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Efficient Updating of Consolidation-Induced Responses by Auxiliary Bayesian Approach

Hua-Ming Tian[1], Zi-Jun Cao[2], Dian-Qing Li[3], and Xiao Chen[4]

[1]State Key Laboratory of Water Resources and Hydropower Engineering Science, Institute of Engineering Risk and Disaster Prevention, Wuhan University, Wuhan, P.R. China
E-mail: huamingtian@whu.edu.cn
[2]State Key Laboratory of Water Resources and Hydropower Engineering Science, Institute of Engineering Risk and Disaster Prevention, Wuhan University, Wuhan, P.R. China
E-mail: zijuncao@whu.edu.cn (Corresponding Author)
[3]State Key Laboratory of Water Resources and Hydropower Engineering Science, Institute of Engineering Risk and Disaster Prevention, Wuhan University, Wuhan, P.R. China
E-mail: dianqing@whu.edu.cn
[4]Science and Technology Management Department, China Railway Siyuan Survey and Design Group Co.,Ltd, Wuhan, P. R.China
Email: 004556@crfsdi.com

Abstract: Predicting the consolidation-induced responses (e.g., settlement and excess pore pressures) is a challenging task due to the existence of soft soils and various geotechnical-related uncertainties. To reduce these uncertainties, observational data obtained at different monitoring moments can be used to update these responses, for example, through Bayesian methods. Nevertheless, Bayesian updating of consolidation behaviors of soft soils can be computationally demanding when sophisticated computational models are involved, and a great number of model evaluations are required for the updating given a monitoring dataset. This becomes more challenging if multiple different datasets (e.g., those sequentially obtained at different monitoring moments) are concerned, for each of which a Bayesian updating run is needed. This paper develops a novel simulation-based Bayesian framework that allows efficient updating of soft soil behaviors based on different datasets. It consists of two major components: (1) driving Bayesian analysis to generate problem-specific information on the response evaluations based on the monitoring dataset obtained at early monitoring moments; and (2) target Bayesian analysis to update the soft soil behaviors given new datasets obtained at latter monitoring moments by making use of the information generated in the first step, which requires negligible computational efforts. A consolidation example of clay is adopted to demonstrate the efficiency and rationality of the proposed approach. Effects of monitoring datasets with different types and locations on the updated response are also investigated.

Keywords: Bayesian updating; BUS; consolidation response; monitoring dataset.

1 Introduction

Predicting the time-dependent responses (e.g., settlement and pore pressure) concerning soft soils in engineering practice is complicated due to various geotechnical-related uncertainties, e.g., those for properly characterizing soft soils (Kelly et al. 2018). To reduce these uncertainties and understand the complex behaviors of soft soils, in situ monitoring data combined with sophisticated numerical model is frequently adopted, for example, in the deterministic or probabilistic inverse analysis (e.g., Rahimi et al. 2019; Tian et al. 2022). Bayesian approaches are widely adopted in probabilistic inverse analyses due to their capability of quantifying various uncertainties and combining newly obtained dataset with prior knowledge in a rigorous and consistent manner, and they have gain extensive popularity for updating the geotechnical responses (Kelly and Huang 2015; Rahimi et al. 2019).

However, Bayesian updating of geotechnical structure responses (e.g., settlement and pore pressure) can be computationally demanding. First, Bayesian updating requires to determine the posterior distribution of uncertain parameters and update the corresponding responses given a dataset. These impose a computational burden when computationally expensive numerical models are involved for response prediction. In such case, analytical posterior distributions of uncertain model parameters may not exist and a great number of model evaluations are required for populating posterior samples in simulation-based Bayesian inferences. The computational efforts become more profound if multiple different datasets (e.g., those sequentially obtained at different monitoring moments) are concerned and repeated runs of Bayesian updating are inevitable.

This study presents an auxiliary Bayesian framework for efficient updating of consolidation-induced responses based on different datasets. It first performs driving Bayesian analysis to generate problem-specific information using the monitoring dataset obtained at early monitoring moments. As more monitoring data is acquired later, target Bayesian analysis is conducted to update the consolidation responses by making use of the information from driving Bayesian analysis, which requires negligible computational efforts. This paper first describes the Bayesian framework for consolidation-induced response updating, followed by descriptions of the auxiliary Bayesian approach. Finally, a consolidation example is employed to illustrate the proposed approach.

2 Bayesian updating of consolidation-induced responses

Let $\boldsymbol{y}_j = [y_1, y_2, \ldots, y_J]$ denote a vector of J measurements (e.g., settlement and/or pore pressure induced by consolidation) available at monitoring moment j. Then, the updated responses $\tilde{Y}_t$ at tth monitoring moment can be calculated under the Bayesian framework as:

$$\tilde{Y}_t = \int M_t(\boldsymbol{\theta}) f(\boldsymbol{\theta}|\boldsymbol{y}_j) \mathrm{d}\boldsymbol{\theta} \tag{1}$$

where $\boldsymbol{\theta}$ represents a vector of uncertain model parameters; $M_t(\boldsymbol{\theta})$ is the prediction model for consolidation responses; and $f(\boldsymbol{\theta}|\boldsymbol{y}_j)$ denotes the posterior probability distribution function (PDF) that quantifies the updated knowledge on $\boldsymbol{\theta}$ given $\boldsymbol{y}_j$, and it is expressed as:

$$f(\boldsymbol{\theta}|\boldsymbol{y}_j) = K_j L(\boldsymbol{\theta}|\boldsymbol{y}_j) f(\boldsymbol{\theta}) \tag{2}$$

where K_j is a normalizing constant; $f(\boldsymbol{\theta})$ is the prior PDF reflecting the available knowledge on $\boldsymbol{\theta}$ without $\boldsymbol{y}_j$; and $L(\boldsymbol{\theta}|\boldsymbol{y}_j)$ denotes the likelihood function that describes the probabilistic relationship between the observed data (e.g., y_l) and simulated responses (e.g., $M_l(\boldsymbol{\theta})$ corresponding to y_l) through model error (e.g., ξ_l). For demonstration, assuming ξ_l (l = 1, 2, …, J) are independently and normally distributed with zero mean and a standard deviation of σ_{ξ_l}, then $L(\boldsymbol{\theta}|\boldsymbol{y}_j)$ can be written as:

$$L(\boldsymbol{\theta}|\boldsymbol{y}_j) = \prod_{l=1}^{J} \frac{1}{\sqrt{2\pi}\sigma_{\xi_l}} \exp\left(-\frac{\left(y_l - M_l(\boldsymbol{\theta})\right)^2}{2\sigma_{\xi_l}^2} \right) \tag{3}$$

Evaluating Eq. (1) is usually challenged due to the unavailability of analytical expressions of posterior distribution $f(\boldsymbol{\theta}|\boldsymbol{y}_j)$ and a significant number of computational model evaluations for generating samples following $f(\boldsymbol{\theta}|\boldsymbol{y}_j)$. Moreover, because monitoring dataset $\boldsymbol{y}_j$ changes at different monitoring moments, repeated Bayesian analyses are unavoidable, which further causes the computational difficulty. Therefore, this study develops an auxiliary Bayesian framework for efficient updating of consolidation-induced responses considering datasets obtained at different monitoring moments. As shown in Fig. 1, the proposed approach contains two major components: (1) driving Bayesian analysis to generate problem-specific information using the monitoring data $\boldsymbol{y}^*$ (namely the driving dataset) obtained at early monitoring moments; and (2) target Bayesian analysis to update the consolidation responses when more $\boldsymbol{y}_j$ are sequentially obtained, as respectively described below.

3 Auxiliary Bayesian analysis

3.1 Driving Bayesian analysis

This subsection employs the BUS method to perform driving Bayesian analysis with $\boldsymbol{y}^*$. BUS addresses Bayesian updating problems (e.g., Eq. (2)) with structural reliability methods by defining a failure event F^* as (Straub and Papaioannou 2015):

$$F^* = \{\ln(w) - \ln(c^* \cdot L(\boldsymbol{\theta}|\boldsymbol{y}^*)) \le 0\} \tag{4}$$

where w follows standard uniform distribution within [0, 1]; c^* is a constant that satisfies $c^* L(\boldsymbol{\theta}|\boldsymbol{y}^*) \le 1$, and $L(\boldsymbol{\theta}|\boldsymbol{y}^*)$ is the likelihood function, for example, see Eq. (3) given $\boldsymbol{y}_j = \boldsymbol{y}^*$.

Generating posterior samples or, equivalently, failure samples falling in F^*, can be achieved using reliability analysis methods (see Fig. 1(a)). In this study, Subset Simulation (SuS) is employed. SuS converts the occurrence probability of a rare event (e.g., F^*) into the estimation of larger conditional probabilities of a sequence (e.g., m) of nested intermediate events, and employs Markov Chain Monte Carlo to efficiently generate conditional samples falling in each intermediate event (see Fig. 1(b)). After completing SuS, a total of m + 1 subsets Ω_i (i = 0, 1, …, m, see Fig. 1(c)) divided from prior sample space are generated. These m + 1 subsets can be used to calculate $P(F^*)$ as (Au 2007; Tian et al. 2022):

$$P(F^*) = \sum\nolimits_{i=0}^{m} P(F^*|\Omega_i) P(\Omega_i) \approx \sum\nolimits_{i=0}^{m} \sum\nolimits_{k=1}^{N_i} I_{F^*}(\boldsymbol{\theta}_k|\Omega_i) P(\Omega_i) / N_i \tag{5}$$

where $P(F^*|\Omega_i)$ is the probability of F^* conditional on Ω_i, and it can be calculated as $\sum_{k=1}^{N_i} I_{F^*}(\boldsymbol{\theta}_k|\Omega_i)/N_i$, and $I_{F^*}(\boldsymbol{\theta}_k|\Omega_i)$ is an indicator function taking as one if F^* occurs given the kth sample from Ω_i or zero otherwise; N_i is the number of samples in Ω_i, and it equals to $N(1\text{-}p_0)$ for i = 0, 1, …, m-1, and $N{\cdot}p_0$ for $i = m$, where N and p_0 are the number of samples and conditional probability in each level of SuS, respectively; $P(\Omega_i)$ is the occurrence probability of Ω_i and it is estimated as $p_0^i(1\text{-}p_0)$ for $i = 0, 1, \ldots, m\text{-}1$, and p_0^i for $i = m$ (Au 2007).

During the driving Bayesian analysis, a total of $N_{\text{total}} = N + N(m\text{-}1)(1\text{-}p_0)$ samples are generated, denoted as "driving samples" in this study. For each sample within Ω_i, the consolidation response predictions $M_j(\boldsymbol{\theta})$ at different monitoring moments are calculated through the numerical model. The next subsection takes advantage of these information (i.e., subsets Ω_i ($i = 0, 1, \ldots, m$), and the simulated responses for each sample within these subsets) to facilitate target Bayesian updating of consolidation responses as different $\boldsymbol{y}_j$ are available.

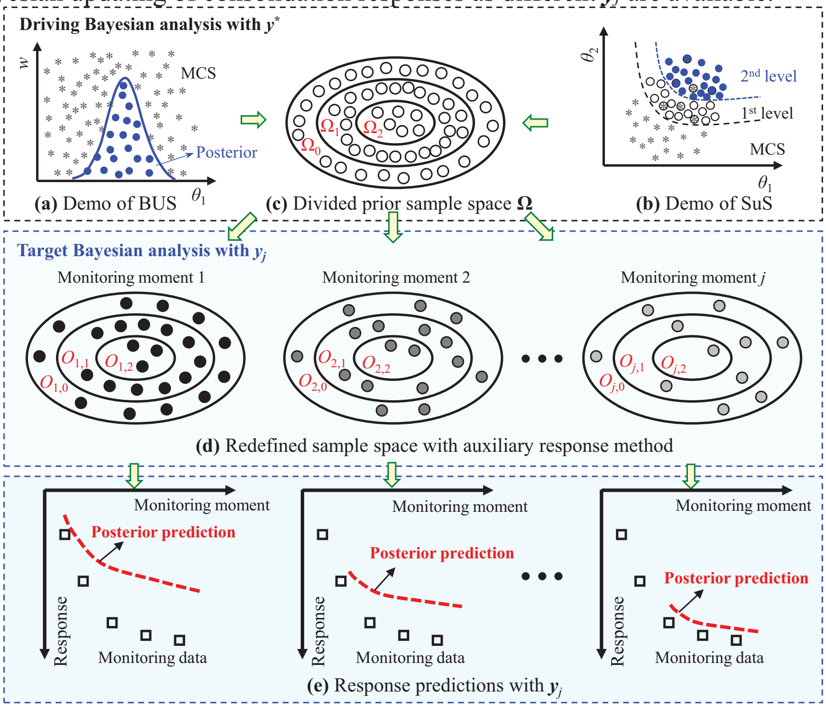

Figure 1. Auxiliary Bayesian framework for consolidation-induced response updating (Modified from Tian et al. 2022)

3.2 Target Bayesian analysis

As monitoring data (e.g., $\boldsymbol{y}_j$) appears, posterior samples and the corresponding updated consolidation responses (e.g., $\tilde{Y}_t$) can be obtained using target Bayesian analysis. In the context of BUS, posterior samples are equivalent to failure samples satisfying the region F_j defined as:

$$F_j = \{\ln(w) - \ln(c_j \cdot L(\boldsymbol{\theta}|\boldsymbol{y}_j)) < 0\} \tag{6}$$

Herein, c_j is a constant satisfies $c_j L(\boldsymbol{\theta}|\boldsymbol{y}_j) \leq 1$ and $L(\boldsymbol{\theta}|\boldsymbol{y}_j)$ is the likelihood function given by Eq. (3).

As indicated by Eq. (6), $\tilde{Y}_t$ in Eq. (1) is actually the expected prediction of consolidation responses conditional on the failure event F_j, i.e., $\tilde{Y}_t = E(\tilde{Y}_t|F_j)$. Based on the obtained subsets from driving Bayesian analysis (i.e., Ω_i, $i = 0, 1, \ldots, m$), $\tilde{Y}_t$ can be formulated using the theorem of total probability as (Au 2007):

$$\tilde{Y}_t = E(\tilde{Y}_t|F_j) = \sum_{i=0}^{m} E(\tilde{Y}_t|\Omega_i, F_j) P(\Omega_i|F_j) \tag{7}$$

where $E(\tilde{Y}_t|\Omega_i,F_j)$ represents the mean response prediction at the tth monitoring moment estimated based on the samples in Ω_i and satisfying F_j; $P(\Omega_i|F_j)$ is the conditional probability of Ω_i given F_j. Let $O_{j,i}$ denote the conditional event that the sample falls in Ω_i given the occurrence of F_j (see Fig. 1(d)). Then, $P(O_{j,i})$ or $P(\Omega_i|F_j)$ is calculated using the Bayes' theorem as:

$$P(O_{j,i}) = P(\Omega_i|F_j) = P(F_j|\Omega_i)P(\Omega_i) / P(F_j) \tag{8}$$

where $P(F_j|\Omega_i)$ is the conditional probability of F_j given Ω_i, and it is calculated as $\sum_{k=1}^{N_i} I_{F_j}(\boldsymbol{\theta}_k|\Omega_i)/N_i$, in which $I_{F_j}(\boldsymbol{\theta}_k|\Omega_i)$ is the indicator function with respect to F_j for the kth sample in Ω_i; $P(F_j)$ is the failure probability of F_j and it can be estimated as:

$$P(F_j) = \sum_{i=0}^{m} P(F_j|\Omega_i)P(\Omega_i) = \sum_{i=0}^{m} \sum_{k=1}^{N_i} I_{F_j}(\boldsymbol{\theta}_k|\Omega_i)P(\Omega_i) / N_i \tag{9}$$

Substituting Eqs. (8) and (9) into Eq. (7) gives the estimate of $\tilde{Y}_t$ based on $\boldsymbol{y}_j$ (see Fig. 1(e)):

$$\tilde{Y}_t = \sum_{i=0}^{m} E(\tilde{Y}_t | O_{j,i}) P(O_{j,i}) \approx \sum_{i=0}^{m} \sum_{k=1}^{N_{j,i}} M_t(\boldsymbol{\theta}_k | O_{j,i}) P(O_{j,i}) / N_{j,i} \quad (10)$$

where $M_t(\boldsymbol{\theta}_k|O_{j,i})$ (k = 1, 2, …, $N_{j,i}$) represents the model prediction at the tth monitoring moment given the kth samples in $O_{j,i}$, and $N_{j,i}$ is the sample size of $O_{j,i}$. Moreover, given $O_{j,i}$ (i = 0, 1, …, m), one can also obtain the posterior statistics of $\boldsymbol{\theta}$, for example, the posterior mean value of $\boldsymbol{\theta}$ considering $\boldsymbol{y}_j$, $\boldsymbol{\mu}_{\boldsymbol{\theta}}(\boldsymbol{y}_j)$ as:

$$\boldsymbol{\mu}_{\boldsymbol{\theta}}(\boldsymbol{y}_j) = \sum_{i=0}^{m} E(\boldsymbol{\theta}|O_{j,i}) P(O_{j,i}) \quad (11)$$

where $E(\boldsymbol{\theta}|O_{j,i})$ represents the mean values of $\boldsymbol{\theta}$ estimated based on the samples in $O_{j,i}$.

It is worthwhile to point out that the above process (see Eqs. (8) – (10) or Fig. 1(d) – (e)) are evaluated based on the same set of driving samples (see Fig. 1(c)) and the corresponding evaluated consolidation responses throughout all monitoring moments concerned. This indicates that evaluating Eq. (10), including determination of $M_t(\boldsymbol{\theta}_k|O_{j,i})$ and $O_{j,i}$, requires negligible computational efforts because there is no need to repeatedly evaluate the numerical model during the calculations and major computational efforts have been completed during the driving Bayesian analysis. This significantly improves the computational efficiency of Bayesian updating with sequentially obtained datasets at different monitoring moments, as demonstrated in the following.

4 Illustrative example

This section applies the proposed approach to updating the consolidation-induced responses (e.g., settlement and excess pore water pressure) under instantaneous loading condition (see Fig. 2). This study aims to predict the settlement and excess pore pressures at various locations considering measurements from various time instances, from which the ground settlement can be estimated with an analytical solution as (Knappett 2012):

$$s_t = m_v H \gamma_f H_f \left(1 - \sum_{q=0}^{\infty} \frac{2}{M^2} \exp(-M^2 T_v) \right) \quad (12)$$

where $M = \pi(2q+1)/2$; T_v is the dimensionless time factor calculated as $T_v = c_v T/H^2$, and T is the time; c_v is the consolidation coefficient; H is the thickness of the soil layer; m_v is the coefficient of volume compressibility; H_f and γ_f are the thickness and unit weight of the fill, respectively. Meanwhile, excess pore pressure at various depths z below the top of clay layer can be evaluated as (Knappett 2012):

$$u_{t,z} = \sum_{q=0}^{\infty} \frac{2\gamma_f H_f}{M} \left(\sin \frac{Mz}{H}\right) \exp(-M^2 T_v) \quad (13)$$

Four uncertain variables $\boldsymbol{\theta} = \{m_v, H, \gamma_f, c_v\}$ are considered, and prior statistics of these parameters are summarized in Table 1. Following Kelly and Huang (2015) and Rahimi et al. (2019), real monitoring data is synthesized with "true values" of $\boldsymbol{\theta}$ shown in the last column of Table 1, and measurement errors of settlement and pore pressures are normally distributed with constant standard deviations of 0.02 m and 1.0 kPa, respectively. Moreover, H_f = 3m and q = 9 are selected for calculating s_t and $u_{t,z}$, which are used as model predictions $M(\boldsymbol{\theta})$ for Bayesian updating, as illustrated below.

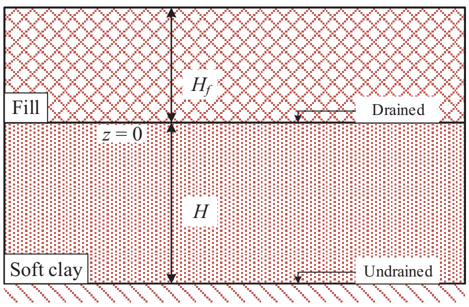

Figure 2. Geometry and boundary of one-dimensional consolidation example

Table 1. Prior statistics of uncertain variables for one-dimensional consolidation (Rahimi et al. 2019)

Soil parameters	Prior statistics			True value
	Distribution	Mean	COV	
m_v, 1/kPa	Lognormal	0.001	0.40	0.0014
H, m	Lognormal	5.0	0.10	5.50
γ_f, kN/m^3	Lognormal	20.0	0.10	22.0
c_v, m^2/year	Lognormal	40.0	1.00	80.0

4.1 Response updating considering settlement only

This subsection performs Bayesian updating considering settlement measurements only. For this purpose, driving dataset $\boldsymbol{y}^*$ is selected as the data from the first monitoring moment (i.e., $\boldsymbol{y}^* = \boldsymbol{y}_{s,1} = [y_{settle,1}]$), and driving Bayesian analysis is performed with p_0 = 0.1 and N = 10000. This generates 19000 driving samples falling in three subsets (i.e., Ω_0, Ω_1 and Ω_2 with 9000, 9000 and 1000 samples, respectively). Moreover, response predictions $M_j(\boldsymbol{\theta})$ corresponding to these 19000 samples are also calculated. As new dataset appears, driving samples and their response predictions (e.g., settlement) are used in target Bayesian analysis to facilitate the settlement updating.

Consider, for example, at monitoring moment $j = 2$, settlement measurements $\boldsymbol{y}_{s,2} = [y_{settle,1}, y_{settle,2}]$ are available. Then, the updated settlement $\tilde{Y}_t$ given $\boldsymbol{y}_{s,2}$ can be estimated according to Eq. (10), as shown by the line with crosses in Fig. 3 (a). For comparison, Fig. 3 (a) also displays the measurement and prior settlement prediction by circles and solid line, respectively. Prior prediction in this study is estimated as the mean value of simulated responses (e.g., settlement) calculated based on 1000 prior samples. It is found that the updated settlement $\tilde{Y}_t$ deviates from prior prediction and plots closer to measurements. This implies that the updated $\tilde{Y}_t$ is greatly updated by incorporating $\boldsymbol{y}_{s,2}$.

To validate the proposed approach, conventional Bayesian updating approach is conducted to populate posterior samples given a dataset (e.g., $\boldsymbol{y}_{s,2}$) and estimate the corresponding updated responses. Note that many datasets (e.g., those from different monitoring moments) are involved in this study. This necessitates repeated conventional Bayesian updating analyses, denoted as "RCBU" herein. For illustration, BUS with SuS is adopted, in which N = 10000 and $p_0 = 0.1$. Fig. 3 (a) gives the updated settlement from RCBU by squares as well. The updated settlements from the proposed approach and RCBU agree well with each other, which validates the proposed approach.

Under the proposed approach, one can also obtain the updated settlement considering other values of $\boldsymbol{y}_{s,j}$. For example, Fig. 3 (b) - (c) give the updated settlement considering $\boldsymbol{y}_{s,4}$ and $\boldsymbol{y}_{s,10}$, respectively. As observed from Fig. 3 (a) - (c), the updated settlement $\tilde{Y}_t$ (see line with crosses) gradually approaches measurements by incorporating more datasets, and $\tilde{Y}_t$ given $\boldsymbol{y}_{10}$ has an excellent agreement with measurements (see Fig. 3 (c)). This observation is further validated by the RCBU results shown by squares in Fig. 3.

It is worthwhile to emphasize that the updated settlements at different monitoring moments (e.g., j = 2, 4 and 10) in Fig. 3 are obtained based on the same driving samples. This indicates that no additional samples are simulated for settlement updating with new dataset and model evaluations are avoided as new dataset is available, which leads to significant computational savings. For example, calculations to obtain results in Fig. 3 can be finished within a second in a desktop computer. In contract, RCBU has to regenerate the posterior samples and reevaluate the model for predicting settlements. Therefore, the proposed approach achieves great computational efficiency for response updating considering sequentially acquired monitoring datasets.

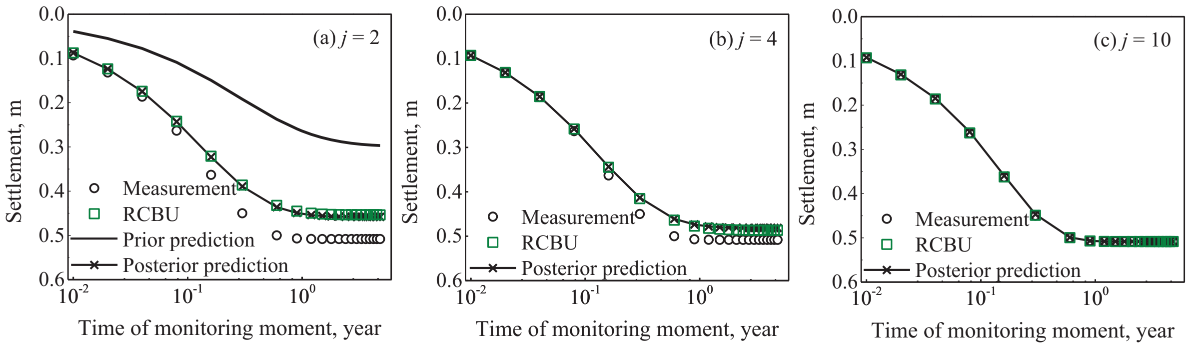

Figure 3. Updated settlements considering settlement measurements from different monitoring moments

4.2 Response updating considering settlement and pore pressure

This subsection further demonstrates the proposed approach by considering more types of measurements (e.g., settlement and excess pore water pressure) from different locations. Herein, settlement at z = 0 and excess pore pressures at z = 1 and 5 m are considered. Similarly, driving Bayesian analysis is first conducted under the proposed approach, in which N = 10000 and $p_0 = 0.1$, and driving data $\boldsymbol{y}^*$ is selected as $\boldsymbol{y}^* = \boldsymbol{y}_{sp,1} = [y_{settle,1}, y_{pore,1,z=1}, y_{pore,1,z=5}]$ from the first monitoring moment. After obtaining the driving samples and their corresponding simulated responses (e.g., settlement at z = 0 and pore pressures at z = 1 and 5 m), the updated settlement and excess pore pressures considering different $\boldsymbol{y}_j$ can be calculated with relative ease according to Eq. (10).

For example, Fig. 4 (a) - (c) depict the updated responses $\tilde{Y}_t$ considering $\boldsymbol{y}_{sp,2}$, $\boldsymbol{y}_{sp,4}$ and $\boldsymbol{y}_{sp,10}$ at j = 2, 4 and 10, respectively. As shown in Fig. 4 (a), the updated settlement given $\boldsymbol{y}_{sp,2}$ (see black line with crosses) is generally consistent with measurements (see black circles), which is more accurate compared with prior prediction (see black line). The same observation is obtained for excess pore pressures at z = 1 m and 5 m shown by green and blue colors, respectively. As more datasets are included in Bayesian updating, the updated responses converge to their respective measurements. For example, when $\boldsymbol{y}_{sp,4}$ is considered at j = 4, the updated settlement and excess pore pressures match well with their measurements (see Fig. 4 (b)).

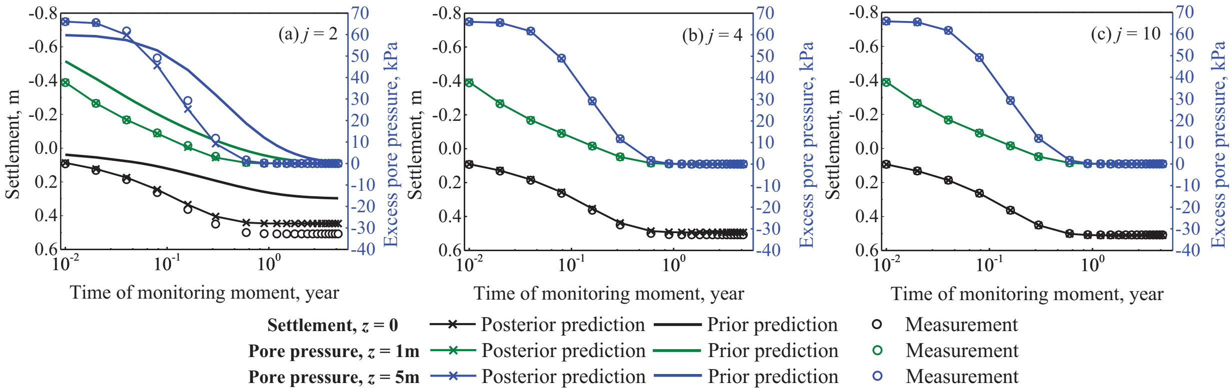

Figure 4. Updated responses considering measurements of various sources at different monitoring moments

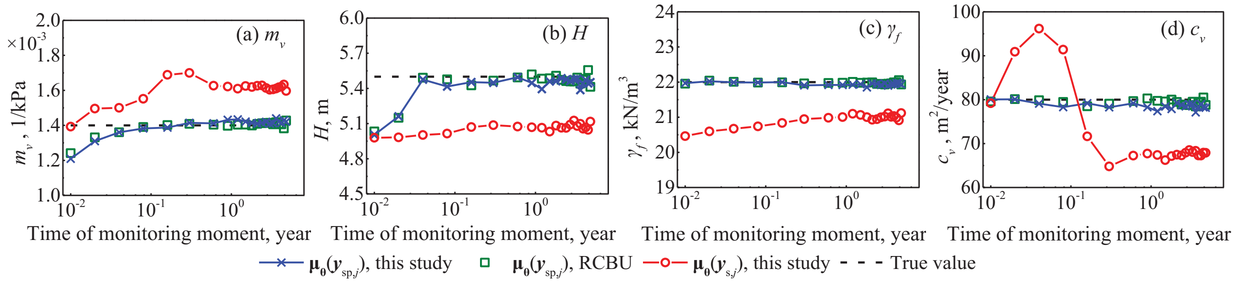

Figure 5. Mean value of posterior samples considering measurements from different sources

In addition to the updated responses, Fig. 5 (a) - (d) gives the mean posterior values $\boldsymbol{\mu}_{\boldsymbol{\theta}}(\boldsymbol{y}_{sp,j})$ of four parameters concerned (i.e., m_v, H, γ_f and c_v) according to Eq. (11) considering $\boldsymbol{y}_{sp,j}$ (i.e., settlement and pore pressures) throughout all monitoring moments, as denoted by line with crosses. For comparison, "true" values of these four parameters are given by dash lines. It is found that as j increases or the amount of incorporated dataset $\boldsymbol{y}_{sp,j}$ becomes greater, $\boldsymbol{\mu}_{\boldsymbol{\theta}}(\boldsymbol{y}_{sp,j})$ gradually converges to their true values for m_v, H, γ_f and c_v, as illustrated by Fig. 5 (a) - (d). This is further validated by the RCBU results shown by squares, which agrees well with those from the proposed approach. In addition, posterior mean values considering settlement measurements only, i.e., $\boldsymbol{\mu}_{\boldsymbol{\theta}}(\boldsymbol{y}_{s,j})$, are also plotted in Fig. 5 by line with circles. Obviously, $\boldsymbol{\mu}_{\boldsymbol{\theta}}(\boldsymbol{y}_{s,j})$ do not converge to their respective "true" values though the updated settlement has a good match with measurements (see Fig. 3 (c)). This demonstrates the advantageous performance by incorporating various types of measurements (e.g., settlement and excess pore pressure) from different locations. Nonetheless, the proposed approach provides an efficient vehicle for performing Bayesian updating considering measurements of different sources sequentially obtained at different monitoring moments.

5 Summary and conclusions

This paper proposed an auxiliary Bayesian approach for efficient updating of consolidation-induced responses. The proposed approach first performs driving Bayesian analysis using BUS and subset simulation (SuS) to generate diving samples and evaluate their corresponding responses based on the monitoring dataset obtained at early monitoring moments. Then, target Bayesian analysis is conducted as new dataset becomes available, which avoids regenerating new samples from posterior distribution given the new dataset and reevaluating the corresponding numerical model.

The proposed approach was illustrated using a consolidation example of clay. Results obtained from the proposed approach were compared and validated with those from repeated Bayesian updating analyses given different monitoring datasets. It was shown that the proposed approach requires only one simulation run for driving Bayesian analysis, and achieves great computational efficiency for target Bayesian analysis given a new dataset. Moreover, effects of monitoring datasets of different sources (e.g., settlement and/or pore pressures at various locations) on the updated response were also explored. It was demonstrated the value of considering various types of measurements from different locations. The proposed approach provides an efficient tool for Bayesian updating considering different monitoring datasets.

Acknowledgments

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References

Au, S. K. (2007). Augmenting approximate solutions for consistent reliability analysis. *Probabilistic Engineering Mechanics*, 22(1), 77-87.

Knappett, J. (2012). Craig's soil mechanics (Vol. 8). *London: spon press*.

Kelly, R. and Huang, J. (2015). Bayesian updating for one-dimensional consolidation measurements. *Canadian Geotechnical Journal*, 52(9), 1318-1330.

Kelly, R. B., Sloan, S. W., Pineda, J. A., Kouretzis, G., and Huang, J. (2018). Outcomes of the Newcastle symposium for the prediction of embankment behaviour on soft soil. *Computers and Geotechnics*, 93, 9-41.

Rahimi, M., Shafieezadeh, A., Wood, D., Kubatko, E. J., and Dormady, N. C. (2019). Bayesian calibration of multi-response systems via multivariate Kriging: Methodology and geological and geotechnical case studies. *Engineering Geology*, 260, 105248.

Straub, D. and Papaioannou, I. (2015). Bayesian updating with structural reliability methods. *Journal of Engineering Mechanics*, 141(3), 04014134.

Tian, H. M., Cao, Z. J., Li, D. Q., Du, W., and Zhang, F. P. (2022). Efficient and flexible Bayesian updating of embankment settlement on soft soils based on different monitoring datasets. *Acta Geotechnica*, 17(4), 1273-1294.