

Probabilistic Approach for Evaluating Relationship between Rock Drilling Energy and P-Wave Velocity

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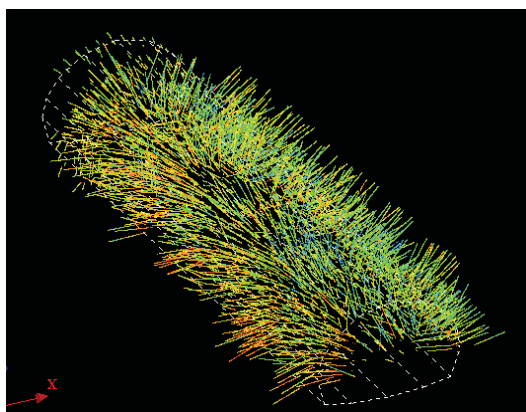
Abstract: The computer drilling jumbo in rock tunnelling can collect a large amount of drilling data on blasting and rock bolting holes in almost real-time. Subsequently, the three-dimensional spatial distribution of the rock drilling energy data can be easily and immediately obtained during tunnel excavation. However, in order to utilize these data for tunnel support design, it is necessary to convert those properties into physical rock properties such as deformation modulus, compressive strength, and P-wave velocity. In this study, we proposed a statistical transformation model to obtain the three-dimensional spatial distribution of the P-wave velocity of rock mass from the rock drilling energy data. The proposed model is used to determine rock properties in a finite difference simulation simulation of actual tunnel excavation, and the applicability of the transformation model is demonstrated by comparing numerical simulation results with observation data.

Keywords: Rock property; uncertainty; tunneling; drilling energy coefficient; bivariate Gaussian probability distribution.

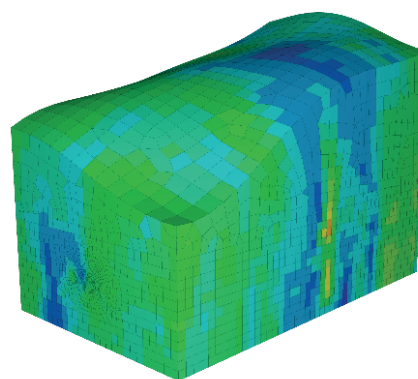
1 Introduction

In tunnel construction, with the spread of computer jumbos, the drilling energy coefficient E_v , which is the fracture energy required for drilling, is automatically obtained together with the geo-coordinates and is being used to evaluate the geological properties surrounding a tunnel. Figure 1 (a) shows a spatial distribution of actual E_v data collected by a computer jumbo, and Figure 1 (b) shows an E_v distribution interpolated using kriging for FE tunnel excavation simulation. Through these visualizations, engineers can intuitively understand the natural heterogeneity of actual rock mass, and this information is useful for more realistic tunnel simulations. This is a good example of the advancement in ICT technology in the rock engineering field.

Koizumi et al. (2020) proposed a practical table to determine some mechanical properties of rock mass from only E_v data for finite element simulations of tunnel excavation (Table 1). In the table, the relationships between parameters are not based on actual observation data but on the experiences of tunnel engineers. Therefore, a data-centric transformation model should be proposed for a more consistent tunnel design.



(a) Drilling data on rock bolting holes



(b) Spatial distribution of interpolated E_v data

Figure 1. Distribution of drilling energy coefficient E_v .

Table 1. Relations between E_v and some mechanical parameters of intact rocks.

Class	E_v (J/cm ³)	V_p (km/s)	E (GPa)	c (kPa)	ϕ (degree)
1	0 ~ 100	2.11	0.150	200	30.0
2	100 ~ 125	3.02	0.357	378	33.0
3	125 ~ 150	3.30	0.468	473	34.5
4	150 ~ 175	3.55	0.638	638	36.4
5	175 ~ 200	3.78	0.822	822	38.2
6	200 ~ 225	3.99	0.992	992	39.9
7	225 ~ 250	4.19	1.304	1,304	41.5
8	250 ~ 300	4.47	1.750	1,750	43.7
9	300 ~ 350	4.81	2.294	2,294	46.5
10	350 ~	5.12	2.793	2,793	49.0

* E : Elastic modulus, c : cohesion, ϕ : internal friction angle

To build a data-centric transformation model, a pairwise dataset of E_v and another rock property are necessary, and we have collected the pairwise datasets of E_v and the corresponding P-wave velocity data V_p from several actual sites. In order to consider the variability of rock properties, we proposed a probabilistic transformation model based on a bivariate Gaussian probability distribution.

2 Probabilistic transformation model between E_v and V_p

This section outlines the proposed probabilistic transformation model between E_v and V_p , and the details of data used for the modelling and how to build the model can be found in Shuku et al. (2022a).

The transformation model is based on a bivariate standard normal distribution defined as:

$$\mathcal{N}(\underline{\mathbf{x}} | \mathbf{C}) = \frac{1}{\sqrt{(2\pi)^M |\mathbf{C}|}} \exp\left\{-\frac{1}{2} \underline{\mathbf{x}}^T \mathbf{C}^{-1} \underline{\mathbf{x}}\right\} \quad (1)$$

where $\underline{\mathbf{x}}$: standard normal variable vector, \mathbf{C} : correlation matrix, M : the number of parameters (= 2 in this study), T : matrix transpose, $^{-1}$: matrix inverse. The original data of E_v and V_p do not follow a normal distribution, and the Johnson system (Johnson 1949) was used to transform E_v and V_p data into the data which follow standard normal distributions. The usefulness of the Johnson system for geological data analysis has been extensively studied by Ching and Phoon (2013, 2014, 2015, 2016, 2017, 2019), and the data analysis method in this study relies on those pioneering works.

The transformation model is shown in Figure 2. In the figure, the red continuous line indicates the mean value, the dashed lines indicate a 95% confidence interval and the cross marks indicate data. It is clear that the proposed model can reasonably capture the nonlinear relationship between E_v and V_p .

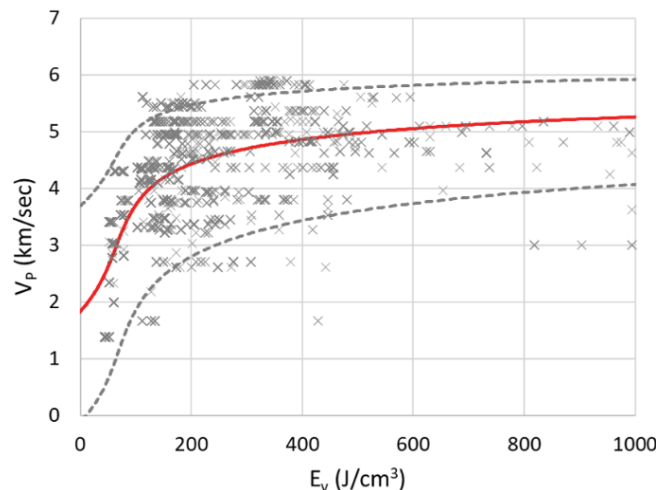


Figure 2. Transformation model between E_v and V_p (Shuku et al. 2022a)

3 Application of probabilistic transformation model to actual tunnel excavation

While the coefficient of fracture energy E_v is not used in estimating other rock properties, V_p is commonly used to estimate uniaxial compressive strength σ_c and Young's modulus E . This implies that σ_c and E can be indirectly estimated from E_v data via two transformation models. This section uses two probabilistic transformation models, one is shown in Figure 2 and another is found in Shuku et al. (2022b), to estimate rock properties used for a simulation of tunnel excavation from E_v data. Shuku et al. (2022b) compiled a Japanese multivariate database of nine rock properties, unit weight, porosity, uniaxial compressive strength, Brazilian tensile strength, Young's modulus, Poisson's ratio, cohesion, and internal friction angle, from 49 studies and constructed a probability density function (PDF) model of nine rock parameters based on the following multivariate standard normal distribution.

$$\mathcal{N}(\mathbf{z} | \mathbf{C}) = \frac{1}{\sqrt{(2\pi)^M |\mathbf{C}|}} \exp\left(-\frac{1}{2} \mathbf{z}^T \mathbf{C}^{-1} \mathbf{z}\right) \quad (2)$$

where \mathbf{z} : standard normal variable vector, M : the number of parameters, T : matrix transpose, $^{-1}$: matrix inverse; \mathbf{C} : a correlation matrix. The estimated rock properties that correspond to $\pm 1\sigma$ (Table 2) were used for a tunnel simulation, and the estimated tunnel displacements were compared to actual observation data for demonstration.

3.1 Numerical setup

A commercially available finite difference code, FLAC3D (Itasca Consulting Group, inc. 2019), was used for the simulation of tunnel excavation. The finite difference mesh for 3D analysis is shown in Figure 3. The tunnel diameter and length are 14 and 240 m respectively. The center of the tunnel in the longitudinal direction was set as the observation point and the results of the numerical analysis were compared with the actual measurement data during the tunnel excavation. The rock cover depth at the observation point was about 87 m.

The geotechnical properties used in the tunneling analysis are shown in Table 2. First, E_v (J/cm^3) was transformed into V_p (km/sec) value using Eq. (1), and density, elastic modulus, cohesion, internal friction angle were estimated using Eq. (2). By using two transformation models, the input parameters for finite difference simulation can be estimated using only E_v data. Traditionally, geotechnical properties have been assigned using a linear and deterministic approach. However, the numerical analysis in this study considers using a nonlinear and probabilistic approach (i.e., using rock properties that correspond to the mean relationship and $\pm 1\sigma$ interval data).

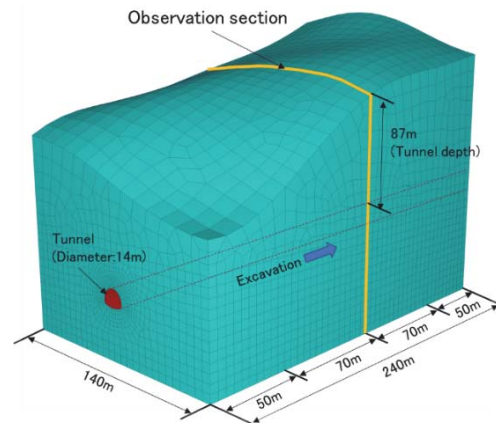


Figure 3. FLAC3D mesh

Table 2. Input parameters estimated by two transformation models

	-1σ	Mean	$+1\sigma$
density [$\times 10^{-3}$ kg/m ³]	0.1531 E_v + 1.2627	0.1492 E_v + 1.6541	0.123 E_v + 2.0538
modulus of elasticity [MPa]	- 0.0007 E_v^2 + 1.0214 E_v - 5.4408	- 0.013 E_v^2 + 17.539 E_v - 102.54	- 0.1753 E_v^2 + 197.24 E_v - 2502.5
Poisson's ratio [-]	- 0.026 E_v + 0.4827	- 0.019 E_v + 0.346	- 0.017 E_v + 0.2503
Cohesion [MPa]	- 3e-6 E_v^2 + 0.0038 E_v + 0.2704	- 3e-5 E_v^2 + 0.0328 E_v + 2.0712	8.6919 E_v + 7.3324
friction angle [°]	2.4526 E_v + 8.5722	2.7722 E_v + 25.032	2.1586 E_v + 44.589
tensile strength [MPa]	- 6E-06 E_v^2 + 0.0066 E_v + 1.0248	- 1E-05 E_v^2 + 0.0148 E_v + 3.1037	- 2E-05 E_v^2 + 0.0251 E_v + 7.6644

3.2 Numerical analysis result

Figure 4 shows the simulated results of crown settlement, tunnel leg settlement and convergence data (horizontal displacement inside the tunnel) with the mean and $\pm 1\sigma$ confidence interval. The measured data are also illustrated in the figure for comparison. Although the simulation result with the mean values of the parameters, which is shown as “predicted” in the figure, cannot predict measured time-displacement curves. However, all the confidence intervals capture the actual measurement data well. The benefit to use probabilistic model is we can discuss best or worst scenarios through parameter study based on the probabilistic models. Deterministic simulations generally used in practice cannot provide the information of uncertainty of simulation results quantitatively. The confidence intervals given by the proposed probabilistic model seem to be too wide.

By collecting additional information, the confidence intervals can be smaller via Bayes’ rule. For example, the measured settlement data can be used for updating the proposed probabilistic model using Bayesian filtering such as Kalman filter and Particle filter. The methodology that assimilates observation data into numerical models is called “data assimilation”, and it will enable a more accurate assessment of the ground behavior during actual tunnel excavation and aid to determine the appropriate tunnel support pattern and excavation procedure.

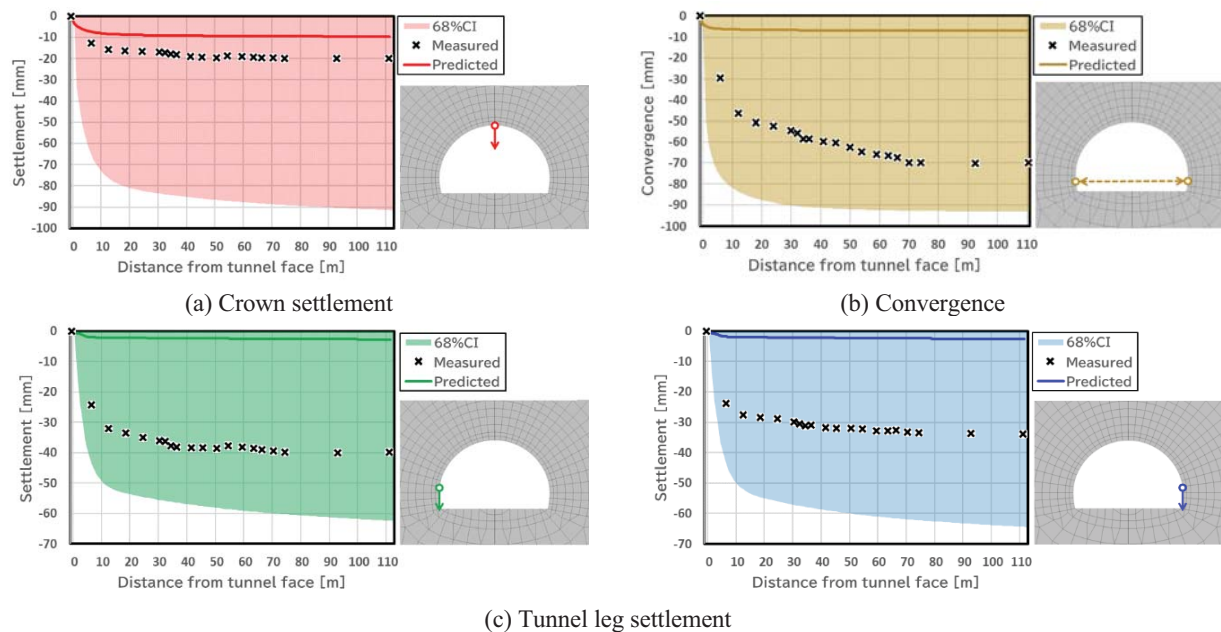


Figure 4. Results of numerical simulation and actual measurement data

4 Conclusions

In this study, we proposed a probabilistic transformation model between E_v and V_p and performed a probabilistic numerical simulation of tunnel excavation to demonstrate the proposed model. Although the existing deterministic approach cannot provide the information of uncertainty of simulation result, the simulations based on the proposed probabilistic transformation model can evaluate the confidence intervals of the numerical simulation. The proposed approach can be useful in decision making in tunnel construction problems which usually include large uncertainty.

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