Proc. of the 8th International Symposium on Geotechnical Safety and Risk (ISGSR)
Edited by Jinsong Huang, D.V. Griffiths, Shui-Hua Jiang, Anna Giacomini and Richard Kelly

doi: 10.3850/978-981-18-5182-7_15-003-cd

ISGSR 2022

A Study of Data-Driven Seismic Response Analysis Based on the Identification of Temporal Evolutionary Law in Dynamic Systems

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Abstract: This study examines the application of dynamic mode decomposition (DMD), one of a data-driven time-series analysis, to seismic response analysis. In this study, a DMD data-driven seismic response analysis model was formulated based on the assumption to observe the seismic bedrock in earthquake engineering (SBEE) and surface layer waveforms. Additionally, we modeled the transfer function between SBEE and surface layer as the equation of motion for a single degree of freedom model (SDOF), which provides a physical meaning to the data-driven model. Then, the possibility of extracting the physical meaning of the ground is discussed by considering the relationship between the DMD model and SDOF through numerical experiments. First, we prepared five cases of soil layer configurations and 25 artificial input seismic waveforms, and seismic response analysis was performed using the one-dimensional equivalent linearized overlapping reflection theory SHAKE. Then, we examine whether the DMD learning model using SHAKE results is described the soil nonlinear response feature as the equivalent linearization parameters (i.e., convergence stiffness and convergence damping constant). Finally, considering the correspondence between the convergence stiffness and convergence damping constants obtained from the SHAKE calculation and DMD model, the possibility of developing a data-driven model that integrates engineering knowledge was discussed.

Keywords: mode decomposition; time series analysis; seismic response analysis; dynamic system

1 Introduction

Recent breakthroughs in numerical analysis and Internet of Things (IoT) technologies have made it possible to utilize a vast amount of high-dimensional data. The technique for extracting essential information from such a vast amount of data and elucidating relevant principles and phenomena is called data-driven science and is expected to be useful in solving social problems in various fields (e.g., Kitchin 2014). Machine learning, including deep-learning, is one of the technologies that belong to data-driven science, which has rapidly developed these performance and tends to be applied in the field of civil engineering, particularly in the field of infrastructure maintenance and management. On the other hand, in applying data-driven approaches to physical phenomena, issues have been pointed out regarding the disappearance of physical laws and difficulties in interpreting the mechanisms of phenomena. We developed a data-driven analysis method for this problem that makes the dynamics' knowledge accumulated in the target field (hereafter referred to as domain knowledge) function effectively. In previous studies (Otake et al. 2019, 2021), the application of mode decomposition (eigen-orthogonal decomposition) to numerical analysis results (spatiotemporal data of geophysical indices) improved the computational efficiency of reliability analysis while maintaining the spatiotemporal characteristics of the soil seismic behavior. This study examines the use of dynamic mode decomposition (DMD) (e.g., Mauroy et al. 2019, Arai et al. 2021), a mode decomposition method that focuses on the time evolution characteristics of time-series data to seismic response analysis.In the DMD method, time evolution modes are functionalized by exponential functions in order to extract dynamic features from time series data. This model allows the characteristics of the damping and periodic vibration components of each mode and the stability of the system to be described mathematically.

This study developed a data-driven ground response analysis method using DMD. Based on the learning of the records of surfacelayer observations during small- and medium-scale earthquakes observed under normal conditions and the observation records of the outcrop bedrock in the vicinity, a data-driven model for ground response prediction was constructed to apply it to wide-area earthquake micro-zoning, risk assessment, immediate damage prediction, etc. This study is positioned as basic research and focuses on one-dimensional ground response analysis. By examining the relationship between data-driven models and the equations of motion for a single mass system, the possibility of developing models that integrate domain knowledge is discussed by providing physical meaning to the data-driven models. The specific development issues are listed below.

a) Handling of non-stationarityof input seismic waveform

The dynamic behavior targeted by the DMD is assumed to be stationary in its dynamics. However, input seismic waveforms acting on SBEE are inherently non-stationary in amplitude and frequency, and dealing with this non-stationarity is a major challenge. In this study, an extension of the DMD algorithm, dynamic mode decomposition with control (DMD-C) (e.g., Kutz et al. 2016), was considered to account for the non-stationarity of the ground seismic response. Furthermore, by utilizing DMD-C, we attempt to construct a time evolution prediction model that considers non-stationarity.

b） Modeling Nonlinearity in Geomaterials

The formulation of DMD assumes that the spatiotemporal information under consideration is a linear dynamical system. However, geomaterials show nonlinear seismic responses even in small strain regions. A method to account for this material nonlinearity has been proposed using Koopman operator analysis (e.g., L.Brunton et al. 2019), a theory that analyzes the properties of nonlinear dynamical systems under infinite-dimensional linear operator behavior. Recently, this methodology was developed to provide theoretical justification and generalization of DMD. Data analysis has the potential to functionalize the mathematical properties of the physics of interest and derive models that consider the causal effects of dynamic physical phenomena. However, Koopman operator theory has not yet developed a general-purpose model and has not yet been widely applied in engineering.

Based on the above discussion, in this study, five cases of soil layer configurations and 25 artificial input seismic waveforms were prepared, and seismic response analysis was performed using the one-dimensional equivalent linearized multiple reflection theory SHAKE (Schnable P.B. et al. 1972). Based on the relationship between the input seismic waveform and the calculated seismic waveform at the surfacelayer, we attempted to model the ground amplification characteristics using DMD. The possibility of extracting the physical meanings of the ground is discussed by considering the relationship between the DMD model and equations of motion. Because DMD assumes that the system under consideration is linear, as described in the next section, the nonlinearity in geomaterials appears in the DMD model as an equivalent linearization. Therefore, considering the correspondence between the convergence stiffness and convergence damping constants obtained from the SHAKE calculation and DMD model, the possibility of developing a data-driven model that integrates engineering knowledge is discussed.

2 Research Methods

2.1Relation between the equation of motion of a Single Degree of Freedom model and DMD

In this study, we constructed a data-driven model that focuses on the seismic response relationship between the SBEE and surface layer, as shown in Figure 1.

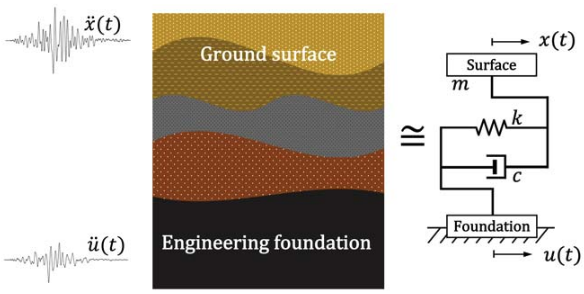

Figure1. Conversion to SDOF (Single Degree of Freedom) model

First, in the SDOF model, the equation of motion is described by the following equation:

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{u} \tag{1}$$

where x is the displacement of the surfacelayer, u is the displacement of the SBEE, m is the mass of the masses, k is the stiffness of the surface layer, and c is the damping constant of the surface layer. If the displacement of the surfacelayer at a certain time step t_k of a discrete-time system with time evolution interval Δt is x_k and the displacement of the SBEE is u_k, the following relationship holds based on the Euler method.

$$x_{k+1} = x_k + \dot{x}_k \Delta t \tag{2}$$

$$\begin{cases} \dot{x}_k = \frac{x(t_k+\Delta t)-x(t_k)}{\Delta t} = \frac{x_{k+1}-x_k}{\Delta t} \\ \ddot{x}_k = \frac{\dot{x}(t_k+\Delta t)-\dot{x}(t_k)}{\Delta t} = \frac{\dot{x}_{k+1}-\dot{x}_k}{\Delta t} \\ \ddot{u}_k = \frac{\dot{u}(t_k+\Delta t)-\dot{u}(t_k)}{\Delta t} = \frac{\dot{u}_{k+1}-\dot{u}_k}{\Delta t} \end{cases} \tag{3}$$

Using the above equation, the equation of motion can be described discretely. However, if it is applied in its original form to the data-driven model based on DMD described below, reconstruction by DMD sometimes diverged, and a general-purpose model could not be derived. Therefore, we extend the equation of motion as follows. (However, due to space limitations, we omit the process of deriving this equation.)

$$\begin{cases} x_{k+1} = x_k + \dot{x}_k \Delta t + \lambda \dot{u}_k + \mu(\dot{u}_{k+1} - \dot{u}_k) \\ \dot{x}_{k+1} + \frac{c}{m} x_{k+1} = \dot{x}_k + \frac{c - k\Delta t}{m} x_k - (\dot{u}_{k+1} - \dot{u}_k) + \xi \dot{u}_k \end{cases} \tag{4}$$

Rewriting Eq.(4) in matrix notation, we obtain the following equation:

$$\begin{bmatrix} x_{k+1} \\ \dot{x}_{k+1} \end{bmatrix} = \begin{pmatrix} 1 & \Delta t \\ -\frac{k}{m}\Delta t & 1 - \frac{c}{m}\Delta t \end{pmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} + \begin{pmatrix} \lambda & \mu \\ \xi - \frac{c}{m}\lambda & -1 - \frac{c}{m}\mu \end{pmatrix} \begin{bmatrix} \dot{u}_k \\ \dot{u}_{k+1} - \dot{u}_k \end{bmatrix} \tag{5}$$

where $\mathbf{x}_k = [x_k \quad \dot{x}_k]^T$ is the vector of the state quantities, and $\mathbf{u}_k = [\dot{u}_k \quad \dot{u}_{k+1} - \dot{u}_k]^T$ is the vector of the forcing terms. Additionally,$\lambda, \mu,$ and $\xi$are the model parameters. We also define the matrices that characterize the time evolution of Eq. (5) as follows.

$$\mathbf{A} = \begin{pmatrix} 1 & \Delta t \\ -\frac{k}{m}\Delta t & 1 - \frac{c}{m}\Delta t \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \lambda & \mu \\ \xi - \frac{c}{m}\lambda & -1 - \frac{c}{m}\mu \end{pmatrix} \tag{6}$$

Matrices **A** and **B** will be referred to as operator matrices hereafter. From the above, Eq. (5) can be expressed as follows:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k \tag{7}$$

assuming that the operator matrices **A** and **B**have time-independent constants, the seismic response at the surfacelayer can be calculated incrementally if the forcing term vector $\mathbf{u}_k$ is known. In this study, only the observed waveforms at the SBEE and the surfacelayer are assumed to be known. The operator matrix is unknown and the DMD-C algorithm, which is described in the next section, is used to identify the operator matrix from the observed waveforms.

2.2DMD-C based operator matrix identification

First, we define a data matrix $\mathbf{X}$ that contains the state quantity vectors in a chronological order.

$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_0 & \mathbf{x}_1 & \cdots & \mathbf{x}_m \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{n \times (m+1)} \tag{8}$$

where n is the number of dimensions of the state quantity (here,$n = 2$),and m is the number of time-seriesdata. Next, the data matrices $\mathbf{X}_1, \mathbf{X}_2$ are created by shifting the data matrix $\mathbf{X}$ by a unit time step (Δt).

$$\mathbf{X}_1 = \begin{bmatrix} | & | & & | \\ \mathbf{x}_0 & \mathbf{x}_1 & \cdots & \mathbf{x}_{m-1} \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{n \times m}, \quad \mathbf{X}_2 = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_m \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{n \times m} \tag{9}$$

Similarly, create a data matrix $\boldsymbol{\Upsilon}$ for the forcing term vector.

$$\boldsymbol{\Upsilon} = \begin{bmatrix} | & | & & | \\ \mathbf{u}_0 & \mathbf{u}_1 & \cdots & \mathbf{u}_{m-1} \\ | & | & & | \end{bmatrix} \in \mathbb{R}^{l \times m} \tag{10}$$

where l is the number of dimensions of the forcing term (here $l = 2$). From the above, we obtain the following equation:

$$\mathbf{X}_2 = \mathbf{A}\mathbf{X}_1 + \mathbf{B}\boldsymbol{\Gamma} = [\mathbf{A} \quad \mathbf{B}] \begin{bmatrix} \mathbf{X}_1 \\ \boldsymbol{\Upsilon} \end{bmatrix} = \mathbf{G}\boldsymbol{\Omega} \tag{11}$$

from the above equation, the operator matrix is obtained as

$$\mathbf{G} = \mathbf{X}_2 \boldsymbol{\Omega}^{-1} \tag{12}$$

However, the matrix $\boldsymbol{\Omega}$ is generally non-square because it comprises observed data, and its regularity is not guaranteed. Therefore, to obtain a matrix, such that Eq. (12) holds as an approximation, we arrive at the minimization problem of the following equation:

$$\mathbf{G} \triangleq \underset{\mathbf{A},\mathbf{B}}{\operatorname{argmin}} \|\mathbf{X}_2 - \mathbf{G}\boldsymbol{\Omega}\|_{\mathrm{F}} \tag{13}$$

where $\|\cdot\|_F$ is the Frobenius norm. Eq. (13) can be computed based on the singular value decomposition. Perform singular value decomposition on matrix $\boldsymbol{\Omega}$ to obtain the approximate matrix $\widetilde{\mathbf{G}}$of matrix $\mathbf{G}$.

$$\mathbf{\Omega} \approx \widetilde{\mathbf{U}}\widetilde{\mathbf{\Sigma}}\widetilde{\mathbf{V}}^{*} \tag{14}$$

$$\widetilde{\mathbf{G}} = \mathbf{X}_2\widetilde{\mathbf{V}}\widetilde{\mathbf{\Sigma}}^{-1}\widetilde{\mathbf{U}}^{*} \tag{15}$$

where "*" is the complex conjugate transpose. Next, to derive the approximate matrices $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$ of the operator matrices $\mathbf{A}$and$\mathbf{B}$, the left singular matrix $\widetilde{\mathbf{U}}^{*}$ in Eq. (15) can be divided into the following two elements:

$$\widetilde{\mathbf{U}}^{*} = [\widetilde{\mathbf{U}}_1^{*} \quad \widetilde{\mathbf{U}}_2^{*}] \tag{16}$$

where $\widetilde{\mathbf{U}}_1^{*} \in \mathbb{R}^{r\times n}, \widetilde{\mathbf{U}}_2^{*} \in \mathbb{R}^{r\times l}$, and r is the number of dimensions after dimension contraction. Singular value decomposition is also performed on the data matrix $\mathbf{X}_2$.

$$\mathbf{X}_2 \approx \widehat{\mathbf{U}}\widehat{\mathbf{\Sigma}}\widehat{\mathbf{V}}^{*} \tag{17}$$

Using the results of Eq. (16) and Eq. (17), we can compute the approximate matrices $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$ of the operator matrix as

$$\widetilde{\mathbf{A}} = \widehat{\mathbf{U}}^{*}\mathbf{X}_2\widetilde{\mathbf{V}}\widetilde{\mathbf{\Sigma}}^{-1}\widetilde{\mathbf{U}}_1^{*}\widehat{\mathbf{U}} \tag{18}$$

$$\widetilde{\mathbf{B}} = \widehat{\mathbf{U}}^{*}\mathbf{X}_2\widetilde{\mathbf{V}}\widetilde{\mathbf{\Sigma}}^{-1}\widetilde{\mathbf{U}}_2^{*} \tag{19}$$

Using Eq. (18) and Eq. (19), the reconstruction of the state quantities yields

$$\mathbf{X}_2 \approx \widetilde{\mathbf{A}}\mathbf{X}_1 + \widetilde{\mathbf{B}}\mathbf{\Upsilon} \tag{20}$$

In this process, the operator matrix is identified. The identified operator matrix is related to the stiffness k of the surfacesoil and the damping constant c of the surface soil and is expected to have equivalent linear parameters that depend on the observed waveform.

3 Research results

This study examined the effectiveness of DMD-C in developing data-driven ground response analysis methods by evaluating the identification accuracy of the operator matrix in nonlinear dynamical systems.

3.1Verification Method

SHAKE analysis method approximates the nonlinear behavior of the ground using a linear analysis based on equivalent linear parameters (convergence parameters). Equivalent linear parameters (convergence stiffness G_e, convergence damping ratio h_e) of the ground of interest can be obtained along with the surfacelayer response waveform (SHAKE analysis result) and the SBEE waveform (input waveform). The DMD-C operator matricesare identified based on the surfacelayer response and SBEE waveforms obtained from the SHAKE analysis. The ground parameters derived from the operator matrices identified by the DMD-C are compared with the equivalent linear parameters obtained by SHAKE.

3.2 Estimation conditions

In this study, five patterns of soil configurations, as shown in **Figure 2**, were considered for the analysis. Where ρ is the soil density and V_s is the shear wave velocity.

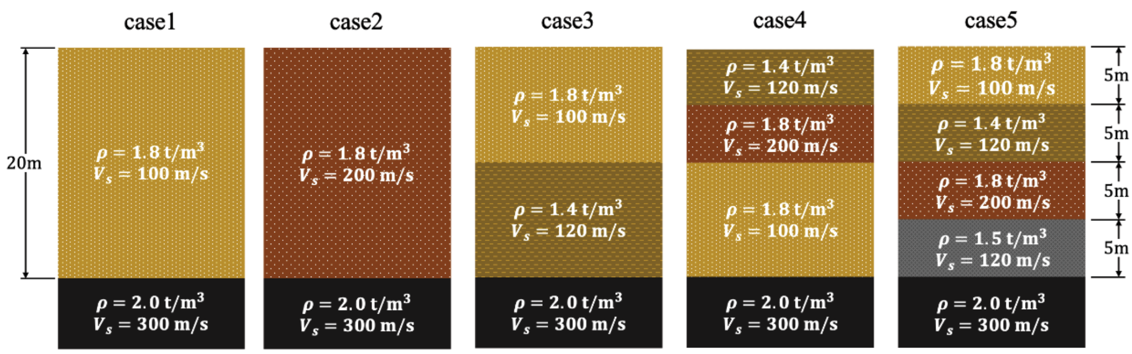

Figure2. Analysis conditions

SHAKE's analytical model divides the surfacelayer vertically into 20 sections at 1 m intervals, and the SBEE is assumed to be semi-infinite. The dynamic deformation characteristics of the ground are based on the PWRI equation (PWRI 1998), which is commonly used in Japan. The simulated earthquake ground waveforms input to the SBEE were 25 waves created based on the statistical waveform synthesis method of Itoi et al. (Itoi et al. 2014). The model of Itoi et al. is based on the statistical waveform synthesis method of Rezaeian and Kiureghian

(Rezaeian and Kiureghian 2008, 2010), which is not detailed due to space limitations, and is based on 44 earthquakes that occurred in Japan between 1997 and 2011 (only crustal earthquakes), which is based on records (about 4,000) observed in the United States and Canada. Although they are limited to crustal earthquakes, they can generate simulated seismic waveforms with various characteristics. Among the simulated waveforms generated based on this method, waveforms with maximum absolute accelerations of $0.5\sim100\text{cm/sec}^2$ were randomly selected for validation, assuming small- to medium- earthquake ground waveforms. The limitation of small- and medium-scale seismic waveforms assumes that the system will be implemented in the future. We predicted various types of earthquake ground waveforms, including large earthquakes, based on actual earthquake observation records instead of learning based on numerical analysis.

Based on the SBEE input of 25 wave inputs of seismic waveforms ($j = 1,2,\dots,25$), SHAKE analyses were performed for each of the five cases of ground conditions, and the convergence parameters of the surface layer waveform and surface soil (convergence stiffness $G_{e-shake}^{(j)}$, convergence damping ratio $h_{e-shake}^{(j)}$) were calculated. The analytical model of SHAKE is divided into 20 sections, but the average value of all sections is defined as the convergence parameter. The operator matrices $\mathbf{A}^{(j)}$ and $\mathbf{B}^{(j)}$ are identified by DMD-C from a set of 25 surfacelayer and SBEE waveforms obtained from the SHAKE analysis. From the elements of the operator matrix $\mathbf{A}^{(j)}$, the parameters of the single mass equation corresponding to the j-th waveform (stiffness $k^{(j)}$, damping constant $c^{(j)}$) can be obtained. These parameters are converted into convergence stiffness $G_e^{(j)}$ and convergence damping ratio $h_e^{(j)}$ based on the following and compared with the convergence parameters (convergence stiffness $G_{e-shake}^{(j)}$ and convergence damping ratio$h_{e-shake}^{(j)}$) from the SHAKE analysis.

$$G_e^{(j)} = \frac{k^{(j)}}{m^{(j)}}\rho H_1 H_2 \tag{21}$$

$$h_e^{(j)} = \frac{c^{(j)}}{2\sqrt{m^{(j)}k^{(j)}}} \tag{22}$$

where H_1 is the height of the surfacesoil in the SHAKE analysis model (in this case, $H_1 = 20\text{m}$) and H_2 is the apparent height when the model is replaced by the SDOF model. The center of gravity of the surface soil is assumed as follows:

$$H_2 = H_1/2 \tag{23}$$

3.3 Verification results

Figure 3shows the scatter plots of $G_e^{(j)}$ and $G_{e-shake}^{(j)}$, $h_e^{(j)}$ and $h_{e-shake}^{(j)}$. There exists a approximately 1:1 relationship between the two. Figure 4 shows an excerpt from the reconstruction of state quantities using Eq. (20) using the operator matrix identified by the DMD-C. Apparently, the ground surface layer waveform obtained by SHAKE is completely reconstructed. This implies that the time evolution matrix obtained by the DMD model reflects the effects of the equivalent linearized stiffness and damping. However, as shown in Figure 3, cases 2 and 4 have a slightly smaller gradient owing to a bias, although a proportional relationship exists. Although the details are omitted, in these two cases,we consider that the primary response mode in the groundis considered to be an unreasonable approximation tothe first-order in the SDOF model. The model does not require any information on the ground and is designed to predict the future based only on the surface layer observation records. We believe that the response mode characteristics in the ground are not being considered and the system being approximated by the first-order mode of a single mass point system leads to results different from the other cases. Although a bias from the 1:1 line exists, a clear proportional relationship between the convergence value of SHAKE and DMD result is maintained. This gradient represents the inherent response characteristics at each cross-section and indicates that it can be extended to an extrapolated estimation model.

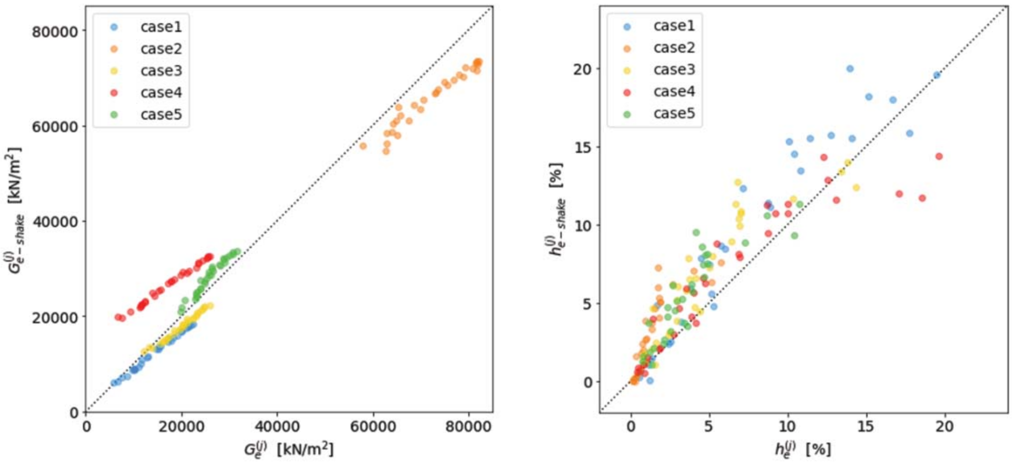

Figure3. Identification accuracy of equivalent linear parameters

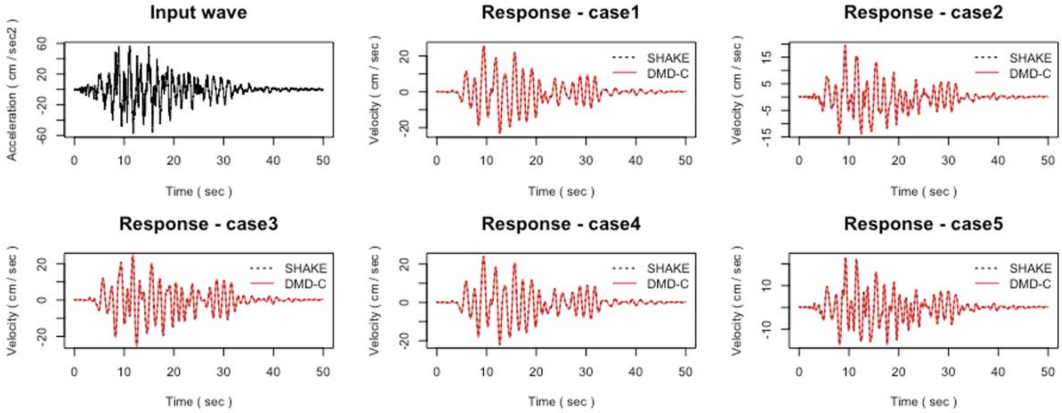

Figure4. Reconstruction of seismic response

4 Conclusion and Future Prospects

In this study, we constructed a model that considers the nonlinearity of the ground response and the nonstationarity of time evolution, which present challenge for the effective implementation of DMD. The analysis results show that the geotechnical properties obtained by applying DMD to soils with different physical properties and geologic structures are generally proportional to those of the SHAKE analysis, indicating that DMD can appropriately evaluate the geotechnical properties, and the reconstructed geotechnical response is in good agreement with the SHAKE analysis results. From the above, the following conclusions were reached regarding the issues to be addressed when applying DMD:

a) Handling theunsteadiness of the input seismic waveform: By introducing DMD-C, an extension of the DMD algorithm, we have demonstrated that it a model considering the nonstationarity of earthquake waveform can be constructed by inputting earthquake waveform as an external force term.

b)Modeling nonlinearity in geomaterials: It was confirmed that the operator matrix obtained by the DMD-C reflects the geotechnical properties. It was also shown that for the strain levels covered in this study, equivalent linearization can be used to evaluate the nonlinearity of geomaterials.

In the future, the equivalent linear parameters obtained by the DMD will be fitted to the Ramberg-Osgood model function to analyze the ground response to unlearned input earthquake ground waveforms. We also plan to construct a wide-area seismic waveform prediction model by horizontally extending the seismic waveform prediction model through simultaneous learning of densely distributed multi-point seismic waveform observation records.

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