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# CPT Interpolation and Driven-Pile Capacity Calculation Based on Kriging Method

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**Abstract:** Site investigation plays an important role in the design of geotechnical structure. However, due to the limitation of funds, the site data is not comprehensive enough to describe the distribution characteristics of soil parameters, which is the major sources of geotechnical uncertainty. This paper proposes a three-dimensional (3D) Kriging method to interpolate the cone penetration test (CPT) data at untested points. 5 theoretical autocorrelation models are compared for the best estimation of the vertical and horizontal scale of fluctuation. A pile design problem introduced in the TC304/TC309 is considered. The applied results illustrate that the well-designed Kriging method is suitable for 3D CPT interpolation and pile design with the spatially variable soil.

Keywords: CPT; Kriging method; Pile capacity; 3D interpolation.

## 1 Introduction

The soil properties are often spatially variable and due to the environmental and financial constraints, a lack of site investigation is very common in geotechnical engineering, leading to a great design uncertainty (Fenton and Griffiths (2008), Li et al. (2016a)). Therefore, a simple and reliable soil parameters interpolation method based on sparse site investigation is necessary for general engineering purposes.

Kriging method, proposed by Krige (1951), is a best, linear unbiased estimate of a random field between known data through a weighted linear combination of the values at each observation point. In order to obtain the best 3D soil random field description, five commonly used spatial autocorrelation theoretical models are considered in Kriging method. Besides, a linear trend of soil parameters with depth is assumed in this paper for better spatial variable description (Vanmarcke, (1977)). A driven-pile capacity design problem based on CPT data introduced in the TC304/TC309 in the 6th National Symposium on Engineering Risk & Insurance Research (NSERIR6) is considered as an example to illustrate the proposed interpolation method. The data are extracted from the A-CPT/232/2500m$^2$ dataset in the 304dB, and the locations of piles and available six CPTs are shown in Figure 1. This site is a section of the South Parklands lying at the southern extremity of the central business district of the city of Adelaide, South Australia (Jaksa (1995)), and the ground conditions comprise a crust of calcareous clay and underlain by Keswick clay.

This paper is organized as follows: First, the basic idea of Kriging method and 5 autocorrelation functions are introduced. Then, the CPT processing and method parameters calibration are illustrated. Finally, the target CPT data is interpolated and the bearing capacity is calculated.

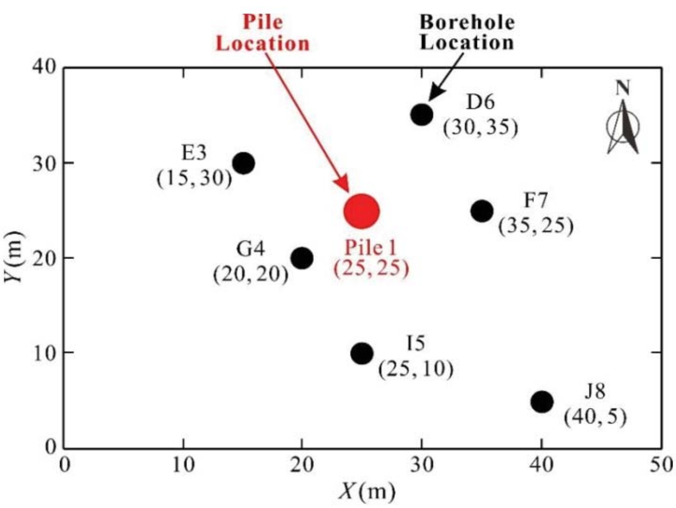


**Figure 1.** Locations of the designed pile and CPTs

## 2 Methodology

### 2.1 Ordinary Kriging

Ordinary kriging is a best, linear unbiased estimation used to estimate soil properties at unsampled locations, and it can take the correlation among existing datasets into account. The results calculated using Kriging is a linear combination of the observations:

$$\hat{R} = \sum_{k=1}^{n} \beta_k R_k \tag{1}$$

where $\hat{R}$ is the unobserved value, and $R_k$ is the measured value. The coefficients $\beta_k$ are determined through the covariance between the measurements and target point. The covariance matrix proposed by Firouzianbandpey (2015) is adopted in Kriging to avoid the calculation error of semi-variogram matrix parameters:

$$K\beta = M \tag{2}$$

$$K = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} & g_1(x_1) & g_2(x_1) & \cdots & g_m(x_1) \\ C_{11} & C_{11} & \cdots & C_{11} & g_1(x_2) & g_2(x_2) & \cdots & g_m(x_2) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ C_{11} & C_{11} & \cdots & C_{11} & g_1(x_n) & g_2(x_n) & \cdots & g_m(x_n) \\ g_1(x_1) & g_1(x_2) & \cdots & g_1(x_n) & 0 & 0 & \cdots & 0 \\ g_2(x_1) & g_2(x_2) & \cdots & g_2(x_2) & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ g_m(x_1) & g_m(x_2) & \cdots & g_m(x_m) & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{3}$$

$$\mu(X) = \sum_{i=1}^{m} a_i g_i(x) \tag{4}$$

$$M^T = \{C_{1x}\, C_{2x} \quad \cdots \quad C_{nx} \quad g_1(x) \quad g_2(x) \quad \cdots \quad g_n(x)\} \tag{5}$$

$$\beta^T = \{\beta_1 \quad \beta_2 \quad \cdots \quad \beta_n \quad -\eta_1 \quad -\eta_2 \quad \cdots \quad -\eta_m\} \tag{6}$$

where $K$ is a matrix with elements explained in Eq. (3), it could be inverted and used repeatedly at different spatial points to build up the best estimate of the random field. $C_{ij}$ is the covariance matrix among observation point; $g_i(x_i)$ is a function with *i-1* power, which is used to simulate the mean value in a regression analysis. $M$ is a vector compositing of covariance between observation point and intermediate spatial point x. The elements $\eta_i$ are Lagrangian parameters that used to make sure the results unbiased. Owning to a finite number of measurements, the errors associated with Kriging interpolation should be emphasized. According to Firouzianbandpey(2015), the variance of estimated values can be expressed as

$$\sigma_E^2 = E[R(x) - \hat{R}(x)]^2 = \hat{\sigma}_R^2 + \beta_n^T (K_{n\times n}\beta_n - 2M_n) \tag{7}$$

where $\beta_n$ and $M_n$ are the first *n* elements and $K_{n\times n}$ is the upper $n \times n$ submatrix, in this study, *n* is the number of measured CPT points.

### 2.2 Spatial correlation structure

The well-estimated vertical and horizontal scales of fluctuation $\theta$ and theoretical autocorrelation function is necessary to describe random field of CPT data and Kriging function. According to general practice (Vanmarcke, 1977,1983; Fenton, 1999a; Lloret-Cabot et al., 2014), we assume that the soil is spatially statistically homogeneous, i.e., the mean value, covariance, correlation structure and high-order moments are independent of the position in the soil body. Moreover, we assume that soil is isotropic in horizontal direction (x, y direction in Figure 1.), so we can use a 2D coupled autocorrelation function to describe 3D random field.

The spatial variability is often separated into a deterministic trend which can be explained on a physical basis and a residual variable around the trend, which is expressed as:

$$Q(x,y,z) = \bar{Q}(x,y,z) + R(x,y,z) \tag{8}$$

where $Q(x,y,z)$ is the value at location (x, y, z), and $\bar{Q}(x,y,z)$ is the trend calculated by ordinary least squares (OLS) and $R(x,y,z)$ is a mean-zero residual component which contains correlation information between different locations.

For 1D random field, the covariance $C(\tau)$ between $X(z)$ and $X(z+\tau)$ is estimated by a biased estimator as:

$$\hat{C}(\tau_j) = \frac{1}{n}\sum_{i=1}^{n-j+1}(x_i - \hat{\mu}_x)(x_{i+j-1} - \hat{\mu}_x), j = 1,2,...,n \tag{9}$$

where the lag is $\tau_j = (j-1)\Delta z$, and the correlation between $X(z)$ and $X(z+\tau)$ is normalized as:

$$\hat{\rho}(\tau_j) = \frac{\hat{C}(\tau_j)}{\hat{C}(0)} \tag{10}$$

For 3D random field, the correlation between $X(x_i, y_j, z_m)$ and $X(x_i+\tau_i, y_j+\tau_j, z_m+\tau_m)$ can be decomposed based on the isotropic in horizontal direction:

$$\hat{\rho}(\tau_{xi}, \tau_{yj}, \tau_{zm}) = \hat{\rho}(\tau_{xi}, \tau_{yj}) \times \hat{\rho}(\tau_{zm}) \tag{11}$$

When we calculate the correlation from one certain CPT drilling, the lag of x, y direction $\tau_{xi} = \tau_{yj} = 0$, and $\hat{\rho}(\tau_{xi}, \tau_{yj}) = 1$, 3D correlation degenerates into one dimension, which can be calculated by Eq. (10) similarly. When the data at the same depth is considered to calculate the correlation coefficient, the lag of z direction $\tau_{zm} = 0$ and $\hat{\rho}(\tau_{zm}) = 1$, the lag in horizontal direction should be reformed as:

$$\tau_{ij} = \sqrt{\Delta x_i^2 + \Delta y_j^2} \tag{12}$$

Therefore, the 3D correlation coefficient can be transformed into the product of two 1D correlation coefficients by solving the covariance in Z and horizontal direction separately.
Various methods are available to estimate the scale of fluctuation. The simplest approach is probably to estimate θ based on the best fitting the theoretical autocorrelation model to the estimated correlation function (Vanmarcke 1977; Uzielli, Vannucchi, and Phoon 2005; Fenton 1999). In this study, 5 classical theoretical functions are considered to fit the calculated correlation (Uzielli, Vannucchi, and Phoon 2005,and YueQ 2018), including (a) single exponential (SNX); (b) cosine exponential (CSX); (c) second-order Markov (SMK); (d) squared exponential (SQX); (e) liner exponential cosine (LNCS). The analytical expressions of the five functions and the formulae relating the scales of fluctuation to the model parameters are shown in Table 1.

**Table 1.** Autocorrelation models and relations between scale of fluctuation and characteristic model parameters

| Autocorrelation Model | Equation | Scale of fluctuation |
|---|---|---|
| SNX | $\rho(\tau) = \exp(-k_{SNX}\lvert\tau\rvert)$ | $\theta = 2 / k_{SNX}$ |
| CSX | $\rho(\tau) = \exp(-k_{CSX}\lvert\tau\rvert)\cos(k_{CSX}\tau)$ | $\theta = 1 / k_{CSX}$ |
| SMK | $\rho(\tau) = (1 + k_{SMK}\lvert\tau\rvert)\exp(-k_{SMK}\lvert\tau\rvert)$ | $\theta = 4 / k_{SMK}$ |
| SQX | $\rho(\tau) = \exp\left[-(k_{SQX}\tau)^2\right]$ | $\theta = \sqrt{\pi} / k_{SQX}$ |
| LNCS | $\rho(\tau) = (1 + k_{LNCS}\lvert\tau\rvert)\exp(-k_{LNCS}\lvert\tau\rvert)\cos(k_{LNCS}\lvert\tau\rvert)$ | $\theta = 1 / k_{LNCS}$ |

## 3 Data process and interpolation analysis

### 3.1 CPT normalization

Because of the significant influence of the effective overburden stress on CPT measurements (Moss et al. (2006)), various methods have been proposed for normalizing CPT data to account for this effect. In this study, the dimensionless, normalized cone penetration resistance (qc1N) proposed by Robertson & Wride (1998) is adopted.

$$q_{c1N} = \left(\frac{q_c}{P_{a2}}\right) C_Q \tag{13}$$

where $q_c$ is the measured cone tip penetration resistance and $C_Q = \left(\frac{P_a}{\sigma'_{v0}}\right)^n$ is a coefficient of correction for overburden stress; the variable stress exponent n takes vales of 0.50, 1.00 and 0.70 for cohesionless, cohesive and intermediate soils respectively; $\sigma'_{v0}$ is the effective vertical stress. According to Jaksa (1995), no groundwater level encountered in site investigation, $\sigma_{v0} = \gamma z$ and $\gamma = 18kN/m^3$ ; $P_a$ and $P_{a2}$ are reference pressure in the same units as $\sigma'_{v0}$ and $q_c$ . An upper bound of $C_Q = 1.7$ is recommend for data at shallow depths according to Youd et al., (2001). The normalized friction ratio is given by Wroth, (1984):

$$F_R = \frac{f_s}{q_c - \sigma_{v0}} \times 100 \tag{14}$$

### 3.2 Interpolation results

The ordinary least squares method is used to model a linear trend function for the normalized data, and the corresponding trend functions for normalized CPT are:

$$\bar{Q}_{\ln q_{c1N}} = 0.0040x - 0.0074y - 0.0807z + 3.8510 \tag{15}$$

$$\bar{Q}_{\ln(F_R)} = -0.0065x - 0.0053y + 0.0746z + 1.9074 \tag{16}$$

where x, y and z are the coordinates in the east, north, and depth directions, respectively and the residuals are shown in Figure 2.

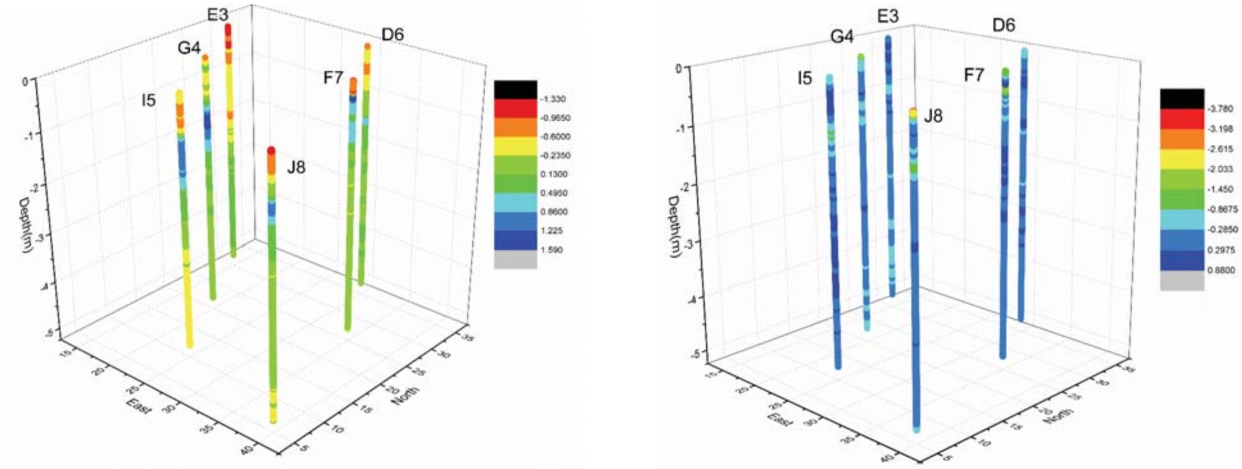


**Figure 2.** Residual of normalized CPT results, (a) $\ln(q_{c1N})$, (b) $\ln(F_R)$

The residuals are used to characterize the autocorrelation function and autocorrelation coefficient values at various separation distances are calculated according to Eq. (9), (10). Table 2 summarizes the coefficient $R^2$ used to describe the goodness of fitting and the scale of fluctuation calculated by each theoretical equation. A single exponential equation is found a good model for $\ln(F_R)$ in both vertical and horizontal direction but not for vertical in $q_{c1N}$ .

**Table 2.** Coefficient of determination $R^2$ and the scale of fluctuation for residual of normalized tip resistance and side friction in vertical and horizonal direction

| Autocorrelation Model | Residual of normalized tip resistance | | | | Residual of normalized side friction | | | |
|---|---|---|---|---|---|---|---|---|
| | Vertical | | Horizontal | | Vertical | | Horizontal | |
| | $R^2$ | θ | $R^2$ | θ | $R^2$ | θ | $R^2$ | θ |
| SNX | 0.86 | 0.818 | 0.95 | 15.1 | 0.88 | 0.432 | 0.97 | 11.99 |
| CSX | 0.92 | 0.707 | 0.88 | 2.25 | 0.72 | 0.292 | 0.94 | 2.15 |
| SMK | 0.89 | 0.840 | 0.93 | 15.87 | 0.73 | 0.374 | 0.95 | 13.02 |
| SQX | 0.89 | 0.842 | 0.90 | 15.3 | 0.74 | 0.345 | 0.95 | 13.09 |
| LNCS | 0.75 | 0.50 | 0.88 | 2.34 | 0.59 | 0.16 | 0.94 | 2.22 |

In this study, the empirical approach proposed by Schmertmann (1978) is adopted for driven pile design:

$$Q_u = q_p A_p + \sum_{i=1}^{n} f_{pi} A_{si}, \; q_p = (q_{c1} + q_{c2})/2 < 15Mpa, \; f_p = k_c f_s \le 120kPa \tag{17}$$

where $Q_u$ is the ultimate axial pile capacity, $q_p$ is the unit end bearing, $A_p$ is the pile end area ($0.126m^2$), $A_{si}$ is the surface area along the pile shaft ($6.28\times10^{-3}m^2$), and $f_p$ is the unit shaft friction, $q_{c1}$ is the average $q_c$ over a distance below the pile tip range 0.7d to 4d, $q_{c2}$ is the average $q_c$ over a distance of 8d over the pile tip under the rule of minimum path suggested by Schmertmann (1978) , and $k_c$ is a coefficient of correction given through curve. According to Jaksa (1995), the data used to calculate the capacity of pile located in (25,25) is not only the CPT data in F5, but the average $q_c$ from CPTs data near it. Figure 3 shows the result with error range and the average data calculated by real data. It can be seen that the range of error is wide, the difference range from 2m to 3m between real value and interpolated data in both qc and fs is significant, that may be caused by the violent fluctuation of soil parameters in the horizontal direction of the original site. The data beyond 5m interpolated by Kriging has no fluctuation but a trend, it seems that the horizontal correlation between original data and interpolated data is more important, so when CPTs at the same depth are missing, it is difficult to obtain accurate interpolation information.

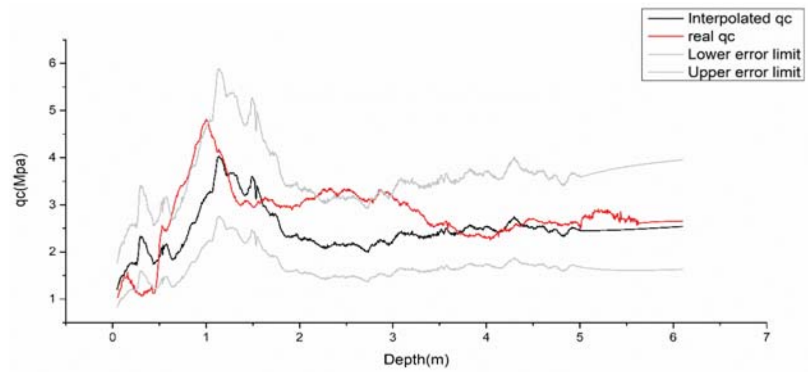


**Figure 3.** (a) the interpolated qc with error range and the real data

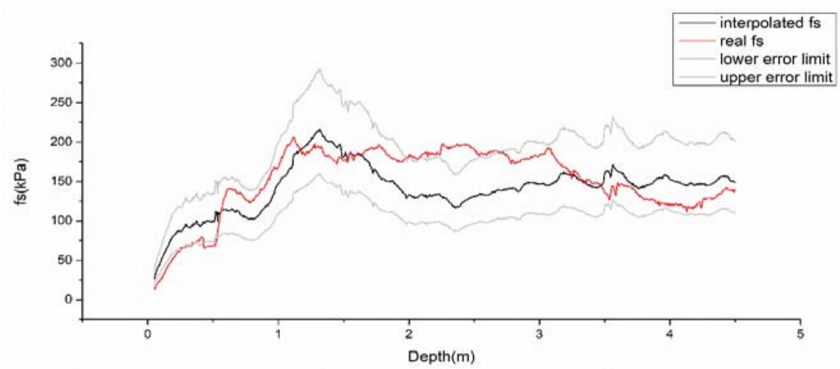


**Figure 3.** (b) the interpolated fs with error range and the real data

Though the interpolated data is not exactly the same as real data, the empirical approach used to calculate capacity of pile can reduce the error through the rule of minimum path. Table 3 shows the results calculated by Eq. (17) with relative error. It can be seen that the relative error of the capacity of pile is just 5.34%, which is reduced significantly by the rule of minimum path and coefficient of correction of sleeve friction. The real data at the depth of two to three meters does not satisfy the rule of minimum path, so it has no contribution to the bearing capacity of the pile.

**Table 3** Comparison of pile bearing capacity and relative errors

| Calculation items | Result calculated by interpolated data | Result calculated by real data | Relative error (%) |
|---|---|---|---|
| $q_{c1}(Mpa)$ | 2.473 | 2.652 | 6.75% |
| $q_{c2}(Mpa)$ | 2.195 | 2.304 | 4.73% |
| $\sum_{i=1}^{n} f_{pi} A_i (kN)$ | 341.86 | 359.62 | 4.94% |
| $Q_u(kN)$ | 635.16 | 671.01 | 5.34% |

## 4 Conclusion

This paper adopts ordinary Kriging model for 3D CPT data interpolation at unsampled locations, providing a simple and reliable method for geotechnical engineering. The applied results demonstrate that Kriging method with a well-estimated autocorrelation function is suitable for spatially variable soil parameters interpolation in the condition of limited measurements. Though the interpolated result is not exactly the same as in situ one, the rule of minimum path adopted by the empirical method can significantly reduce the relative error of bearing capacity of the driven pile.


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