables.

3.1Proposed approach

3.1.1 Limit state function based on the hydraulic head

In this approach,a combined method is used in the PTK with two python scripts and the executable of DgFlow. The PTK computes the critical hydraulic head in DgFlow (H_C), which is not automatically computed (see section 2.1.1). This is done by first computing the critical head based on the analytical assessment model of Sellmeijer ($H_{C,Sell}$) using the first python script and subsequently by running the executable of DgFlow around this value (from $H_{C,sell} - 0.75$ m until $H_{C,sell} + 0.75$ m) in steps of 0.1 m to find H_C. The second python script contains the limit state function from Section 2.1.1 to define failure.

3.1.2 Limit state function based on the pipe length

This second approach is similar to the previous one. The first python script computes the occurring water level(Δh) as defined in Equation 3, which is needed to calculate the pipe length in DgFlow. The last python script contains the limit state of Section 2.1.2 and calls DgFlow,which uses the pipe length to define failure.

3.2Schematization

This study focuses on finding a robust and fast probabilistic method to conduct probabilistic calculations with DgFlow. Therefore, a simplified and small base schematization is used to reduce the computational time. In practice, longer computational times than found using the base schematization are expected when using more realistic cases.

The base schematization (DB) contains a 40 m wide dike, a 40 m long impermeable foreshore, and 50 m long impermeable hinterland. Because all three components (foreshore, dike, and hinterland) are impermeable only the aquifer is modeled. This reduces computational time. The aquifer is 25 m thick. The river boundary condition enters horizontally in the aquifer (blue line in Figure 1) and exits the model horizontally at the boundary or through the ditch (red lines in Figure 1).The mesh size is 1 m on the pipe, 0.5 m on the ditch, and 5 m everywhere else with a transition size of 0.2. This leads to a model with approximately 460 mesh elements and 80 pipe elements. The schematization is provided in Figure 1.

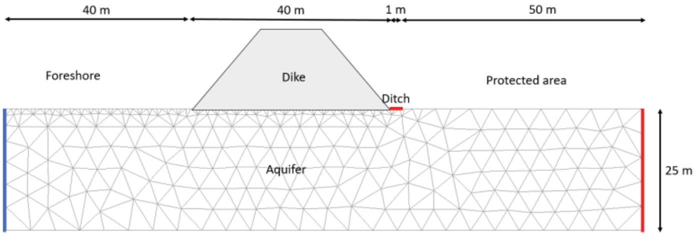

Figure1. Illustration of the base schematization.

The parameters of the base schematization are presented in Table 1. Due to model issues, the grain size and permeability of the aquifer are given a truncated lognormal distribution. The river water level is taken from a location along the Rhine River and is defined using an empirical distribution. The deterministic value of the polder groundwater level is chosen as such that the reliability index of the base schematization equals approximately 3, to reduce computational time.

Table1. Parameters used for this study.

Variable name	Distribution	Parameters
Coefficient of White - η (-)	Deterministic	0.25
Density of sand underwater - γ_{sub} (kN/m^3)	Deterministic	16.5
Density of water - γ_{w} (kN/m^3)	Deterministic	10
Bedding angle - θ (°)	Deterministic	37
Reference d70-value - d_{70m} (m)	Deterministic	0.000208
70-percentiel of the aquifer - d_{70} (m)	Truncated lognormal	μ = 0.0002; σ = 2.4E-05;min = 0.00013; max = 0.00029
Intrinsic permeability aquifer - k (m^2)	Truncated lognormal	μ = 6.13E-11; σ = 3.065E-11;min = 9E-12; max = 2E-10
Seepage length - L (m)	Deterministic	40
Thickness of the aquifer - D (m)	Deterministic	25
Modelfactor - m_p (-)	Normal	μ = 1; σ= 0.1
Reduction factor heave - r_c (-)	Deterministic	0.3
Waterlevel at the inner side - h_{polder} (m)	Deterministic	13.5
Blanket layer thickness hinterland - d(m)	Deterministic	2

4 Results

4.1 Based on the hydraulic head limit state

The failure probability of the base schematization based on the hydraulic head limit state is found using the three different probabilistic methods mentioned above. The MC and MCIS are computed with 4 parallel runs, i.e. using 4 cores. Table 2 summarizes the results. As expected, the MC analysis has the longest computational time followed by MCIS and FORM. The reliability index of MC is assumed to be as the most accurate. Based on this, the MCIS slightly over-estimates the reliability index and FORM underestimates it. However, all three methods give very similar results and are therefore considered as sufficient. The influence factors of the four parameters are similar for all three methods. From these results, FORM is the most suitable(i.e. sufficiently reliable and fast)for this limit state function.It gives in an acceptable time a slight underestimation of the reliability index.

4.2 Based on the pipe length limit state

The failure probability of the base schematization based on the pipe length is found using MCIS. Performing another MC analysis is not deemed necessary due to the high computational time found previously. FORM is not suitable due to the nonlinearity of the limit state function, which gave unstable results. Table 2shows the results of MCIS. These results provide an overestimation of the reliability index found with MC above. This is probably because this limit state function uses a model factor for the hydraulic head instead of for the pipe length. The computational time is the same as the one of MCIS. With these results, it can be concluded that using the pipe length in the limit state function has no added value when calculating the failure probability for this base schematization.For a more complex schematization, where the computation for a single head drop takes more time, the difference in computational time between approach 1 and approach 2 increases.

Table2. Results using the limit state function based on the hydraulic head.

Limit state function	Hydraulic head limit state			Pipe length limit state
Probabilistic method	FORM	MC	MCIS	MCIS
Realibility index (-)	2.84	2.93	2.95	3.13
Convergency (-)	0.00378	0.0999	0.0997	0.0999
Computational time (HH:MM:SS)	00:05:26	80:51:42	01:13:17	01:21:30
Number of simulations (-)	25	59000	780	780
Influence factor m_p (%)	5.,993	6.053	8.441	9.720
Influence factor h_r (%)	77.752	78.226	77.986	79.766
Influence factor d_{70} (%)	1.412	0.923	1.184	1.166

Influence factor k (%)	14.843	14.758	12.389	9.348
Parallel runs	1	4	4	4

5 Robustness test

The results from Section 4,with a small and simple model, indicate that a FORM analysis, using the limit state function based on the hydraulic head,provides fast and accurate results. To test the robustness of this method a more complex schematization is used. The schematization has the same dike width, two times longer foreshore, and a 180 m long hinterland. The blanket layer is no longer impermeable and is, therefore, included in the model as a 2 m thick layer. The aquifer has the same thickness but now contains two layers with different permeabilities, see Figure 2. Anisotropy is applied to the upper layer of the aquifer. The river and polder boundary conditions are now also applied at the top of the blanket layer, allowing for seepage through the blanket. The same parameters are applied to the model as in Table 1, except for the permeabilities, see Table 3.

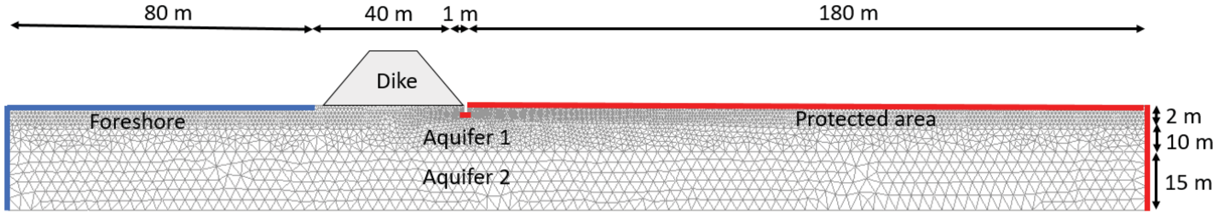

Figure2. Illustration of the more complex schematization.

Table3.Permeabilities used in the more complex schematization.

Variable name	Distribution	Parameters
Intrinsic permeability blanket layer - k (m^2)	Deterministic	9.8E-13
Intrinsic permeability aquifer layer 1 - k_1 (m^2)	Truncated lognormal	μ = 6.14E-11; σ = 3.07E-11; min = 9.00E-12; max = 2.00E-10
Intrinsic permeability aquifer layer 2 -k_2 (m^2)	Truncated lognormal	μ = 7.67E-11; σ = 1.00E-11; min = 9.00E-12; max = 2.30E-10

Two different types of meshes are applied to the model, a finer mesh (visible in Figure 2) and a coarser mesh.The finer mesh contains 7035 mesh elements and 130 pipe elements. The coarser mesh contains 3,522 mesh elements and 73 pipe elements. When computing a deterministic DgFlow calculation, the finer mesh gives a 3.67 m critical head and a 15.53 m critical pipe length. With the coarser mesh similar results are found; 3.60 m critical gradient and 14.59 m critical pipe length. In Figure 3 the hydraulic heads and pipe growth of the fine mesh at the critical head is illustrated.

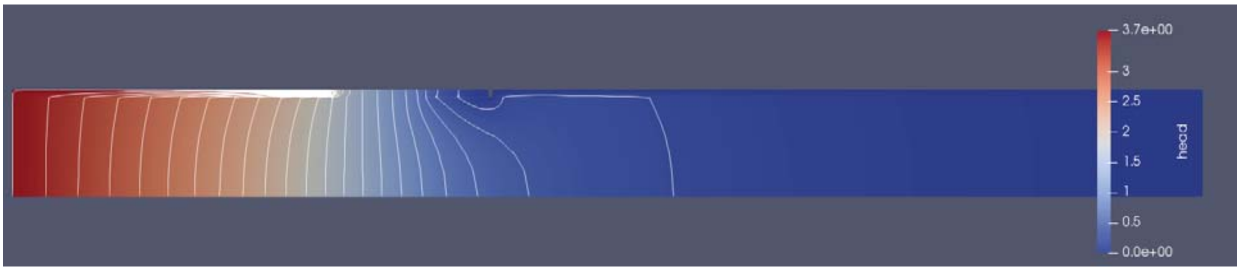

Figure3. Illustration hydraulic head and pipe growth for a head drop of 3.67 m.

In addition to the FORM analysis, as is done in Section 4, a FORM analyses with a starting method called 'direction' is evaluated. This is done because, due to the complexity of the schematization,it is expected that the conventional FORM without starting method could give some problems. The results are provided in Table 4. For both types of mesh, the computations with 'direction' as starting method have a high computational time but similar reliability index (hence, no increase in performance). The finer mesh gives as predicted a higher computational time and a higher reliability index. From these results, it can be concluded that the FORM without starting method is preferable and gives no computational problems. However, this conclusion needs to be confirmed in a follow up analysis.

Table4. Results using the more complex schematization all based on FORM.

	Finer mesh	Finer mesh	Coarser mesh	Coarser mesh
Starting method	None	Direction	None	Direction
Reliability index (-)	3.33	3.35	3.25	3.25
Convergency (-)	0.00768	0.0408	0.00863	0.00828
Computational time (HH:MM:SS)	11:31:05	19:02:09	00:41:58	01:27:42
Number of simulations (-)	23	33	4	8
Influence factor m_p (%)	13.848	13.673	12.761	12.909
Influence factor h_r (%)	74.226	72.512	72.673	73.050
Influence factor d_{70} (%)	0.000	1.960	1.976	1.992
Influence factor k_1 (%)	1.830	1.882	1.945	1.940
Influence factor k_2 (%)	10.096	9.974	10.645	10.110
Parallel runs	1	1	1	1

6 Conclusions and recommendations

In this study, a first attempt to compute a BEP probabilistic computation using a FEM model called DgFlow is made using the PTK of Deltares. This is done to find the most suitable probabilistic method and limit state function for such analyses, which can be implemented in the successor of DgFlow called D-Geo Flow 2022. The analysis concludes that a FORM routine in combination with a search algorithm to find the critical head in DgFlow gives fast and reliable results. It is advised to extend this study further by computing FORM calculations with different complex schematizations to investigate whether larger meshes, larger time steps in DgFlow, different forms of anisotropy, other FORM settings, or more stochastic parameters affect the performance. Furthermore, it is advised to benchmark these against MC.

Acknowledgments

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