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A Bayesian Framework for Settlement Predictions of Immersed Tunnels

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Abstract: In immersed tunnels, the considerable settlement that can develop during their service period may induce structural damage. Accordingly, the effective predictions of settlement are crucial for an immersed tunnel in service. This paper proposes a Bayesian updating framework for predicting the settlement of immersed tunnels based on a beam on elastic foundation model (BEFM) and using knowledge gathered from field observations to promote prediction accuracy. The procedure combines the differential evolution transitional Markov chain Monte Carlo (DE-TMCMC) algorithm and finite element solving algorithm to perform effective samplings and settlement computations. Field application in the Hong Kong-Zhuhai-Macao (HZM) immersed tunnel is used to demonstrate the effectiveness of the proposed framework.

Keywords: Settlement; Immersed tunnel; Bayesian updating framework; beam on elastic foundation model; Hong Kong-Zhuhai-Macao tunnel

1 Introduction

Many immersed tunnels have been reported to suffer a considerable amount of settlement during their long-term service period (Grantz, 2001a, 2001b). Various problems, such as leakage, concrete cracking, and damage to joint waterproofing gaskets, are likely to arise as significant settlement develops, which may significantly interfere with the normal operation of an immersed tunnel (Xie et al., 2014; Zhang and Broere, 2019). Accordingly, accurate predictions of settlement are vital during an immersed tunnel's service period.

Two common methods for estimating the settlement of immersed tunnels are the finite element method (FEM) and the beam on elastic foundation model (BEFM). However, neither method can guarantee an accurate prediction due to the uncertainty from the complex underwater geological conditions, highly varying sub-soil stiffness, measurement error, etc. A feasible strategy is to combine the prediction model with a back analysis framework, in which knowledge drawn from field observations can be used to promote prediction accuracy. However, combining the FEM with a back analysis framework can be very difficult, because immersed tunnels are usually very large in scale and the corresponding modeling work can be highly time-consuming and computationally intensive. Due to this constraint, only several selected tubes can be modeled in these FEM analyses, and the interactions between all tubes cannot be fully considered. For example, because of the limits imposed by computational efficiency, the 3D FEM model of the HZM tunnel established by Song et al. (2018) comprises only four tubes. In contrast, the BEFM can provide a full-length assessment of settlement and can be readily coupled with a back analysis framework. Therefore, this paper proposes using the BEFM as a prediction model in this context. The discussion in Section 2 will present further details about BEFM for immersed tunnels.

Many methods are currently available for back analysis of geotechnical problems, such as the least-squares method (e.g., Finno and Calvello, 2005), artificial neural networks (e.g., Yu et al., 2007), multi-objective optimization (e.g., Sun et al., 2018), and the Bayesian method (e.g., Juang et al., 2013; Qi and Zhou, 2017). The Bayesian method treats soil parameters as random variables and can update input parameters' probability distribution function based on field observations, in contrast to most of the other methods, which regard the soil parameters as constants and can only perform deterministic analysis (Juang et al., 2013; Qi and Zhou, 2017). Many successful applications of the Bayesian updating approach in geotechnical problems have been reported, including deformation analysis of deep excavations (Qi and Zhou, 2017) and tunnel convergence predictions (Feng et al., 2019). Accordingly, this study employs the Bayesian approach as the back analysis framework.

The objective of this study is to develop a Bayesian framework for the prediction of settlement in immersed tunnels. The proposed framework is verified with a case study of a famous immersed tunnel, the Hong Kong-Zhuhai-Macao (HZM) tunnel. The remainder of the paper is organized as follows. Section 2 provides a brief introduction to the BEFM for immersed tunnels and the Bayesian approach. Next, the application of the proposed framework in assessing the settlement of the HZM tunnel is demonstrated in Section 3. Lastly, Section 4 presents the study conclusions.

2 Methodology

2.1 BEFM for immersed tunnels

An immersed tunnel can be regarded as a beam placed on an elastic foundation, in which the soil-structure interactions can be simulated by a series of springs based on the assumptions of the Winkler model. The stiffness of these springs is known as the foundation modulus. One way to consider the interactions between tubes involves using vertical springs to simulate the shear joints between tubes (Wei and Lu, 2018). In addition, this investigation employs the linearly varying foundation modulus to consider the continuous variation of soil stiffness, as shown in Fig 1. The governing equation can be expressed as follows (Reddy, 2006):

$$EI\frac{d^4w}{dx^4}=(q-kw)b \tag{1}$$

where EI denotes the bending stiffness of the tubes; x is the coordinate along the tunnel alignment, while w is the corresponding settlement at the coordinate x; q is the load above the tunnel; k is the foundation modulus; and b is the tunnel width.

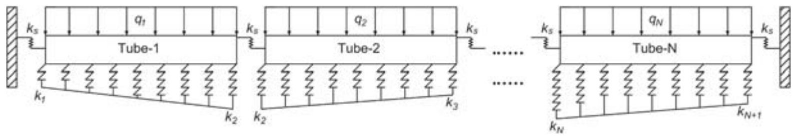

Figure 1. Schematic diagram of the BEFM for settlement estimation of immersed tunnels

In this paper, the foundation moduli k_1–k_{N+1} and shear stiffness of the joints k_s are taken as variables, as no reliable assessments of their values are available. Consequently, the total number of uncertain model parameters needing to be determined is N+2. Notably, the load is assumed to be uniformly distributed along the tube in the BEFM.

The principle of the finite element method (FEM) is incorporated into the BEFM to compute the settlement. An immersed tunnel containing N tubes can be partitioned into 2N+1 beam elements. N beam elements are generated for tubes; the other N+1 beam elements are generated for shear joints.

For each tube, a set of displacement shape functions ϕ^n is used to form the matrix equations, which is expressed in terms of local coordinate $\bar{x}$ (Reddy, 2006), as follows:

$$\phi_1^n=1-3(\frac{\bar{x}}{h_n})^2+2(\frac{\bar{x}}{h_n})^3,\phi_2^n=-\bar{x}(1-\frac{\bar{x}}{h_n})^2,\phi_3^n=3(\frac{\bar{x}}{h_n})^2-2(\frac{\bar{x}}{h_n})^3,\phi_4^n=-\bar{x}[(\frac{\bar{x}}{h_n})^2-\frac{\bar{x}}{h_n}] \tag{2}$$

where n denotes the index of the nth tube, and h_n is the length of the nth tube.

The foundation modulus $k(\bar{x})$ at a local coordinate $\bar{x}$ of the nth tube can be expressed as:

$$k(\bar{x})=k_n+\frac{k_{n+1}-k_n}{h_n}\bar{x} \tag{3}$$

where k_n and k_{n+1} are the foundation moduli at both ends of the nth tube.

By adopting the principle of virtual work, the stiffness matrix $\mathbf{K^n}$ of the nth tube in an immersed tunnel is derived by:

$$\mathbf{K}^n=\mathbf{K}_1^n+\mathbf{K}_2^n=EI\int_0^{h_n}\frac{d^2\phi_i^n}{dx^2}\frac{d^2\phi_j^n}{dx^2}d\bar{x}+\int_0^{h_n}k(\bar{x})\phi_i^n\phi_j^n d\bar{x} \tag{4}$$

where i and j both denote the index of the interpolation function (i=1, 2, 3, 4; and j=1, 2, 3, 4). $\mathbf{K}_1^n$ represents the stiffness matrix of structural rigidity, while $\mathbf{K}_2^n$ denotes the stiffness matrix of soil springs.

The uniform-distributed load q_n of the nth tube is integrated into the calculation as a force matrix, which is computed as:

$$\mathbf{F}^n=\int_0^{h_n}q_n\phi_i^n d\bar{x} \tag{5}$$

where q_n is the uniform load acting on the nth tube.

Each joint can be regarded as an independent beam element whose stiffness matrix can be derived by the linear relationship between the shear force and differential settlement at each joint (corresponding spring constant is k_s).

Once the stiffness matrix and force matrix have been obtained for each beam element, the global matrix equation is then obtained by assembling all matrices of beam elements in order according to the node's force equilibrium and deformation compatibility conditions, which is established as:

$$\mathbf{K}\boldsymbol{\Delta} = \mathbf{F} \tag{6}$$

where $\mathbf{K}$ is the global stiffness matrix. $\boldsymbol{\Delta}$ and $\mathbf{F}$ denote the vector of generalized displacements (i.e., displacement and rotation angle) and vector of generalized forces (i.e., point force and bending moment), respectively (Reddy, 2006).

By adopting matrix operations to compute Δ in Eq (9), the settlement of an immersed tunnel (i.e., vector of displacement in Δ) at each node can be obtained.

2.2 Bayesian approach

This paper adopts the Bayesian updating framework to realize an accurate settlement estimation. The implementation starts with expressing the observed settlement as follows (Juang et al., 2013; Jin et al., 2019a):

$$w_{\text{obs}} = c w_{\text{pre}}(\boldsymbol{\theta}_{\text{m}}) \tag{7}$$

where w_{obs} denotes the observed settlement, while $w_{\text{pre}}(\boldsymbol{\theta}_{\text{m}})$ represents the predicted settlement at the observed point. c is a model bias factor that is assumed to be a one-mean Gaussian random variable with a standard deviation (SD) of σ_ε. The σ_ε is another unknown parameter in addition to the model parameters $\boldsymbol{\theta}_{\mathbf{m}}$. Accordingly, the uncertain parameters vector $\boldsymbol{\theta}$ includes the model parameters and the prediction error SD σ_ε, i.e., $\boldsymbol{\theta}=[\boldsymbol{\theta}_{\mathbf{m}}^T, \sigma_\varepsilon]^T$.

The vector $\boldsymbol{\theta}$ and its uncertainty can be derived from a posterior probability density function (PDF), which is expressed as follows based on Bayes' theorem (e.g., Yuen, 2010; Zhou et al., 2018, 2021):

$$f(\boldsymbol{\theta} \mid \mathbf{W}) = \frac{f(\boldsymbol{\theta}) f(\mathbf{W} \mid \boldsymbol{\theta})}{f(\mathbf{W})} \tag{8}$$

where $f(\boldsymbol{\theta})$ is the prior PDF of $\boldsymbol{\theta}$, representing the user's judgment or previous knowledge before the data $\mathbf{W}$ are observed. $f(\mathbf{W})$ is a normalization factor that guarantees unity for the cumulative probability over the entire range of $\boldsymbol{\theta}$. $f(\mathbf{W}|\boldsymbol{\theta})$ is the likelihood function expressing the level of data fitting. If the prediction errors at different observed points are statistically independent, the likelihood function can then be defined as (Juang et al., 2013; Jin et al., 2019a):

$$f(\mathbf{W} \mid \boldsymbol{\theta}) = (2\pi\sigma_\varepsilon^2)^{-\frac{M}{2}} \exp\left\{ -\frac{\sum_{i=1}^{M} \left[\frac{w_{\mathbf{obs}}^{(i)}}{w_{\mathbf{pre}}^{(i)}(\theta_{\text{m}})} - 1\right]^2}{2\sigma_\varepsilon^2} \right\} \tag{9}$$

where M denotes the total number of the observed points.

The posterior PDF is usually obtained from sampling techniques. This study has adopted an efficient and robust sampling technique, known as the differential evolution transitional Markov chain Monte Carlo (DE-TMCMC) algorithm. The basic idea behind this algorithm is to construct a series of intermediate PDFs that converge to the target posterior PDF from the initial prior PDF (Ching and Chen, 2007). Additionally, the differential evolution technique is integrated into the DE-TMCMC algorithm to overcome possible local convergence (Jin et al., 2019a, 2019b). This algorithm can be enacted by the following steps:

(1) In the initial stage (s=0), generate samples $\{\theta_{0,k}$:k=1,2,…,$N_0\}$ from the prior PDF $f_0(\boldsymbol{\theta})$.

(2) Compute the plausibility weight $w(\theta_{s,k}) = f\left(\mathbf{Y} \mid \theta_{s,k}\right)^{p_{s+1}-p_s}$ for k=1,2,…,N_s; next, choose p_{s+1} by comparing the COV (coefficient of variation) of $w(\theta_{s,k})$ with a prescribed threshold. Here, $\theta_{s,k}$ denotes the kth sample that belongs to stage s.

(3) Select the resampling index l from the set [1,2,…,N_s] using the sequential importance sampling method, where each l is assigned probability $w(\theta_{s,l}) / \sum_{l=1}^{N_s} w(\theta_{s,l})$.

(4) Generate the new samples with Eqs. (10) and (11), employing the Metropolis-Hastings algorithm to judge whether to accept these new samples:

$$\theta_{s,l}^{\text{new}} = \theta_{s,l}^{c} + d\theta_{s,l} \tag{10}$$

with

$$d\theta_{s,l} = (1+\lambda)\gamma[(\theta_{s}^{\text{best}} - \theta_{s,l}^{c}) + (\theta_{s,a} - \theta_{s,b})] + \zeta \tag{11}$$

where $\theta_{s,l}^{\text{new}}$ is the new sample, $\theta_{s,l}^{c}$ is the current sample, $d\theta_{s,l}$ is the jump between the new sample and current sample, and θ_{s}^{best} is the sample corresponding to the maximum weight in the current iteration. More details about Eq. (11) can be found in Jin et al. (2019a, 2019b).

Repeat steps (2) to (4) until level N_s has been reached, i.e., $P_{Ns} = 1$. At this stage, samples $\{\theta_{N_s,k} : k = 1, 2, ..., N_{N_s}\}$ are asymptotically distributed as $f(\boldsymbol{\theta}|\mathbf{Y})$ (Ching and Chen, 2007).

3 Application in the HZM tunnel

The HZM tunnel, which is a part of the HZM linkage in southern China, has a total length of 6087.3m. This tunnel has 33 underwater tubes with 2 buried sections located at both ends, comprising 35 tubes in total (CCCC, 2012). Table 1 lists the length of each tube. Rectangular concrete tubes with a width of 37.95m are employed in the HZM tunnel, and the bending stiffness of each tube is calculated as 1.05×10^{11} kN*m^2 (Mu, 2011).

Figure 2 shows the geological and foundation profile of the immersed tunnel (CCCC, 2012). Most of the tunnel's tubes rest upon soft layers; moreover, significant discrepancies are found between different layers. Additionally, various foundation treatments are used to prevent excessive settlement of tubes located in the mucky soil (Hu et al., 2018). Thus, the highly variable stiffness of the foundation along the tunnel alignment makes estimating the settlement of the immersed tunnel extremely challenging.

Table 1. Lengths of tubes in the immersed tunnel

Tube	BW	E1, E2	E3–E26, E31	E27, E28	E29, E30	E32, E33	BE
Length (m)	193	112.5	180	157.5	177	135	230.7

Note: E1–E33 signify tubes from the west artificial island to the east artificial island. BW and BE denote the buried sections at two ends, respectively.

This study employed 7 sets of observed settlements and corresponding loads for analysis, covering the period from April 22, 2019, to November 16, 2020. Two observed points were located at both ends of each tube, yielding 70 points for the 35 tubes. To check the effectiveness of the developed prediction model, the data on November 16, 2020, is used as the test set, while the rest of the data (6 sets of data in total) are designated as the training sets to conduct model calibration with the Bayesian approach.

The total load used in this study was composed of the self-weight of the tunnel body (the anti-floating safety factor was assumed to be 1.1), backfill load, back siltation load, and building weight above the buried sections at both ends. The back siltation load was considered to vary with thickness of back siltation.

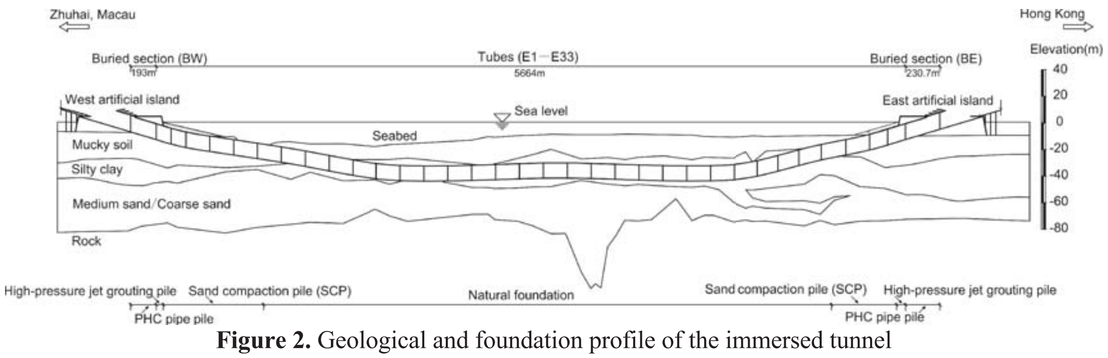

Figure 2. Geological and foundation profile of the immersed tunnel

The length of the joints is 0.8m (Song et al., 2018), whereas the length of each tube is shown in Table 1. Out of a total 71 beam elements, 35 are generated for tubes, while the other 36 are generated for joints. The total number of uncertain parameters is 38, including the foundation moduli k_1–k_{36}, shear stiffness k_s, and SD of model prediction error σ_ε. The prior PDFs of all uncertain parameters are assumed to follow uniform distributions, and the lower and upper bounds are determined based on previous research and experiments (Steenfelt et al., 2013; Lu, 2018; Song et al., 2018; Jin et al., 2019a), as summarized in Table 2. Additionally, the number of samples was set to 10^5 for each round of sampling in this study.

Table 2. Prior bounds of uncertain parameters

Parameters	k_1–k_{36} (kPa/m)	k_s (kN/m)	σ_ε (mm)
Lower bound	1×10^2	1.2×10^7	0
Upper bound	5×10^3	1×10^5	1

Table 3. Updated results of uncertain parameters

Parameters	k_1–k_{36}	k_s	σ_ε
Mean values	157.44–1520.1 kPa/m	3.86×10^{-6} kN/m	1.22×10^{-1} mm
COVs	1.08×10^{-2}–7.86×10^{-2}	1.86×10^{-2}	2.81×10^{-2}

With the prior knowledge and field observations, the posterior PDFs of the uncertain parameters could be obtained via the Bayesian approach with the DE-TMCMC algorithm. Table 3 lists the updated results of these uncertain parameters. The relatively small COVs indicate a significant reduction in model uncertainty after Bayesian updating.

The predictions were then made with Monte Carlo simulation using 10^6 samples in this case. Figure 3 presents the ratios of predicted settlement to observed settlement (R_w=$w_{\text{pre}}/w_{\text{obs}}$) along the tunnel alignment. The values of R_w were around 1.0, with a varied range of 0.77–1.34. The coefficient of determination (R^2) between the predictions and observations reached 89.44%, indicating that a good prediction performance was achieved. Additionally, the uncertainty of the predicted settlement could also be captured with the corresponding COVs (coefficients of variation) in the range of 1.09×10^{-2}–3.66×10^{-2}. In short, the proposed framework proved effective in the settlement predictions of the HZM tunnel.

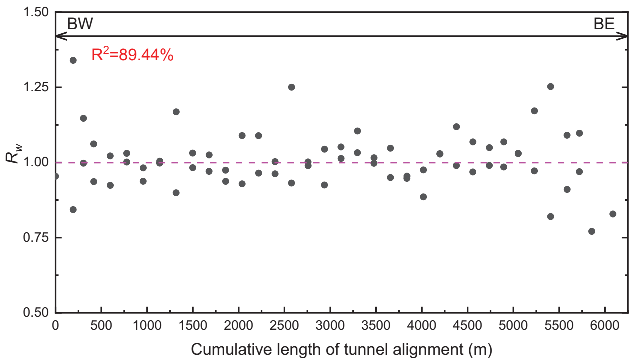

Figure 3. Ratios of predicted settlement to observed settlement along the cumulative length of the tunnel alignment

4 Conclusions

A Bayesian framework that adopted the BEFM as the prediction model was developed to predict the settlement of immersed tunnels in this paper. The DE-TMCMC algorithm and finite element solving algorithm were combined to perform effective samplings and settlement computations. The proposed framework was verified with data from a field study of the HZM immersed tunnel. A good agreement was found between the predicted settlement and observed settlement, with an R^2 of 89.44%. In summary, the developed framework provides a valuable tool to estimate the settlement of immersed tunnels, which, in turn, supports potential risk prevention.

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