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Response Surface Based-Robust Design of Supported Excavation Considering Multiple Failure Modes

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Abstract: The robust geotechnical design (RGD) is widely used in retaining wall, foundation and deep excavation because it can balance construction cost and structural safety well. For deep braced excavation, it was well recognized as a complex system design in geotechnical community because its multiple failure modes and high uncertainty. In this study, responses surface based-robust design method (RSM-RGD) was applied in the design of supported excavation. Different from traditional robust design, the responses surfaces as an alternative model of finite element model was used to establish failure functions. Both one kind of ultimate limit failure mode and two kinds of serviceability limit failure modes of retaining system were taken into account. Via proposed method, a multi-objective optimization framework was modeled to find the optimal design with lower construction cost and higher robustness and safety. Finally, the whole steps of proposed method were illustrated by a case study. The result indicated that the multi-objective optimization model and genetic algorithm has an efficient effort on solving Pareto frontiers.

Keywords: responses surface method; system reliability; robust geotechnical design; multi-objective optimization.

1 Introduction

High uncertainty is an inherent behavior of soil and rock physical and mechanical parameters, which is well recognized by the geotechnical engineering community(Phoon and Kulhawy 1999). Robust geotechnical design (RGD) method was also an efficient method to carry optimization design(Juang and Wang 2013). RGD method aimed to choose the optimal design with reasonable economic cost and high safety. Nowadays, there are some studies that develops RGD method in geotechnical design, such as rectangular spread foundations (Juang et al. 2014), shield-driven tunnels (Gong et al. 2014), and braced excavations (Khoshnevisan et al. 2015).

However, for RGD method, limit state functions and correlation between failure modes were key points(Xiao et al. 2020). For reliability analysis with multiple failure modes and random variables, limit states functions were hard to determine directly. Zhong et al. (2020) presented system reliability model of shallow foundations based on bearing capacity, bending and shearing limit states functions. Zhang et al. (2019) and Fu et al. (2019) adopted the limit states functions from the manual of American Society for Mechanical Engineers and analyzed system reliability of pipelines via multiple normal distributions.Lü et al. (2017) adopted moving least square method to determine limit state functions of tunnel allowable settlement, allowable stress and allowable horizontal deformation. Khoshnevisan et al. (2017) firstly analyzed the influence of uncertainty of soil parameters on safety factors of excavation, then established the surrogate model between uncertainty vectors and safety factors via response surface model.

Moreover, multiple objective optimization algorithm also played a significant role in RGD method. NSGA-II algorithm proposed by Deb et al. (2002) was a main method to solve multiple objective optimization problems in engineering. Recently, other new optimization algorithms were developed, such asNon-dominated Sorting Genetic Algorithms(Deb and Jain 2014; Jain and Deb 2014). They have a better performance than NSGA-II on solving optimizations. Nevertheless, there are few studies on application of these algorithm in practice.

In this study, RGD method was applied the design of retaining system of deep excavation.Section 2 introduced multiple response surface model and system reliability model in detail. Section 3 introduced background of case study, arrangement of input variables and verification of surrogate model. Section 4 illustrated the whole framework of RSM-RGD and analyzed the results.

2 Methodology

2.1 Multiple responses surface method (MRSM)

Responses surface method(RSM) was a general way to fit limit state functions in reliability analysis, especially for problems with multiple variables and complex failure function. For robust design, both noise vectors and design vectors are considered as input variables (Khoshnevisan et al. 2017).Noise vectors are parameters of soil properties with high uncertainty, such as soil cohesion, friction angle, and elastic modulus. Design vectors are geometry parameters of retaining systems, such as length and thickness of retaining wall,section area of inner struts, which are easy to be controlled by human beings. Therefore, MRSM was adopted to build limit state function. MRSM is expressed as Eq (1):

$$\begin{aligned} f &= a_0 + a_1x_1 + a_2x_1^2 + \cdots + a_{2m}x_m + a_{2m+1}x_m^2 \\ a_i &= b_0^i + b_1^i y_1 + b_2^i y_1^2 + \cdots + b_{2n}^i y_n + b_{2n+1}^i y_n^2 \quad \left(i = 1, 2, \cdots m\right) \end{aligned} \tag{1}$$

where $x_i$and $y_j$were noise vectors and design vectors, respectively. $f(x_i,y_i)$ were excavation responses such as deflection and bending moment of retaining wall, ground settlement and deformation of exist building.

Traditional RSM only analyzed the correlation between random variables and output, however, MRSM also considered the undetermined coefficients $a_i$as variables. The coefficients were functions of design vectors. The influence of noise vectors and design vectors on excavation responses were characterized in MRSM via combining the undetermined coefficients and input variables.

In order to guarantee the performance of responses surface model, the design points should cover design pool and space of random variables uniformly. The arrangement of design points is shown in Table 1.

Table 1. Calibration points for multiple response surface model

Noise vectors	x_1	x_2	…	x_m	Design vectors	y_1	y_2	…	y_n
1	*M*	*M*	…	*M*	1	μ	μ	…	μ
2	*L*	*M*	…	*M*	2	μ-3σ	μ	…	μ
3	*U*	*M*	…	*M*	3	μ+3σ	μ	…	μ
4	*M*	*L*	…	*M*	4	μ	μ-3σ	…	μ
5	*M*	*U*	…	*M*	5	μ	μ+3σ	…	μ
…	…	…	…	…	…	…	…	…	…
2*m*	*M*	*M*	…	*L*	2*n*	μ	μ	…	μ-3σ
2*m*+1	*M*	*M*	…	*U*	2*n*+1	μ	μ	…	μ+3σ

Note: *L* = Lower bound of design vectors; *M* = mean value of design vectors; *U* = upper bound of design vectors;μ= average of noise vectors; σ= deviation of noise vectors.

From Eq (1) and Table 1, for a multiple responses surface model with *m* design vectors and *n* noise vectors, (2*n*+1)•(2*m*+1) design points were necessary. After determining the number of design points, the corresponding value of limit state functions $f(x_i,y_i)$ were obtained via calculating finite element models. With enough design points and limit state functions value, the undetermined coefficients b_i in Eq (1) were determined.

Finally, the responses surface model was taken to calculate the reliability index via Monte Carlo Method (MCS), as shown in Eq (2).

$$\beta_i = \frac{f_i - \delta_i}{\sigma_i} \tag{2}$$

where β_i is the reliability index of i^{th} failure mode, σ_i is standard deviation of fi, δ_i threshold of i^{th} failure mode.Generally, 100000 times Monte Carlo simulations is enough.

2.2 System reliability model

From section 2.1, there are multiple failure modes, the correlation between failure modes is important. Ignorance of correlation will under-estimate the safety of retaining system (Lü et al. 2017; Xiao et al. 2020). In this paper, the spearman correlation matrix was adopted to represent correlation of excavation responses. Based on previous studies (Fu et al. 2019; Fu et al. 2021), spearman matrix was a suitable index to represent correlation of failure modes. To calculate the system reliability index of excavation, system reliability model was taken herein, which is shown in Eq (3).

$$\begin{aligned} p_f^{sys} &= 1 - \Phi\left(\beta_i, R\right) \\ \beta_{sys} &= \Phi^{-1}\left(1 - p_f^{sys}\right) \end{aligned} \tag{3}$$

where R is spearman correlation matrix of excavation responses, $\beta_i=(\beta_1,\beta_2\ldots\beta_m)^T$ is reliability index of each failure mode, p_f^{sys} and β_{sys} is system failure probability and system reliability index, respectively. $\Phi(\bullet)$ is m-dimensional standard normal cumulative distribution functions.

3 Case study

3.1 Background

In this paper, a deep excavation project in Taipei was adopted to illustrate the proposed method (Xuan et al. 2009). The profile of excavation was shown in Figure 1. From Figure 1, the width of excavation is 35m, the depth of excavation is 18.5m.

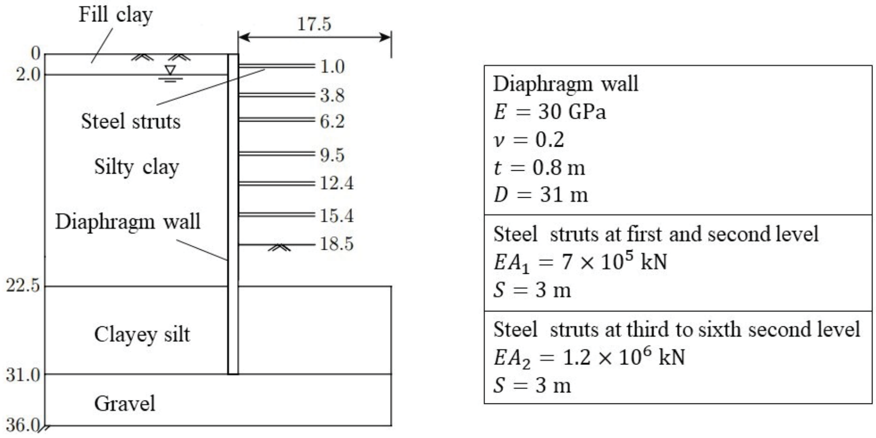

Figure 1. Profile of excavation and original design in Taipei.

In this case, there are five design vectors: width (t) and depth (D) of retaining wall, compressive modulus (EA_1, EA_2) of steel struts, horizontal space of struts (S) and six noise vectors: elastic modulus (E_1, E_2), soil cohesion (c_1, c_2) and friction angle (φ_1, φ_2) of silty clay layer and clayey silt layer. However, for design vectors D and t, quartic functions have a better performance on fitting limit state functions than quadratic functions. For other design vectors and noise vectors, quadratic functions were used. Therefore, there are totally $(2\times3+4\times2+1)\times(2\times6+1)=195$ design points. The arrangement of design points was shown in Table 2.

Table 2. Arrangement of design points.

Design vectors	Design values	Noise vector	Design values	Standard deviation,σ_i	Distribution
		E_1 (MPa)	(10,16,22)	2	lognormal
D (m)	(25,28,31,34,37)	c_1 (kPa)	(25,40,55)	5	lognormal
t (m)	(0.6,0.8,1.0,1.2,1.4)	φ_1 (°)	(23,32,41)	3	lognormal
EA_1 (kN)	$(5,6.5,8)\times10^5$	E_2 (MPa)	(16,40,64)	8	lognormal
EA_2 (kN)	$(8,11,14)\times10^5$	c_2 (kPa)	(55,100,145)	15	lognormal
S (m)	(3,4,5)	φ_2 (°)	(25,34,43)	3	lognormal

3.2 Verification of responses surface model

Moreover, 300 random cases were generated to evaluate the performance of responses surface model, as shown in Figure2. From Figure2, R^2 of quartic functions is 0.9769, 0.9733,0.9586, 0.9506. Root mean squared error (RMSE) is 0.503, 9.507, 16.354, 0.504, which satisfies requirements of surrogate models. On the other hand, based on the excavation responses of train set and test set, the correlation between excavation responses was estimated by spearman correlation matrix.

At last, β_i and β_{sys} of original design was obtained from Monte Carlo simulations and system reliability model. β_i equals to 0.73, 5.99, 1.49, 0.64. β_{sys} equals to 0.42.

4 Response surface based-robust design framework

From section 2, robust geotechnical design based on responses surface model (RSM-RGD) was proposed. Robust design aims to find the optimal design with higher robustness and lower cost. For deep excavation, cost mainly included retaining wall (Q_w) and struts (Q_s) (Juang et al. 2014), which is expressed as Eq (4).

$$
\begin{aligned}
&Q = Q_w + Q_s \\
&Q_w = D \cdot t \cdot P_e \cdot U_c \\
&Q_s = W \cdot \rho \cdot U_s \\
&P_e = 2(B + L) \\
&V = S_1 \cdot N_h \cdot N_v + S_2 \cdot N_h \cdot N_v
\end{aligned}
\tag{4}
$$

where $U_c$and $U_s$are the unit cost of concrete and steel, respectively.U_c equals to 330\$/m^3 and U_s equals to 0.6 \$/kg.$W$is total weight of struts, ρ is density of struts, which equals to 7.85×10^3kg/m^3.P_e is the perimeter of excavation, B and L are width and length of excavation.B equals to 35m and L equals to 200m.S_1 and S_2 are the unit area of two type struts. N_h and N_v are row number of struts in vertical and horizontal direction.

Based on the aim of robust design, the multiple objective optimization model can be adopted to solve robust design, which is shown in Eq (5).

$$
\begin{aligned}
&\min \beta_{sys} = f\left(x_i, y_i\right) \\
&s.t. \begin{cases} \beta_1 > \left[T_1\right] \\ \beta_2 > \left[T_2\right] \\ \quad \cdots \\ \beta_n > \left[T_n\right] \\ x_i \in D \\ y_i \in S \end{cases}
\end{aligned}
\tag{5}
$$

where D is design pool of design vectors, S is random space of noise vectors. $[T_i]$ is threshold of each failure mode.

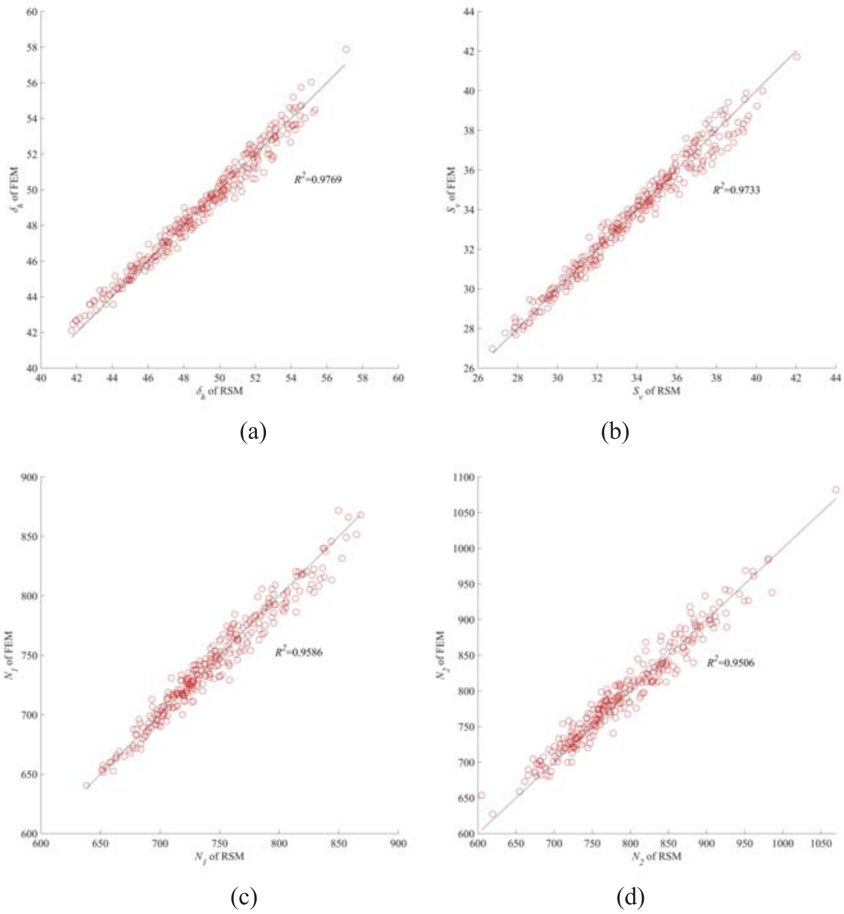

Figure 2. Verification of responses surface model (a) wall deflection (b) ground settlement (c) first type strut (d) second type strut.

In this paper, the target functions are cost and system reliability of deep excavation. The constraints are the limitation of reliability index of single failure mode.The design pool of design vectors and statistical character of noise vectors were shown in Table 2.

The whole framework of RSM-RGD was shown as followings. The main procedures included six steps.

(1) Determine design pool and space of random variables.

(2) Arrange design points and establish responses surface model.

(3) Determine the threshold of failure mode and limit state functions.

(4) Calculate reliability index via Monte Carlo simulation.

(5) Evaluate the spearman correlation matrix and calculate system reliability index via multi-dimensional lognormal distribution.

(6)Obtain Pareto frontier and optimal design based on multiple objective optimization model.

PlatEMO toolbox in MATLAB 2020a, which has a better performance on solving non-linear multiple objective optimizations, was adopted in this paper (Ye et al. 2017).PlatEMO toolbox included many optimization algorithms such as non-dominated Sorting Genetic Algorithms (NSGA-II, NSGA-III, ANSGA-III) (Deb et al. 2002, Deb and Jain 2014, Jain and Deb 2014), Many-objective evolutionary algorithms (MaOEA/IT) (Sun, Xue et al. 2019)and Cellular genetic algorithm (MOCell) (Nebro et al. 2009)etc.In this paper, NSGA-III algorithm was adopted in this paper. The generation of NSGA-III algorithm was 50, the number of design points on Pareto frontiers was 30. The constraint functions were set as β_i>1.5.

5 Results

Via RSM-RGD framework, the design points and Pareto frontiers were obtained, as shown in Figure 3. There is a positive correlation between economic cost and system safety. The design point with the lowest cost and the highest system reliability called "ideal point", which is used to choose optimal design. The optimal design was the design point on Pareto frontiers with longest distance from ideal point.In this case study, the optimal design was marked with blue pentagonal. The design case was D=29m, t=1.2m, EA_1=8 × 10^5kN, EA_2= 1.4 × 10^6kN, S=4m, the economic cost is 3220136$, the system reliability β_{sys} is 3.725.

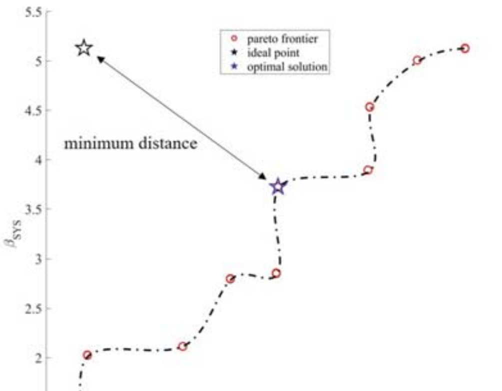

Figure3. Pareto frontiers and optimal design.

In this study, a response surface based robust design method (RSM-RGD) was presented. Both one bearing capacity limit state and two serviceability limit states were taken into account. Based on the analysis above, the following conclusions are arrived at:

(1) The multiple response surface model had a better performance on predicting the value of excavation responses. The reasonable expression of RSM should be lower than fourth power, too complex expression will lead to overfitting.

(2) In order to improve efficiency on multiple objective optimization model, discrete optimization was a better way.NSGA-III algorithm was also good at solving multiple objective optimization model. For two-objective optimization model, 30 generation samples are suggested to determine optimal solutions.Via discrete optimization and NSGA-III algorithm, the Pareto frontiers were obtained easily.

(3) For series systems, the system reliability depends on the failure mode with minimum reliability index. System reliability model is a comprehensive method to estimate integral safety of braced excavations, which takes less calculation efforts than considering failure modes one by one.

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