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Probabilistic Seismic Hazard Analysis Considering Local Ground Motion Observations: Bayesian Approach

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Abstract: This study proposed the methodology of Bayesian seismic hazard analysis, including a case study on the site in Taiwan. As many other Bayesian works, this application also used the "observation" to update the "prior information" to obtain the "posterior information." Then according to the posterior information, the seismic hazard or the annual rate of PGA exceedance can be estimated for the study site. In addition to the details of the Bayesian updating, the method was demonstrated with a case study using the data from Taiwan.

Keywords: Bayesian method, seismic hazard analysis, Taiwan

1 Introduction

1.1 Annual rate of PGA exceedance

For estimating the occurrence probability of a certain event (e.g., catastrophic earthquake) within a given period of time (e.g., 50 years), we need to estimate its annual rate (e.g., 0.1 per year) in the first place, and then (usually) use the Poisson point process to estimate the event's occurrence probability within the given time window. For estimating the annual rate of such events, the most straightforward approach is based on the records in the past. For instance, assuming there were 5 catastrophic earthquakes occurring in a region in the past 500 years, then the best-estimate annual rate is considered 0.01 per year. Similarly, for estimating the annual rate of a certain (earthquake-induced) peak ground acceleration (PGA) or spectral acceleration (SA) exceedance (e.g., PGA > 0.1 g) at a site, such a "direct estimation" method or frequentist approach can also be used based on the PGA data recorded or observed at the site. For instance, if 10 events of PGA > 0.1 g have been recorded at a site in the past 50 years, then the best-estimate annual rate is considered 0.2 per year. Understandably, for estimating the annual rate of PGA exceedance using the frequentist method based on the PGA records, the "sample size" matters. That is, the longer the observation window (e.g., 10000 years) is, the more reliable the frequentist estimate will be. It is also noted that because PGA is extracted from earthquake ground motions or acceleration time histories (Figure 1), thus the fundamental data for estimating the annual rate of PGA exceedance using the frequentist approach are the observed earthquake ground motion records.

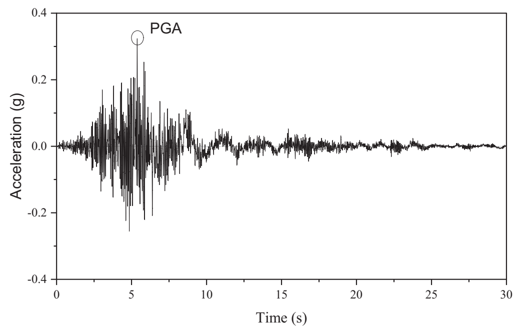

Figure 1. Earthquake ground motion time history and PGA; this illustrates the fundamental data for estimating the annual rate of PGA exceedance using the proposed Bayesian calculation are the ground motion time histories.

However, the history of modern seismic instrumentation arrays for recording earthquake ground motions is no more than tens of years. As a result, sample size is an issue when using the frequentist method to estimate the annual rate of PGA exceedance at a site, especially for large PGA exceedances (e.g., PGA > 1.0 g) with a return period in hundreds of years. Besides, because the frequentist method is based on the observed ground motion records at the site, the method cannot be used for sites where ground motion records are absent.

Under the circumstances, indirect approaches, such as Probabilistic Seismic Hazard Analysis (PSHA) (Cornell, 1968), were proposed for estimating the annual rate of PGA (or SA) exceedance considering a combination of indirect data, mainly including ground motion prediction equations, seismic source models, and earthquake catalogs. For instance, based on the source model **(**Figure 2**)** and using three local PGA ground motion prediction equations (Table 1), Wang et al. (2013) conducted a PSHA study for the site located in the center of Taipei (Taiwan), and estimated the annual rate of PGA > 0.3 g at the site was about 0.0021 per year, mainly based on the seismic source model (Figure 2) and three PGA ground motion prediction equations (Table 1). As mentioned previously, with the best-estimate annual rate from PSHA, we can use the Poisson point process to estimate the occurrence probability of PGA > 0.3 g at the site in the next 50 years is equal to 10%.

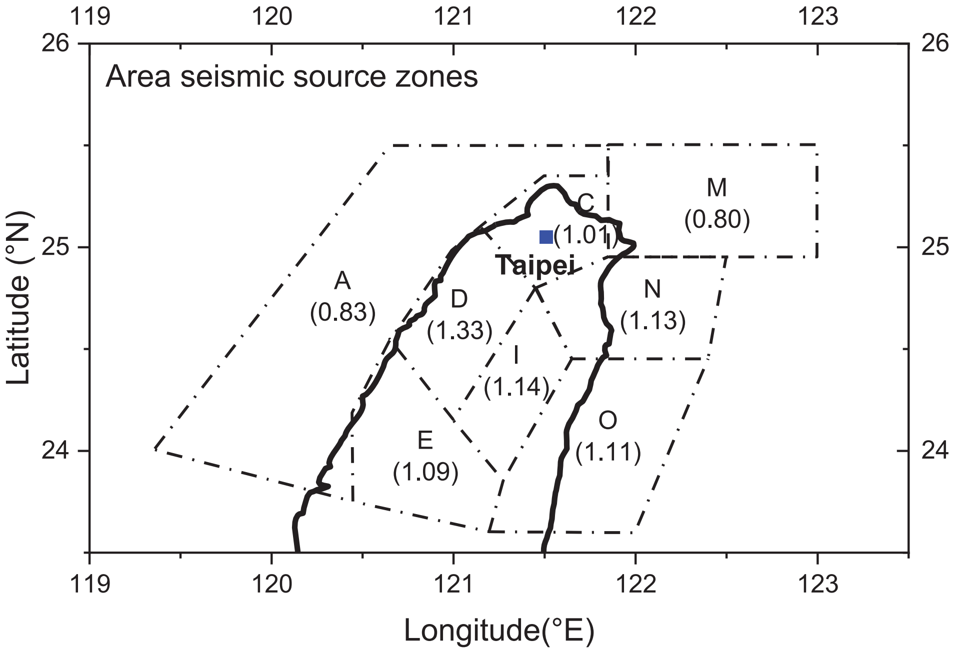

Figure 2. The seismic source model used in a PSHA study for Taipei (Taiwan); the value in the parenthesis is the b-value of the Gutenberg-Richter recurrence law (after Wang et al., 2013).

Table 1. Three ground motion prediction equations used in a PSHA study (Wang et al., 2013) for developing the seismic hazard curve (Fig. 4) at the site in Taipei.

Ground motion prediction equations	Reference
$\ln PGA = -3.248 + 0.943 \times M - 1.471 \times \ln(D + e^{0.648M}) \pm 0.628$	Lin et al. (2011)
$\ln PGA = -3.25 + 1.075 \times M - 1.723 \times \ln(D + 0.156e^{0.62391M}) \pm 0.577$	Cheng et al. (2007)
$\ln PGA = 2.303 \times 0.00215 + 0.581M - \log(D + 0.00871 \times 10^{0.5M}) - 0.00414D \pm 0.79$	Wu et al. (2001)

* *M* = moment magnitude; *D* = rupture distance (source-to-site distance) in km; *PGA* in g.

Since the PSHA method was proposed in the 1960s, a number of case studies have been conducted and published on academic journals (e.g., Petersen et al., 2004; Anbazhagan et al., 2009; Stirling et al., 2011; Kolathayar and Sitharam, 2012; Wang et al. 2013). In addition, it is also worth noting that the U.S. Nuclear Regulatory Commission (USNRC, 2007) has designated PSHA as the standard method in a technical guidance for developing site-specific, earthquake-resistant designs for the safety-related structures of a nuclear power plant.

However, although using PSHA to estimate the annual rate of PGA exceedance based on indirect data has been accepted and commonly employed, the method has also been challenged for its reliability, given that the estimated annual rates of PGA exceedance were not in good agreement with the observed PGA records (e.g., Castanos and Lomnitz, 2002; Musson 2012a, 2012b; Wang, 2012). Furthermore, it is a common practice in PSHA using the logic-tree analysis for "consolidating" the epistemic uncertainties (e.g., which ground motion prediction equations should be used), and this makes it difficult to repeat and validate a PSHA study by others (Krinitzsky, 2002; Bommer and Scherbaum, 2008; Wang and Lin, 2021).

1.2 Bayesian approach

When encountering a statistical or probabilistic estimation, engineers and scientists most likely will use samples (or observation) to obtain the inference, and such a method is referred to as the frequentist approach (as the previous example showing how to obtain the annual rate of PGA exceedance based on the observed PGA records in the past 50 years). Nevertheless, the frequentist inferences might be unrepresentative and unrealistic when the sample size is too small. For instance, when only one sample is available, theoretically speaking the standard deviation of the random variable of interest (e.g., soil's friction angle) is equal to 0, which is unrealistic.

As a result, for obtaining a more accountable inference with limited samples, alternative methods, such as the Bayesian approach, are more commonly used. In essence, the Bayesian approach is to integrate the two sources of data to obtain an inference: 1) the "prior information" from indirect data, and 2) the "observation" from direct data or samples. In other words, the Bayesian approach utilizes the prior information from indirect data to compensate the limited observation or direct data, although the former (i.e., indirect data) is less reliable and relevant to the problem of interest than the latter (i.e., direct data). On the contrary, when the direct data or samples are in abundance, it is uncommon to still use the Bayesian approach to obtain an inference. After all, the indirect data are less relevant to the target problem, and adding them to the abundant direct samples might "contaminate" the resultant inference. The mathematical model of the Bayesian approach is elaborated in Methodology.

A variety of Bayesian applications in engineering geology and geotechnical engineering have been reported. For instance, for estimating the standard deviation of soil friction angle at a study site with only two samples from laboratory tests, Wang and Xu (2015) considered the generic coefficient of variation for soil friction angles based on a global database (Phoon and Kulhawy, 1999) as the prior information, and used the Bayesian approach to integrate the indirect data or prior information with the two project-specific samples or direct data for the estimation. Similarly, Hapke and Plant (2010) utilized the Bayesian approach to estimate the rate of coastal cliff retreat at two sites in southern California, based on the observed retreat rate (as observation) in the past decades, along with the model prediction (as prior information) based on the lithology and geometry of the cliff. Similar examples also include that Ching et al. (2010) proposed an application of the Bayesian approach to estimate the clay's undrained shear strength at study sites, using limited laboratory tests as the observation and several empirical relationships as the prior information. Likewise, Wang et al. (2016) used the Bayesian approach to estimate the return period of major earthquakes induced by an active fault in northern Taiwan, based on the evidence from the sediment sequences as the observation (inferring three major earthquakes having occurred in the past 2600 years), along with four theoretical model estimates as the prior information by different research teams. Last but not least, Cao and Wang (2014) also used the Bayesian approach to estimate the clay's undrained shear strength at a study site based on limited laboratory tests (as the observation), along with the results of in-situ cone penetration tests (as the prior information); Wu and Wang (2020) developed an empirical relationship between fault rupture length and earthquake magnitude for the regions of Taiwan, using hundreds of global data points as the prior information and two local data points as the observation. In a nutshell, no matter what kinds of application are targeted, the Bayesian approach is to obtain an inference by integrating the observation from direct data with the prior information from indirect data, especially when the direct data are very limited.

2 Methodology

2.1 Bayesian method

As mentioned previously, the Bayesian approach aims to integrate the prior information from indirect data with the (project-specific) observation from direct data. Mathematically, the Bayesian approach for integrating the two sources of data is expressed as follows (Ang and Tang, 2007).

$$\Pr''(\theta_i) = \frac{\Pr'(\theta_i)\times\Pr(\varepsilon|\theta_i)}{\sum_{i=1}^{n}\Pr'(\theta_i)\times\Pr(\varepsilon|\theta_i)} \quad (1)$$

where θ_i denotes the i-th estimate for a target question; ε is the (project-specific) observation; $\Pr'(\theta_i)$ denotes the prior probability of θ_i, and $\Pr''(\theta_i)$ denotes the posterior probability of θ_i after updating the information from the observation (ε). The term $\Pr(\varepsilon|\theta_i)$ is referred to as the likelihood function in the Bayesian calculation, which is the probability that the observation (ε) will occur on the condition of the occurrence of θ_i.

2.2 Bayesian formula for this application

Based on the generic Bayesian formula (Eq. (1)), this section elaborates the Bayesian equation for estimating the annual rate of PGA exceedance considering both the prior information (i.e., PSHA prediction based on indirect data) and the observation (i.e., direct PGA records at the site). To be more specific, the prior information is the best-estimate annual rate of PGA exceedance from PSHA based on indirect data, and the observation is how many times the PGA exceedance were observed at the site in the past *t* years.

Given the best-estimate annual rate of PGA > y* (y* is a constant; e.g., 0.1 g) from PSHA is equal to ν per year, therefore the mean rate of PGA > y* within *t* years is equal to νt. Then assuming the event (i.e., PGA > y*) is a rare event that can be properly modeled by the Poisson point process (Kramer, 1996), the probability for the target event to occur *i* times (*i* = 0, 1, 2,…) within *t* years is equal to:

$$\Pr(i) = \frac{e^{-\nu t}\times \nu t^{i}}{i!} \quad (2)$$

As a result, Eq. (2) is the prior probability mass function for the proposed Bayesian calculation.

Next, we would like to formulate the likelihood function (i.e., $\Pr(\varepsilon \mid \theta_i)$) for this Bayesian updating, where ε is the "n-event-in-t-year" observation. Again, considering the rare event (i.e., PGA > y*) can be modeled by the Poisson point process, the likelihood function is equal to:

$$\Pr(\varepsilon|i) = \Pr(\varepsilon: n \text{ events in } t \text{ years}|i) = \frac{e^{-i} \times i^n}{n!} \tag{3}$$

With the prior probability mass function (Eq. (2)) and the likelihood function (Eq. (3)) that have been derived, the final formula for this Bayesian calculation can be expressed as follows by substituting the two (i.e., Eqs. (2) and (3)) into Eq. (1):

$$\Pr''(i) = \frac{\frac{e^{-vt} \times vt^i}{i!} \times \frac{e^{-i} \times i^n}{n!}}{\sum_{i=0}^{\infty}\left\{\frac{e^{-vt} \times vt^i}{i!} \times \frac{e^{-i} \times i^n}{n!}\right\}} \tag{4}$$

where v is the (prior) annual rate of PGA > y* based on indirect data; n and t are in association with the direct data or the observation as n events that were recorded in the past t years.

3 Application and case study

This section presents the case study using the proposed Bayesian calculation to estimate the annual rate of PGA exceedance at a site in Taipei, Taiwan (Figure 3). The direct (the observation) and indirect data (the prior information) used in the Bayesian calculation are elaborated in the following.

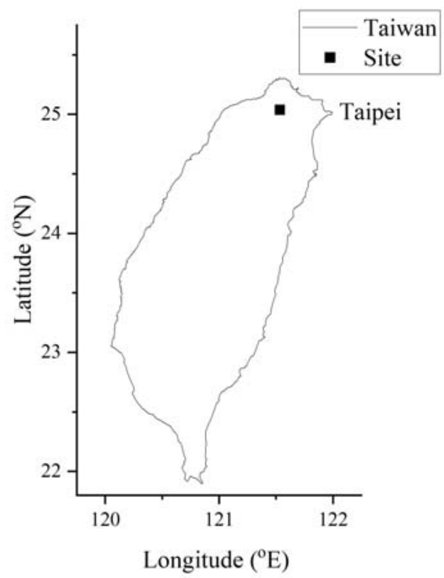

Figure 3. Locations of the study site in Taiwan.

3.1 Hazard curve at the center of Taipei from PSHA

As mentioned previously, PSHA can estimate the annual rate of PGA exceedance at a site based on a combination of indirect earthquake data. For a particular site with the same sources of input data, PSHA practitioners can compute the annual rate (v) for different PGA exceedances. Ultimately, the relationship between the annual rate (i.e., $v_{\text{PGA > y*}}$) and PGA exceedance (i.e., y*) can be illustrated, and such a plot is referred to as the seismic hazard curve for the study site (e.g., Kramer, 1996).

Figure 4 shows the (PGA) seismic hazard curve for the site located in the center of Taipei (Figure 3) from a PSHA study (Wang et al., 2013). (In fact, this PSHA study is the one using the source models (Figure 2) and the three ground motion prediction equations (Table 1) as the main input data.) Based on the seismic hazard curve, the annual rates for PGA > 0.1 g and 0.2 g, for example, are equal to 0.025 and 0.006 per year, respectively.

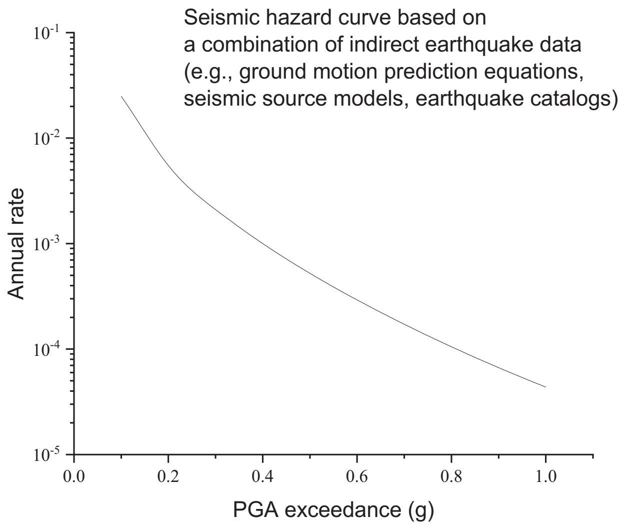

Figure 4. Seismic hazard curve for the site in Taipei (Figure 3).

3.2 Taiwan Strong Motion Instrumentation Program

In the previous section, we illustrated how to obtain the prior information for this Bayesian updating from existing seismic hazard curves or maps. In this section, we went on elaborating how to obtain the observation or the PGA records at the study sites in the past 23 years.

In 1999, a catastrophic earthquake, known as the Chi-Chi earthquake, occurred in central Taiwan. The event killed at least 2000 people and injured more than 4000, and was considered the second-deadliest earthquake in the recorded history of Taiwan (Cheng et al., 2007). Afterward, the government launched the "Taiwan Strong Motion Instrumentation Program" (TSMIP), and its objective is to collect a large amount of ground motion time histories for local earthquake studies by setting up many seismic stations in Taiwan. Currently, around 800 stations have been established in Taiwan (Figure 6) via TSMIP, and the ground motion data induced by major (or big) earthquakes are managed by the Central Weather Bureau (CWB) of Taiwan, and the data are accessible on the website of CWB (CWB, 2022).

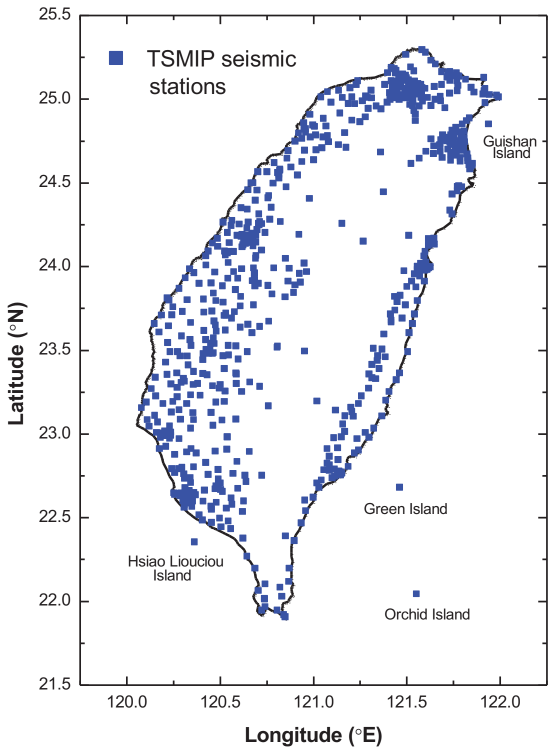

Figure 6. Locations of TSMIP seismic stations in Taiwan.

Since the operation of TSMIP, a local ground motion database has been developed, and more importantly, several earthquake studies were reported based on the TSMIP ground motion database. For instance, Wang et al. (2018) used the database to study the basin effect on local site response in Taipei, identifying the areas in Taipei more susceptible to site amplification owing to the basin effect. Furthermore, new ground motion prediction equations were also developed with the TSMIP ground motion database, which are useful for local seismic hazard studies (Xu et al., 2019).

Therefore, we can obtain the ground motion time histories at the studies sites in the past 23 years (from 1999 to 2022). Table 2 summarizes the "mother" earthquakes of the ground motion data, including magnitudes, distances, epicenters, dates, and the resultant PGA values extracted from the acceleration time histories. For the site in Taipei, the maximum PGA recorded in the past 23 years is 0.117 g, which was induced by the M_w 7.1 earthquake in northeastern Taiwan on March 31, 2002. Therefore, based on the observed PGA records in the past 23 years, the annual rate of PGA > 0.2 g, for example, at the site is equal to 0 using the frequentist method, which should be unrealistic owing to the short observation window or the limited sample size. Also note that the PGA induced by the M_w 7.7 Chi-Chi earthquake is "only" 0.064 g at the site in Taipei, because the epicenter is relatively far (about 147 km) from the site. Because the PGAs induced by each major earthquake (magnitude above 6.0) around Taiwan in the past 23 years were all recorded, there should not be any missing records of substantial PGA exceedance (e.g., PGA > 0.3 g) for the study sites.

Table 2. Summary of the observed PGA records at the study sites since 1999.

Date	M_W	D (km)	Longitude (° E)	Latitude (° N)	Site	PGA (g)
1999/9/21	7.7	147	120.80	23.90	Taipei	0.064
1999/10/22	5.8	207	120.42	23.52	Taipei	0.005
2002/3/31	7.1	110	122.13	24.24	Taipei	0.117
2013/6/2	6.2	144	120.97	23.86	Taipei	0.012
2013/10/31	6.3	165	121.35	23.57	Taipei	0.027
2016/2/6	6.4	257	120.54	22.92	Taipei	0.006
2018/2/6	6.4	107	121.73	24.11	Taipei	0.011

* D = source-to-site distance in km

3.3 Case study: for the site in Taipei

Taking PGA > 0.2 g for example; the annual rate of PGA > 0.2 g at the site in Taipei was estimated at 0.006 per year based on indirect data (e.g., seismic source models (Figure 2)) and ground motion prediction equations (Table 1). Accordingly, Figure 7 shows the prior probability mass function (using Eq. 2) for the annual rate of PGA > 0.2 g based on this prior information. On the other hand, based on the "0-event-in-23-year" observation as no PGA > 0.2 g events having been recorded in the past 23 years, Figure 7 also shows the likelihood function (using Eq. 3) for this Bayesian calculation. Finally, we substituted both into Eq. 4 to obtain the posterior probability mass function (also shown in Figure 7), based on which we obtained the Bayesian inference for the annual rate of PGA > 0.2 g at the site in Taipei as 0.003 per year.

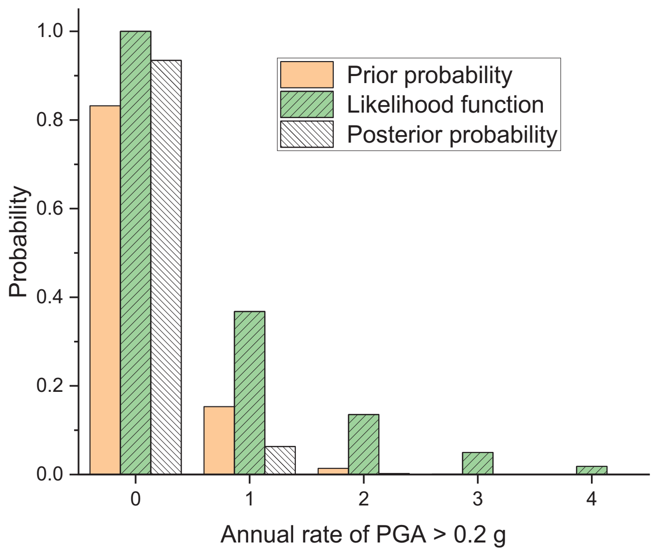

Figure 7. Prior probability mass function, likelihood function, and posterior probability mass function for the annual rate of PGA > 0.2 g at the site in Taipei.

To sum up, for this case study estimating the annual rate of PGA > 0.2 g at the site in Taipei, the prior estimate on the annual rate based on the indirect data was 0.006 per year, and the frequentist estimate based on the direct "0-event-in-23-year" observation was 0. Then integrating the two sources of data using the Bayesian approach, the Bayesian estimate on the annual rate was 0.003 per year, which was in between the prior estimate (i.e., 0.006) based on the indirect data and the (frequentist) estimate (i.e., 0) based on the direct observation. This indicates this Bayesian calculation is robust, producing a reasonable inference that is in between the two individual estimates serving as the inputs of the Bayesian calculation. More discussion over the robustness of the Bayesian calculation is given in one of the following sections.

Repeating the same Bayesian calculations, we can estimate the annual rate for different PGA exceedances (e.g., 0.3 g, 0.4 g, etc.) at the site in Taipei, producing a new seismic hazard curve for the site based on the indirect data and the direct PGA observation. Figure 8 shows the new seismic hazard curve from the Bayesian calculation. It demonstrates that for large PGA exceedances (e.g., PGA > 0.3 g), the Bayesian estimate on the annual rate is lower than the prior estimate based on the indirect data, after updating the "0-event-in-23-year" observation. By contrast, for low PGA exceedances (e.g., PGA > 0.1 g), the Bayesian estimate on the annual rate is larger than the prior estimate based on the indirect data, after updating the "1-event-in-23-year" observation.

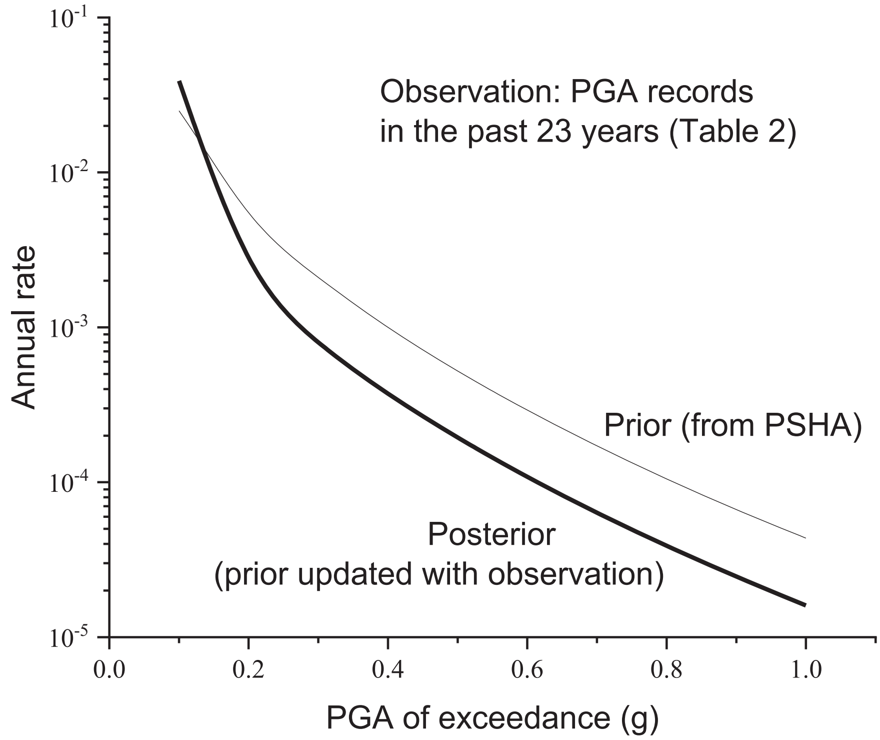

Figure 8. The new seismic hazard curve for the site in Taipei using the proposed Bayesian calculation.

4 Discussions

4.1 Robustness of Bayesian calculation and inference

As mentioned previously, the Bayesian approach is to obtain an inference considering both the prior information (from indirect data) and the observation (from direct data) for a target problem. Therefore, the Bayesian inference should be in between the prior estimate (based on the prior information or indirect data) and the frequentist estimate (based on the observation or the direct data), or the Bayesian calculation should be in question.

As a result, for examining the robustness of the proposed Bayesian calculation, we compared the Bayesian inferences to the other two based on the above case studies. Note that such an examination was also used in other Bayesian studies for validating a Bayesian calculation (e.g., Wang et al., 2016).

Table 3 summarizes the respective estimates for the case study, and it shows that the Bayesian inferences are in between the prior estimate (based on the indirect data) and the frequentist estimate (based on the directly recorded PGA data in the past 23 years). In particular, it shows that when no event was observed in the past 23 years, the new (Bayesian) inference would be lowered after updating the information from the "0-event-in-23-year" observation. By contrast, when 1 event was observed in the past 23 years inferring the frequentist estimate as 0.043 per year, the new (Bayesian) inference would be increased after updating the information from the "1-event-in-23-year" observation.

Table 3. Summary of the estimates on the annual rate based on the prior information (indirect data), the observation (direct data), and the posterior information updated with the proposed Bayesian calculation.

Event	Site	Prior estimate (per year)	Observation: number of events recorded in the past 23 years	Posterior estimate (per year)
PGA > 0.1 g	Taipei	0.025	1(annual rate = 0.043)	0.039
PGA > 0.2 g	Taipei	0.006	0 (annual rate = 0)	0.003
PGA > 0.3 g	Taipei	0.0021	0 (annual rate = 0)	0.0008
PGA > 0.4 g	Taipei	0.0010	0 (annual rate = 0)	0.0004
PGA > 0.5 g	Taipei	0.00052	0 (annual rate = 0)	0.00019
PGA > 0.6 g	Taipei	0.00029	0 (annual rate = 0)	0.00011
PGA > 0.7 g	Taipei	0.00017	0 (annual rate = 0)	0.00006
PGA > 0.8 g	Taipei	0.00011	0 (annual rate = 0)	0.00004
PGA > 0.9 g	Taipei	0.00007	0 (annual rate = 0)	0.00002
PGA > 1.0 g	Taipei	0.00004	0 (annual rate = 0)	0.00001

4.2 Proper use of Bayesian updating

Understandably, direct data are more useful and reliable than indirect data for obtaining an inference. As a result, the reason we added indirect data in the estimating is due to the lack of sufficient direct data, like those Bayesian applications summarized in Introduction. More importantly, as those Bayesian studies, the researchers considered the limited direct data still contained some valuable information for the problems of interest, and they ought to be utilized in conjunction with indirect data, or using the indirect data to compensate the limited direct data (e.g., Ching et al., 2010; Hapke and Plant, 2010; Wang et al., 2016). On the other hand, because the indirect data are less reliable than the direct data, thus when the sample size of direct data is large, it is unnecessary and unwise to add the indirect data into the estimation still. As a result, the proper use of the Bayesian updating should be when the availability of direct data is very limited.

Taking this study for example: hypothetically speaking, if we have the PGA records at the study sites in the past 1000000 years, we can obtain a representative (frequentist) estimate for the annual rate of PGA exceedances at the study sites simply based on the observation, and will not consider any indirect data in the estimation. Nevertheless, the reality is that such an observation is not possible at this moment, so the Bayesian approach can be utilized as using the indirect data as the prior information that is then updated with the observation from the direct data or samples.

5 Conclusions

a)Currently, the annual rate of PGA exceedance is usually estimated with indirect data (e.g., ground motion prediction equations) using the PSHA approach, because the PGA records in the past hundreds of years at a study site are not available.

b)Like other Bayesian studies, this paper proposed a new Bayesian application to estimate the annual rate of PGA exceedance, utilizing the indirect data as the prior information and the direct data as the observation for this Bayesian calculation.

c)In addition to the methodology and algorithm, this paper also presents a case study. All the cases show that the Bayesian estimates on the annual rate of PGA exceedance are in between the frequentist estimate sorely based on the PGA records, and the prior estimate sorely based on the indirect data, which indicates the proposed Bayesian calculation is robust.

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