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Probabilistic Seepage Analysis of Embankments with Different Types of Random Fields

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Abstract: Probabilistic approaches based on reliability analysis have been developed considerably for carrying out dam risk analysis studies in the last years. The random field (RF) theory and Monte Carlo simulation (MCs) are widely used in geotechnical engineering which considers the numerical model with soil spatial variability, and some studies use this method in seepage analysis with RFs. In this study, probabilistic analysis of seepage through a dam was performed. The traditional deterministic seepage analysis method was extended to develop a probabilistic approach that accounts for the spatial variation of the permeability. The analysis of the variability of seepage in an earth dam will be analyzed in different types of random fields: classical stationary RFs, conditional RFs (to consider the position of available measurements), and nonstationary RFs (to consider the mean variation with depth). The numerical model will consider the random fields (RFs) of permeability. The three mentioned types of RFs are all implemented in the same Finite Element software named OptumG2. This article will also compare these three types of RFs according to the effect of the different result parameters of seepage analysis of hydraulic gradient.

Keywords: embankment; seepage analysis; spatial variability; conditional random fields; nonstationary random fields.

1 Introduction

At present, most scholars mainly use random fields theory to explicitly simulate the spatial correlation characteristics of soil parameters in the study area, so the parameters of rock and soil are different in space (Vanmarcke 1984). The stochastic finite element method based on the RF method has made great progress in recent years (Griffiths and Fenton 2004; Guo et al. 2018; Mouyeaux et al. 2018). Zhu reproduces the anisotropic spatial variation structure of geotechnical parameters by setting the fluctuation range in the horizontal and vertical directions (Zhu et al. 2013). However, this method of random field generation has limitations for directional anisotropic random fields. And it ignores the observational data in the study area, resulting in an overestimation of the spatial variability of soil parameters at specific sites (Li et al. 2017). In response to the above problems, some scholars try to describe the anisotropy of geological structures through random field theory, and some scholars use the observation data as constraints, use conditional random fields and establish nonstationary random fields whose mean value varies with depth to describe the spatial variability of geological structures (Guo et al. 2021). Guo compared the calculation results of slope stability of an existing dam considering conditional random fields and unconditional random fields (for dry density, cohesion and friction angle) and got the conclusion that the differences among the reliability results of using different RFs are not noticeable, which is mainly due to the global homogeneity of the considered data.

Scholars have conducted some research on the influence of spatial variability of permeability on slope instability. For example, Griffiths and Fenton (1993) investigated the mean, standard deviation and spatial correlation structure of the soil permeability in the case of two cut-off walls dam and got the results about the effect of the standard deviation and correlation structure of the permeability on the output statistics relating to seepage quantities, exit gradients and uplift pressures. Ahmed (2009) modelled the permeability as a spatially random field following lognormal distribution and got the conclusion that the flow through the dam had markedly increased when the permeability was more strongly correlated in the horizontal direction than the vertical direction. Cho performed a probabilistic analysis of seepage through an embankment on the soil foundation to study the effects of uncertainty due to the spatial heterogeneity of the permeability on the seepage flow (Cho 2012). However, these studies generally use random fields of spatial variation based on the fluctuation range in the horizontal and vertical directions to simulate the spatial variability of geotechnical parameters, and neither conditional random fields nor nonstationary random fields are involved.

In view of the current research status, the research objectives of this article are: (1) Consider three types of RF (unconditional-stationary RF, conditional RF, and nonstationary RF) to represent the spatial variability of hydraulic parameters (permeability). (2)Evaluate the effect of these three types of RF on the variability of seepage through the dam.

2 Presentation of Seepage analysis

Seepage flow depends on several factors that determine the level of fluid movement, and one of the main factors is permeability.According to existing experience, the seepage failure of earth dams mostly occurred during the first impoundment process or under historically high water level conditions. Internal erosion is a particularly dangerous process as it gradually degrades the integrity of a structure in a manner that is often completely undetectable (Fell and Wan 2005). When the seepage gradient J generated by the seepage of the embankment exceeds the maximum allowable gradient J_C of the material of the embankment body (or foundation), the embankment will undergo the initiation of an internal erosion process. Therefore, the calculation of the seepage field is the key to judging whether the embankment has seepage damage. In this paper, the seepage analysis is performed with a steady-state numerical hydraulic model, and the hydraulic gradients are then evaluated from Darcy's law.Darcy's law shows that: the flow velocity is linearly related to the hydraulic gradient, and the flow velocity is proportional to the hydraulic gradient,

$$i=v/k_s.$$

where: i= hydraulic gradient, v= flow velocity, k_s=saturatedpermeability

Due to the large variability and strong non-linearity of soil properties, scholars generally use Monte Carlo techniques instead of traditional reliability analysis methods such as the First-order Reliability Method (Melchers and Beck 2018) and Second-order Reliability Method (Breitung 1984) to solve geotechnical problems. Even though this approach may require a large amount of computational effort, the mathematical formula of Monte Carlo simulation is relatively simple, the calculation precision is relatively high, and this method has the ability to handle almost all complex cases (Rubinstein and Kroese 2016).

In order to reduce the amount of calculation and improve the efficiency of obtaining accurate results, the convergence of the unconditional-stationary model can be analyzed at first before performing the large number of simulations for the different models with three types of RFs. According to the inspection of the convergence of the flow rate using 1000 simulations, it is found that: The estimated mean and standard deviation of the flow rate almost won't be varied with the increase of the number of simulations if the number of simulations is >300.In order to ensure that the calculated results are accurate and efficient, 500 simulations were performed in each type of RFs in this article to ensure that the results of the analysis were not affected by the number of simulations.

The probabilistic analysis of the infiltrations will be carried out by MCS. A sample of random permeability fields (for each type of random field) will be generated and integrated into the hydraulic model to characterize the spatial variability (mean and standard deviation) of the resulting hydraulic gradients.

3 Three types of RFs

Several probabilistic analysis works in geotechnics take into account the spatial variability of material properties with RFs (generally adopting stationary RFs).This paper presents a comparison of three types ofrandom fields (stationary, conditional and nonstationary) from the same data set (32 permeability values).

3.1 Unconditional-stationary RFs

There are several methods to deal with discrete problems of random fields, such as the average local method, center point method, Karhunen-Loeve (K-L) expansion, orthogonal series expansion, etc.This article mainly studies the characteristics of the K-L method to facilitate the subsequent generation of random fields of soil parameters. The K-L method expands the random field into a linear combination of the deterministic function and the product of the spatial coordinates. This method has already been generally used by the scholars to study the seepage analysis which consider the spatial variability of soil properties (Cho 2012; Liuet al. 2017; Mouyeauxet al.2019). Therefore, K-L expansion was used to generate unconditional stationary RFs in this article.

The analysis was performed using the mean value of saturated permeability for the studied earth dam shown in Figure 1. The embankment constructed on a 5m thick soil foundation was 5 m high, with upstream and downstream slopes of 2h:1v, and a water level in the reservoir was 4.5 m above the foundation. The geometry and soil properties were referred to Cho's model (Cho 2012) in the analysis, and they are displayed in Table 1. Where, θ_r and θ_s are the residual and saturated volumetric water contents respectively, m, n, and α are van Genuchten parameters, k_s is the saturated permeability.In order to solve the free-surface problem, a more general saturated/unsaturated approach (van Genuchten model) was be taken whereby the governing equations are valid throughout the domain, and material parameters that vary significantly with the degree of saturation. At the free surface in the embankment, both the pressure and the normal flux are zero. At the same time, the flow rate is also equal to zero on the surface of foundation which at the downstream side.

The flow calculations were performed using the finite element software OptumG2. The results obtained from the deterministic analysis are shown in Figure 2 and Figure 3. As did in previous studies on this dam, the permeability is modeled by means of RFs to consider the variability. According to lots of literature, the soil permeability of the earth dam has been considered a spatially random field that followed a lognormal distribution, with a prescribed mean, variance, and spatial correlation structure (Griffiths and Fenton 1993).

Table1. Statistical properties of soil parameters used for seepage analysis.

Parameter	Mean	COV	Auto-correlation distance
K_s (m/day)	0.864	0.75	L_h=20m, L_v=2m
θ_r	0.045	-	-
θ_s	0.430	-	-
α(kpa^{-1})	1.478	-	-
n	2.68	-	-
Coefficient of anisotropy K_h/K_v	1		

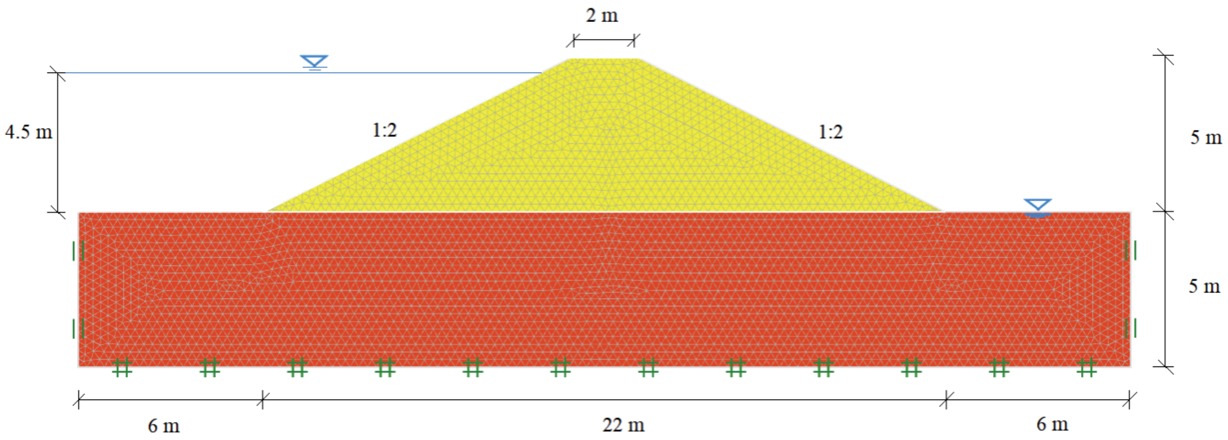


Figure1. The Finite element model in Optum G2.

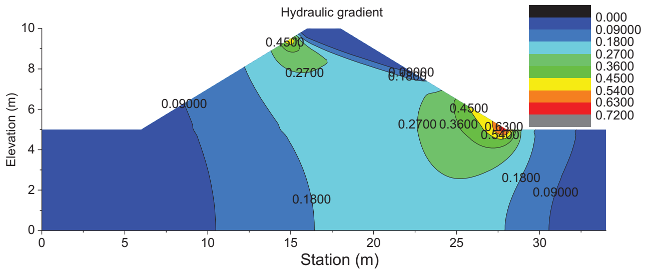


Figure 2. Deterministic analysis - Hydraulic gradient.

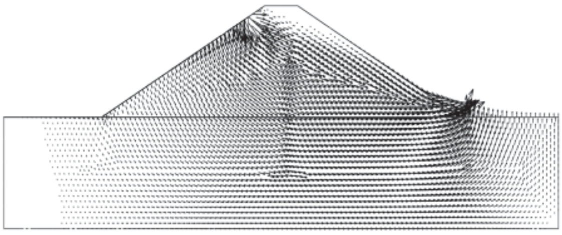


Figure 3. Deterministic analysis - Flow velocity vector.

3.2 Conditional RFs

A conditional random field can be used when soil properties at certain locations are known. Its purpose is to ensure that the simulated random field matches the soil properties of these specific sites exactly. It shows that in each realization of the conditional RFs, the soil properties at these specific locations are constant (measured data), and the soil properties at other locations are random variables correlated to the measured data. For this reason, the ordinary Kriging method is used, as it provides the best estimate of soil properties at unknown points while taking into account the spatial correspondence.

3.3Nonstationary RFs

At present, most of the reliability studies use stationary random fields to characterize the spatial variability of soil parameters. However, a large number of field tests and related test data confirm that soil parameters usually exhibit nonstationary distribution characteristics along with the depth, such as the mean and standard deviation of dry density, undrained shear strength, internal friction angle, and the compressive modulus. They have a tendency to increase in the depth direction (Zhu et al. 2017). However, Rehfeldt et al. (1992) used the borehole flowmeter method for the shallow soil in a test site to clearly point out that the permeability has a decreasing trend with depth. At the same time, Jiang et al. (2010) deduced an empirical and semi-empirical negative exponential trend equation describing the permeability decreasing with depth in natural soils.

In this paper, the measured data will be used to estimate the mean value of each soil layer to evaluate their evolution according to the depth (Figure 4). In practical engineering, the zone of the embankment is artificially landfilled and constructed, so the linear trend of permeability in this area in the vertical direction is not very obvious. However, the naturally formed foundation could more easily present a trend of linear change in the mean value of permeability in the vertical direction. Figure 4 (right) presents the obtained variation of the permeability mean values. The global average value determined by all the 32 measurements is presented as the red vertical line.

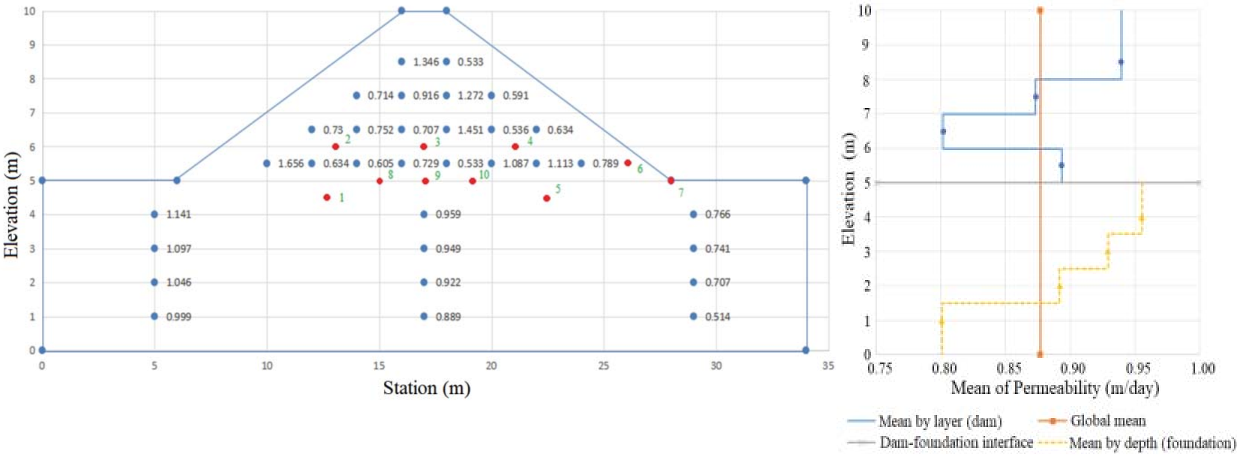


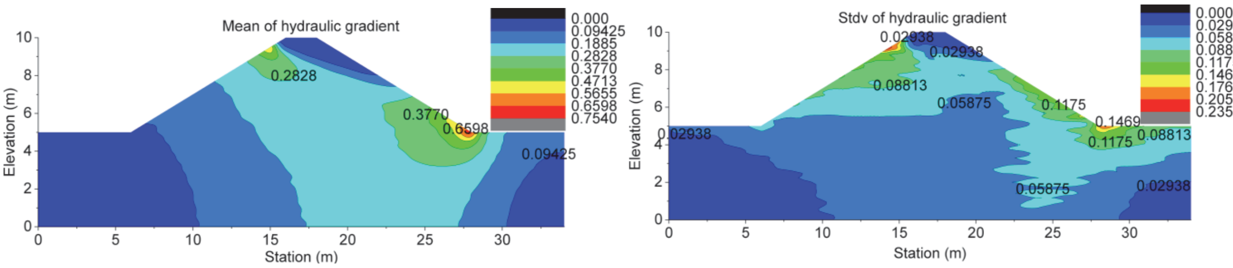
Figure 4. The measurements used for the estimation of variable permeability mean value.

4 Compare the results of seepage analysis-hydraulic gradient

In order to compare the effects of three different types of RFs with the hydraulic gradient analysis results, some results at the locations of our interest should be extracted. In this paper, 10 points are considered to compare the results obtained with the different random fields. These points numbered from 1-10 are presented in Figure 4 (red points). The resultant value of the hydraulic gradient was calculated as the research objective in this section. The paper now specifies that conditional RFs lead to lower standard deviation values than those obtained with stationary RFs, which has also been found by other researchers such as Yang et al. (2019)

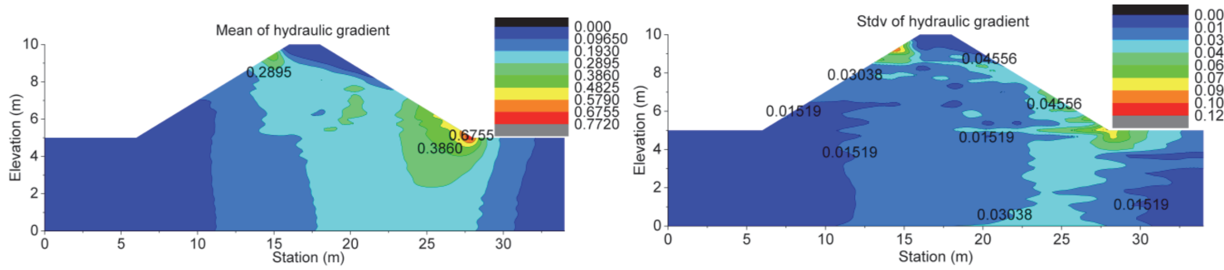
Figures 5 to 7 represent the statistical properties (mean and standard deviation) of hydraulic gradient obtained by Monte-Carlo simulations with unconditional stationary RFs, conditional RFs, and nonstationary RFs, respectively. A sample of the hydraulic gradient is calculated on each node of the mesh. These figures are obtained by computing the mean and the standard deviation of each of these samples. The areas of highest variability of hydraulic gradient (i.e., with the highest standard deviations) are mostly located at the toe of embankment, wherewith the highest mean of hydraulic gradient (as shown by Figures 5 to 7) and highest flow velocity (as shown by Figure 3). The contours of the mean hydraulic gradient in the three random fields are generally consistent, and they are similar to the contour in the deterministic analysis. For variance, the maximum value exists at the toe of the embankment. And the contours of the hydraulic gradient in the case of conditional RFs seem a little different from the other two cases.

By observing Figure 6 (a), it illustrated that the distribution of hydraulic gradient in conditional RFs is different from the other types of RFs, especially the area with larger or smaller values of measured data. For example, there is different small area colored in light blue and light green located in the center part of the dam body. For the area colored in blue, the bigger value of permeability (1.451m/day) is located in this area. The flow velocity is a result parameter related to the overall seepage situation of the earth dam, therefore, the value will not have a significant increase in the surrounding area nearby the several bigger measured data either. According to Darcy's law which has already been presented in chapter 2, an explanation is presented: The estimated mean value of the hydraulic gradient near the area with big measured permeability is significantly smaller than other areas. A similar explanation may be easily adapted that the area color in green at the center part of the dam body. Figure 6 (b) clearly shows that the standard deviation of the hydraulic gradient becomes smaller around the measurement than the area far from the measurement. By comparing Figure 6 (b) to Figure 5 (b) and Figure 7 (b), the standard deviation of the hydraulic gradient of the conditional RFs is smaller than that of the other two types of RFs. It shows that the observation data plays a very important role in the generation of conditional RFs. If it is in practical engineering, this method can be used to estimate the hydraulic gradient of the local area more accurately.



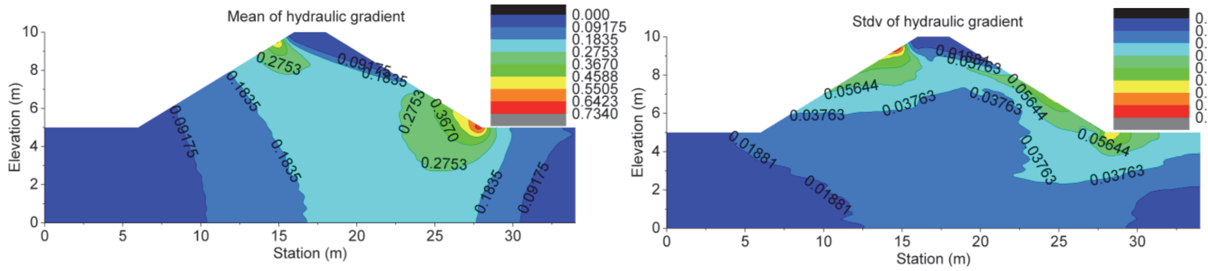
(a) Mean of the hydraulic gradient (b) Standard deviation of the hydraulic gradient

Figure 5. Contours of the hydraulic gradient with unconditional **stationary RFs** obtained by Monte-Carlo simulations.



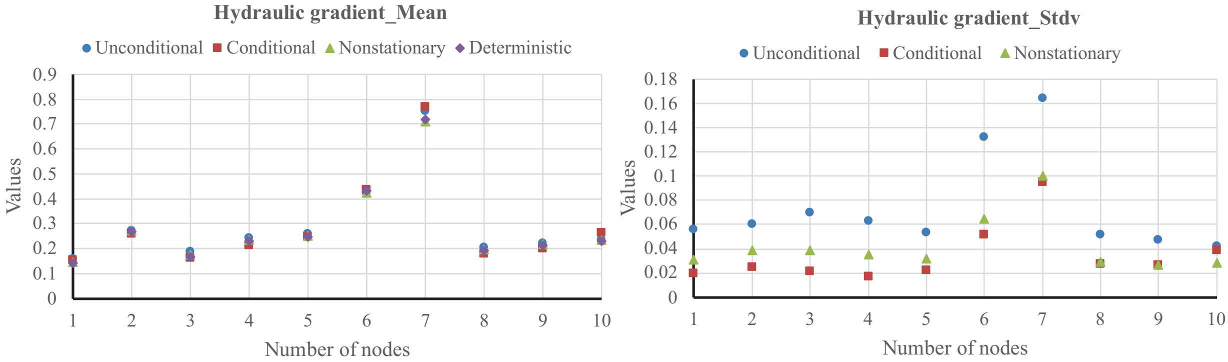
(a) Mean of the hydraulic gradient (b) Standard deviation of the hydraulic gradient

Figure 6. Contours of the hydraulic gradient with **conditional RFs** obtained by Monte-Carlo simulations.



(a) Mean of the hydraulic gradient (b) Standard deviation of the hydraulic gradient

Figure 7. Contours of the hydraulic gradient with **nonstationary RFs** obtained by Monte-Carlo simulations



(a) Mean of the hydraulic gradient (b) Standard deviation of the hydraulic gradient

Figure 8. Comparison of hydraulic gradient among three types of RFs in mean and standard deviation

Figure 8 allows the comparison of hydraulic gradient among three types of RFs on the different nodes in their means and standard deviation. It shows that the mean-field of hydraulic gradient obtained after Monte-Carlo simulations is close to the deterministic one (see. Figure 2). Despite the uncertainties brought by the procedure, the results obtained for the case study after completing the probabilistic modelling of the hydraulic gradient are globally similar in three types of RFs, as illustrated by Figure 8 (a). Figure 8 (b) clearly shows that the standard deviation of the hydraulic gradient which uses unconditionally stationary RFs is significantly larger than the other two types of RFs. Among them, the standard deviation calculated by using the conditional RFs is obviously smaller than the other two cases, so the use of the conditional RFs can greatly reduce the uncertainty of the calculation results and can accurately estimate and calculate the real results of the earth dam.

5 Conclusion and perspective

In this article, three different types of RFs: unconditional stationary RFs, conditional RFs, and nonstationary RFs were used to model soil spatial variability. The parameters of the different RFs are determined by considering the same hypothetical observation data in the theoretical dam. The effects of the different types of RFs on the dam hydraulic gradient are investigated as well. On the whole, the differences between seepage analysis results (mean of hydraulic gradient) using different types of RFs were not significant. The results obtained from this case study show that it's better to use the observation data of the measurements to generate conditional RFs because it will reduce the input uncertainties and decrease the standard deviation of seepage analysis results. For the other two types of RFs, they will overestimate the simulation standard deviation of the underlying random fields of the soil properties. This article only focuses on the results of hydraulic gradient and the effect of different random fields on the variability of other hydraulic properties (velocities, pore pressures, and so on) as well as the effect on the probabilistic assessment of internal erosion will be studied in the next work.

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