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An Efficient Procedure for Seismic Slope Stability Analysis Considering Input Uncertainties and Soil Spatial Variability

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Abstract: This paper presents a probabilistic slope stability analysis considering soil non-homogeneity and seismic loadings. One objective is to propose an efficient procedure for such analysis at least in a preliminary design stage of practical engineering. The procedure includes the Discretization Kinematic Approach (DKA) which can evaluate the non-homogeneous slope stability, the Pseudo-Dynamic Method (PDM) which allows considering the time and space variations of seismic loadings, and an adaptive Polynomial Chaos Kriging (PCK)-based probabilistic analysis method, which permits to construct a fast-to-evaluate metamodel and can provide failure probabilities, sensitivity index and other interesting results within limited simulations. By benefiting from the high computational efficiency of the proposed procedure, a variety of hypothetical cases are analyzed to give some insights into the following three issues: (1) comparison of PDM and the traditional pseudo-static method in a probabilistic framework; (2) sensitivity analysis of seismic intensity parameters (seismic coefficients, period and amplification factor) and soil parameters (unit weight, friction angle, cohesion and non-homogeneity coefficients); (3) discussion of incorporating random fields into this procedure aiming a better soil spatial modeling. The proposed procedure is expected to be practical for seismic landslide risk analysis due to its computational efficiency and versatility. Some insights about the seismic method selection and uncertainty modeling in the probabilistic analysis are provided.

Keywords: Probabilistic seismic analysis; Discretization kinematic approach; Pseudo-dynamic approach; Non-homogeneity; Random field.

1 Introduction

Earthquake is one of major triggers for slope failures and it has received worldwide attention due to its destructive influence, which can lead to a great loss of properties and lives. Accurate estimation of slope stability against seismic effects is necessary to prevent potential engineering problems (Yang and Long, 2016). A high degree of uncertainties and randomness of soil and seismic properties always exist due to the lack of enough measured data, the complex geological conditions, the soil inherent variability and measurement errors (Cho, 2010). Probabilistic analysis is thus more suitable for the slope safety assessment since it can rationally account for the variabilities of involved parameters. Several related works were done (Johari and Khodaparast, 2015; Tsompanakis et al., 2010). However, there are still some limitations, which include:

(1) The Pseudo-Static Method (PSM) was commonly used to analyze the seismic effects on slope stability due to its simplicity (Hamrouni et al., 2018). This method may lead to biased results because it considers the seismic loading on a soil mass as a permanent force without consideration of the seismic nonlinear dynamic behavior (Hou et al., 2019). The Pseudo-Dynamic Method (PDM), which has the capacity to consider the time and space variations of a ground shaking, will be introduced in this probabilistic seismic analysis.

(2) Most of the former probabilistic seismic studies considered homogeneous slopes. It is inconsistent with the data investigated in practice, which shows that the soil exhibits non-homogeneity in its properties. There are mainly three ways to consider this characteristic in probabilistic analyses: (a) multi-layered case of Figure 1(a) (Zhang et al., 2021b), (b) depth-dependency case of Figure 1(b), which means the parameters vary along with depth while the horizontal variability is ignored (Qin et al., 2019), and (c) spatial variability case of Figure 1(c), which can describe the spatial autocorrelation of soil properties in different locations and is closer to real projects.

Xiao et al. (2016) considered the shear strength parameters spatial variation in a slope probabilistic seismic analysis and performed the finite difference method to discuss the random field effects on slope stability. However, it should be noted that using numerical models suffer from a heavy computational burden due to model construction, meshing and calculation. It is thus preferable to perform the deterministic analytical methods instead of numerical methods in a probabilistic analysis since the induced estimations are relatively accurate and the computational time is significantly reduced.

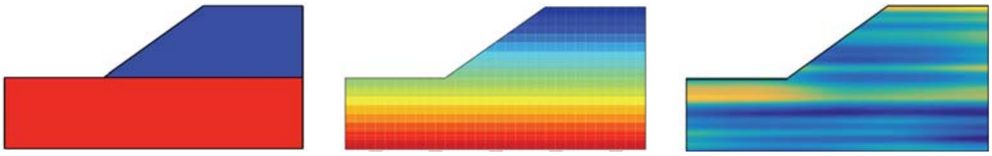

(a) Multi-layered case (b) Depth-dependency case (c) Spatial variability case

Figure 1. Three kinds of non-homogeneity consideration.

The Limit Equilibrium Method (LEM) is widely used in practical engineering due to its simple theory and quick estimation. However, this method may provide unrealistic results due to the assumptions made for the failure surfaces and inter-slice forces (Sun et al., 2018). Alternatively, the upper bound limit analysis method (namely the kinematic approach), which is based on the kinematically admissible velocity field, is also commonly adopted to analyze slope stability. This method considers the plasticity theory and can lead to more rigorous results than LEM, whereas the complex and tedious integral calculations are inevitable when soil non-homogeneity is considered. Discretization Kinematic Approach (DKA) was then proposed to solve this problem by discretizing the traditional log-spiral failure surface into a variety of segments (Du et al., 2021; Mollon et al., 2011). It is implemented in this study to effectively consider the slope soil non-homogeneity.

Moreover, the Monte-Carlo Simulation (MCS) is popular due to its robustness and conceptual simplicity, whereas it lacks computational efficiency, especially for cases with small failure probabilities. In order to overcome the inconvenience, the metamodel adaptive Polynomial Chaos Kriging (PCK) is introduced, which can further improve the calculation efficiency by constructing a fast-to-evaluate surrogate model from an original deterministic one. After the PCK metamodel construction, the MCS is employed to calculate the probabilistic results. Besides, the Global Sensitivity Analysis (GSA) is also performed to measure the considered variables effects on the model response.

This paper proposes an efficient procedure to perform probabilistic seismic stability analyses of slopes. Compared to the former studies, the main advantages of this paper include: (1) the soil spatial variability and the seismic nonlinear dynamic behavior can be considered conveniently; (2) a variety of valuable results (i.e. Failure Probability (P_f), Probability Density Function (PDF), Cumulative Distribution Function (CDF), statistical moments of the system response, sensitivity index) can be provided with an acceptable computational burden. Three discussions are then provided, which include: (1) comparison of PDM and PSM in a probabilistic framework; (2) effects of the considered parameters uncertainties on the model response; (3) accuracy and efficiency of incorporating random fields into the procedure.

2 The proposed procedure

This section gives the proposed procedure for the probabilistic seismic analyses of slopes. Figure 2 depicts the procedure flowchart. It is mainly divided into 3 parts, which include preparation, samples generation & map into the model, and probabilistic analyses. A detailed introduction of this procedure and the corresponding considered methods are as follows:

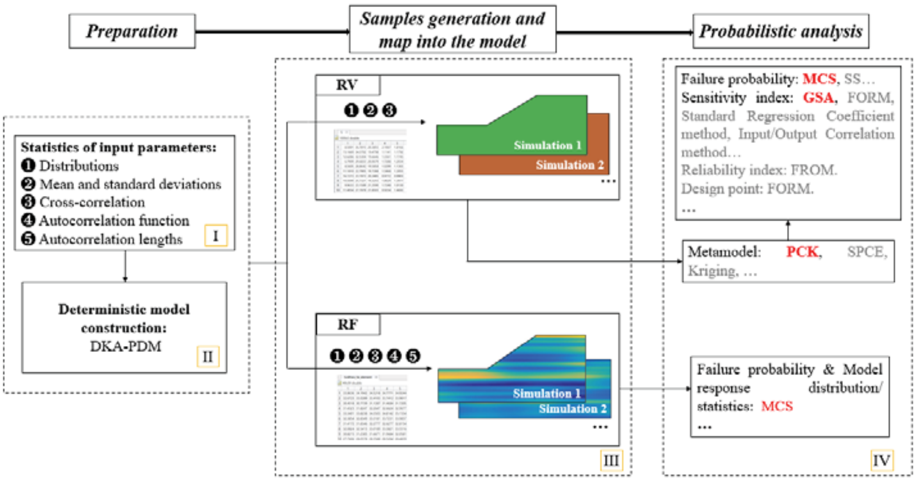
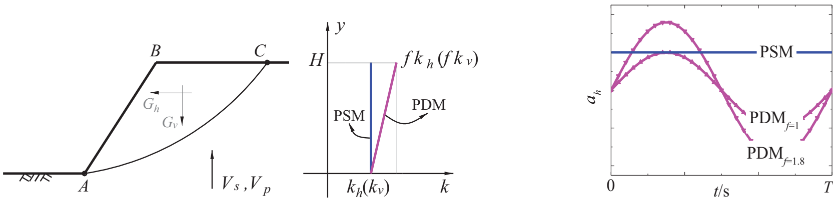


Figure 2. Flowchart of the proposed probabilistic seismic analysis procedure.

Step I: Determine the statistics of input variables. The distribution type and the related parameters (namely the mean value, standard deviation and cross-correlation coefficient) are necessary for a Random Variable case (RV). The autocorrelation function and horizontal & vertical autocorrelation lengths also should be determined if a Random Field case (RF) depicted in Figure 1(c) is considered.

Step II: Develop a deterministic seismic analysis model. The PDM is introduced to consider the space and time variations of a ground shaking as shown in Figure 3. It can be observed that compared with the constant seismic acceleration of the PSM, the PDM considers accelerations amplification, which is assumed to be varied linearly from the base to the crest surface with an amplification factor *f*. The seismic acceleration values are increased as *f* increases. Considering the time variations, a sinusoidal function is taken as the seismic wave for the sake of simplicity, and the horizontal and vertical shakings have the same period *T*.



(a) Space variation (b) Time variation

Figure 3. Space and time variations of a ground shaking using PDM.

DKA is performed to analyze the stability and Figure 4 illustrates the principle of the discretized mechanism. H and β are respectively the slope depth and slope angle. The coordinate system is established with the slope toe (point D) being the origin. The points along the slip surface are determined by a "point-to-point" technique, which means that point P_{i+1} is derived from the previous one P_i. The generation process is performed until the generated points reach the slope crest. More details can be found in Hou et al. (2019).

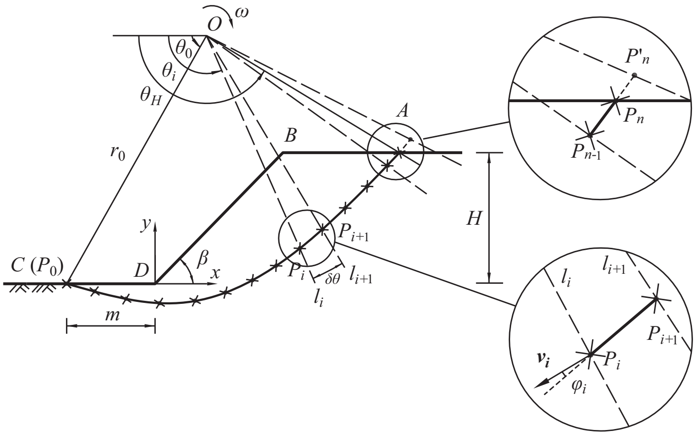


Figure 4. Principle of the discretized failure mechanism.

Step III: Generate several samples using Latin Hypercube Sampling method (LHS) and map the samples into the deterministic model to calculate the corresponding outputs (Safety Factor: F_s). Several input-output sets can then be determined. It is noted that the series expansion method: Karhunen-loève expansion is introduced to generate the random field since it requires the least random variables for a given accuracy and is independent of the finite element discretization compared with other RF discretization methods (such as point discretization and average discretization) (Guo et al., 2019).

Step IV: Perform the probabilistic analysis based on the input-output sets and the limit-state function is determined by $G(x) = F_s - 1$. For the RV case, the metamodel PCK is constructed to improve the calculation efficiency. The MCS and GSA are then respectively performed to obtain the failure probability and sensitivity index without more deterministic simulations (Zhang et al., 2021a). It is pointed out that not only the above-mentioned methods (PCK, MCS and GSA) can be involved in this procedure, other uncertainty-related analyses methods can also be performed when more probabilistic analysis results are necessary, such as the design point of the First Order Reliability Method (FORM). For the RF calculation, only MCS is performed in this study. This is because the random field case is a high-dimensional problem and it is unaffordable for other probabilistic methods to perform.

This procedure aims to provide valuable probabilistic results for the problems considering the soil non-homogeneity and seismic effects within an acceptable computational burden.

3 Problem definition

This section provides information about the reference slope (depth-dependent case of Figure 1(b)) and the corresponding parameters are summarized in Table 1.

Table 1. Summary of soil, seismic and geometric parameters of the reference case.

Parameters	Notation	Values	
		Mean	Coefficient of variation (*COV*)
Soil			
Friction angle at slope crest	φ_1 (°)	10[a]	0.3[a]
Friction angle non-homogeneity coefficient	λ_φ	1.3[b]	0.2[b]
Cohesion at slope crest	c_1 (kPa)	30[a]	0.2[a]
Cohesion non-homogeneity coefficient	λ_c	1.3[b]	0.2[b]
Unit weight	γ (kN/m^3)	20[a]	0.05[a]
Earthquake			
Horizontal seismic coefficient	k_h	0.2[c]	0.25[c]
Vertical seismic coefficient	k_v	0.1[c]	0.25[c]
Period	T (s)	0.3[c]	0.1[c]
Amplification factor	f	1.2[c]	0.15[c]
Shear wave velocity	V_s (m/s)	150[c]	-
Primary wave velocity	V_p (m/s)	280.5[c]	-
Geometry			
Slope height	H (m)	10[a]	-
Slope angle	β (°)	45[a]	-

Note:

[a] Based on the value given by Cho (2010);

[b] Zhang et al. (2021b);

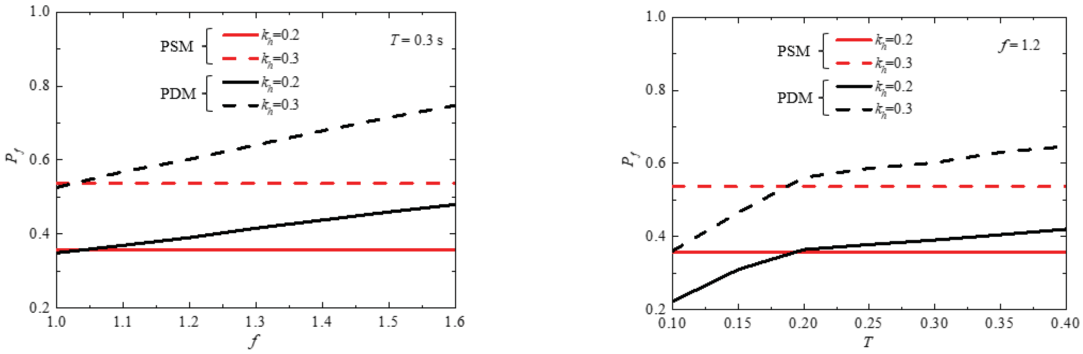
[c] Pan et al. (2019).

It should be noted that the soil is assumed to be increased linearly with depth and λ_φ and λ_c are respectively the non-homogeneity coefficients of friction angle and cohesion. They can be expressed as $\lambda_\varphi = \varphi(c)_2/\varphi(c)_1$, where φ_1 and c_1 are the strength parameters at the slope crest surface, φ_2 and c_2 are the ones at the slope toe. Besides, the lognormal distribution is adopted to model the variabilities of considered random variables.

4 Discussion

4.1 Comparison of two seismic analysis methods

The differences between PDM and PSM versus the seismic amplification factors f and periods T are investigated. 5 soil parameters are considered as random variables, whereas the seismic intensity parameters are set to be deterministic to eliminate their uncertainties effects on the probabilistic results. The results are depicted in Figure 5. It can be observed that with the increase of f and T, the failure probabilities evaluated by PDM are increased while the ones of PSM are unchanged. It highlights the merits of the PDM, which is more realistic to analyze the seismic effect since the time and space variation effects can be considered. Besides, the PSM may overestimate or underestimate the P_f as f and T change; the PDM is then recommended to be employed for the sake of safety. Moreover, the effects of f and T on the P_f values are more significant with the increase of the horizontal seismic coefficient k_h. For example, the P_f is increased by around 0.13 as f increases when $k_h = 0.2$, while its difference can be up to 0.22 for the case of $k_h = 0.3$. Therefore, the seismic parameters determination should be done with attention in practice, particularly for the cases with larger values of k_h.



(a) f (b) T

Figure 5. Difference between PDM and PSM versus f and T

4.2 Sensitivity analysis of soil and seismic intensity parameters

This section aims to investigate the soil and seismic intensity parameters effects on the model response using a Sobol-based global sensitivity analysis. 9 random variables presented in Table 1 are involved and the GSA results, which include First order and total effect indices are presented in Figure 6.

The friction angle φ_1 contributes the most to the model response variation and the corresponding non-homogeneity coefficient λ_φ follows. It reveals that the friction angle and its non-homogeneity play a significant role in the seismic slope stability analysis. The possible explanation is that the friction angle is not only involved in the failure surface generation, but also in the work rates calculation. The sensitivity indices about the cohesion (c_2 and λ_c) are smaller than the friction angle ones and the γ effect is the slightest among the soil parameters. Concerning the seismic intensity parameters, the horizontal seismic coefficient k_h is dominant for the model response variation compared with the vertical one. The effects of period and amplification factor are similar and the corresponding sensitivity indices are smaller than 0.03.

In general, this sensitivity analysis can help designers to determine the 'important' parameters (horizontal seismic coefficient, soil strength parameters and related non-homogeneity coefficients), which should be determined with attention for the design of such cases. Besides, it can be regarded as an indicator for selecting which parameters should be considered in a random field manner to better model the soil spatial variability as discussed in the next section.

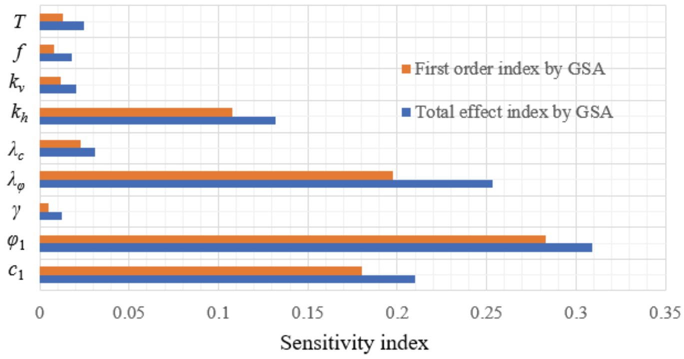


Figure 6. Sensitivity analysis results.

4.3 Random field analysis

This section investigates the feasibility of the proposed procedure for random field problems and the autocorrelation length effects on the model response. The cohesion and fiction angle are modeled by a lognormal random field while other parameters are set to the mean values. The exponential autocorrelation function is used and the horizontal and vertical autocorrelation lengths (l_x and l_y) are respectively equal to 40 m and 3 m. It is noted that 1000 sets of samples generated by LHS are considered for each case in the following discussions.

The analytical method DKA is firstly validated by comparing with a Finite Element Limit Analysis (FELA) and the results are presented in Figure 7. Nearly all scatter points are within ±5% of the 45° line, which allows to validate the effectiveness of the DKA in a random field problem. Besides, the introduction of DKA can reduce the computational time compared to FELA (around 10 s and 36 s respectively).

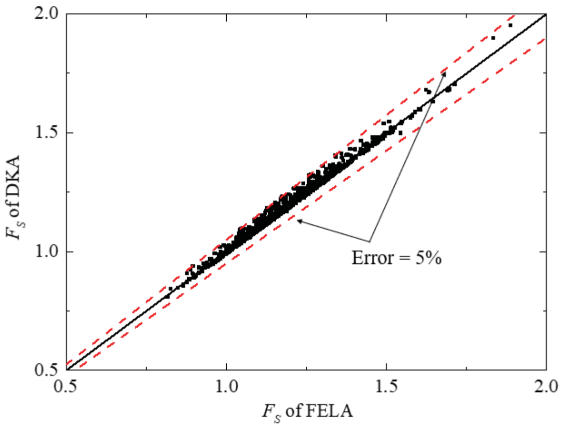
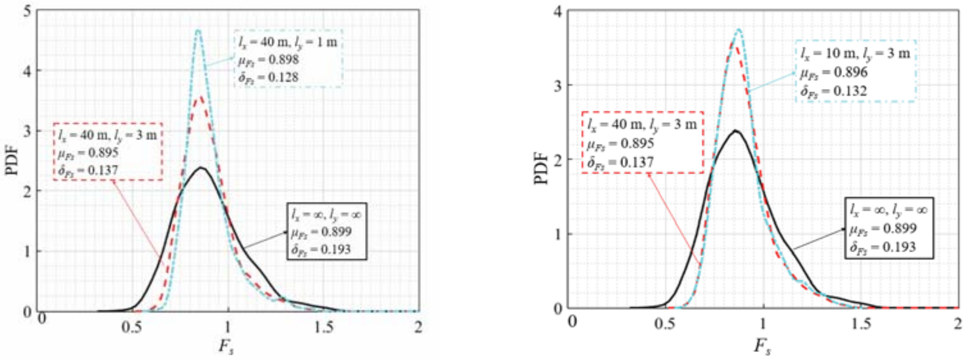


Figure 7. F_s comparison obtained by FELA and DKA models ($k_h = 0$).

The autocorrelation length discussions in terms of PDF and corresponding statistical moments of F_s are presented in Figure 8. With the decrease of the autocorrelation lengths, the F_s PDF is taller and narrower, and its standard deviation is smaller. It means that a smaller autocorrelation length can lead to safety factors with less uncertainty. This is because a small autocorrelation length means the soil shear strength parameters are correlated in a short distance, and more non-homogeneous zones can be generated compared to the larger one. The global average of the shear strength changes less for different simulations, which can result in lower variability of the safety factors. Conversely, the RV approach (i.e. $l_x = \infty$, $l_y = \infty$) gives the most scattered F_s distribution. Therefore, considering soil spatial variability can provide more precise results. Besides, the vertical autocorrelation length effect is remarkable compared to the horizontal one.



(a) l_y (b) l_x

Figure 8. PDF and statistical moments comparison considering different autocorrelation lengths.

5 Conclusion

This paper introduces an efficient procedure for probabilistic seismic analysis of slopes based on the Discretization Kinematic Approach (DKA), Pseudo-Dynamic Method (PDM) and adaptive Polynomial Chaos Kriging (PCK)-based probabilistic analysis method. It can conveniently consider the soil non-homogeneity and the time and space variations of seismic loading, and a variety of valuable results can be provided based on this procedure, which is useful for practical design and construction.

The probabilistic comparison of PDM and the traditional Pseudo-Static Method (PSM) indicates that the PSM may overestimate or underestimate the slope stability. PDM is preferable to be employed due to its rational consideration of the ground shakings. The sensitivity analysis results demonstrate the importance of the horizontal seismic coefficient, soil strength parameters and corresponding non-homogeneity coefficients, which deserve more attention in the related information collections. The feasibility and efficiency of this procedure on a random field problem are also validated and discussed.

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