Proc. of the 8th International Symposium on Geotechnical Safety and Risk (ISGSR)
Edited by Jinsong Huang, D.V. Griffiths, Shui-Hua Jiang, Anna Giacomini and Richard Kelly

Published by Research Publishing, Singapore.
doi: 10.3850/978-981-18-5182-7_07-006-cd

ISGSR 2022

Monitoring Data-Driven Numerical Modeling of Slope Hydraulic Analysis Using Bayesian Updating with Structural Methods

Xin Liu[1] and Yu Wang[2]

[1]Department of Architecture and Civil Engineering, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, China.
E-mail: xliu268-c@my.cityu.edu.hk
[2]Department of Architecture and Civil Engineering, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, China.
E-mail: yuwang@cityu.edu.hk

Abstract: Predicting soil hydraulic responses of a slope under rainfalls is critical for predicting slope instability. This can be performed using a physics-based slope hydraulic model and soil hydraulic parameters. However, the numerical modeling of slope hydraulic analysis may be non-trivial for some reasons. For example, subsurface conditions of the slope are invisible and may vary with time; soil properties are spatially variable; and site investigation data are often quite limited in geotechnical practice. Therefore, a slope can be modelled by a series of candidate slope hydraulic models with different choices of governing equations, boundary conditions, and initial conditions. In addition, soil hydraulic parameters from site investigation at the slope could have large uncertainties. Monitoring data (e.g., rainfall and pore water pressure of soil under rainfalls) represent actual field responses of an existing slope subjected to rainfalls and can provide valuable information for the slope subsurface conditions. This study uses monitoring data from a real slope and Bayesian updating with structural reliability methods (BUS) to select the most suitable slope hydraulic model among a series of candidate models and identify the most appropriate soil hydraulic parameters. Results showed that the most suitable slope hydraulic model not only improves quantification of uncertainties in soil hydraulic parameters, but also accurately predicts soil hydraulic responses under future rainfalls. The proposed method enables a monitoring data-driven way for numerical modeling of slope hydraulic analysis.

Keywords: Bayesian methods; Monitoring data; Numerical modeling; Pore water pressure; Rainfall.

1 Introduction

Predicting soil hydraulic responses (e.g., pore water pressure, PWP) of a slope under rainfalls is critical for predicting slope instability, and thus landslide early warning (Lu and Godt, 2013). This can be performed using a physics-based slope hydraulic model and soil hydraulic parameters. However, the numerical modeling of slope hydraulic analysis may be non-trivial for some reasons. For example, subsurface conditions of the slope are invisible and may vary with time; soil properties are spatially variable; and site investigation data are often quite limited in geotechnical practice. Therefore, a slope can be modelled by a series of candidate slope hydraulic models with different choices of governing equations, boundary conditions, and initial conditions. In addition, soil hydraulic parameters from site investigation at the slope could have large uncertainties (Phoon et al., 2010). Monitoring data (e.g., rainfall and PWP of soil under rainfalls) represent actual field responses of an existing slope subjected to rainfalls and can provide valuable information for the slope subsurface conditions (Zhang et al., 2013).

This study presents a monitoring data-driven method (Liu and Wang, 2021) that utilizes existing monitoring data to select the most suitable slope hydraulic model among a series of candidate models and identify the most appropriate soil hydraulic parameters based on Bayesian updating with structural reliability methods (BUS) (Straub and Papaioannou, 2015). The proposed method is introduced in Section 2. Section 3 shows the application of the proposed method to a real slope with in situ monitoring data. Conclusions are drawn in Section 4.

2 Methodology

Figure 1 presents a framework of the proposed method (Liu and Wang, 2021). For a given slope with available monitoring data of rainfall and PWP, due to a lack of subsurface information and site investigation data in slope practice, candidate slope hydraulic models can be established by considering different model settings (e.g., governing equations, boundary conditions, and initial conditions). The soil hydraulic parameters are taken as uncertain parameters. Their prior distributions can be determined using existing database of soil hydraulic properties. Then, each candidate model is used to perform probabilistic back analysis using BUS to obtain the model evidence and the posterior distribution of uncertain parameters. Repeat this procedure for all candidate models. The most suitable slope hydraulic model and its parameters can be identified using Bayesian model comparison. The slope hydraulic model and parameters can be used to predict soil hydraulic responses under future rainfalls for landslide risk mitigation and early warning.

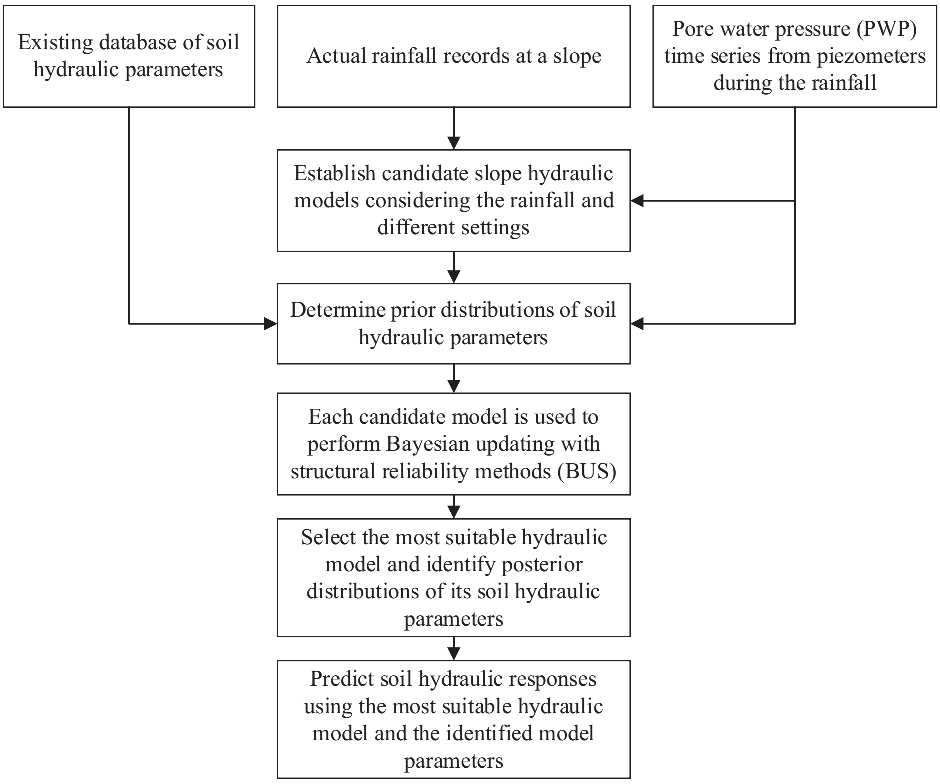


Figure 1. Framework of the proposed method (after Liu and Wang, 2021)

2.1 Slope hydraulic analysis under rainfalls

Physics-based slope hydraulic models are often used to predict soil hydraulic responses under rainfall infiltration, which is associated with the numerical modeling of seepage flow in saturated and/or unsaturated soils subjected to rainfall infiltration. The numerical modeling often includes three components, i.e., governing equations, boundary conditions, and initial conditions.

Table 1 shows various model choices regarding these three components. For example, Green-Ampt model and Richards equation are two popular governing equations. In addition, soil water characteristic curve (SWCC) and hydraulic conductivity function (HCF) are needed to describe behaviors of unsaturated soils. Specifically, SWCC describes the relationship between the matric suction and volumetric water content. HCF represents the variation of hydraulic conductivity with volumetric water content. Both SWCC and HCF are nonlinear, and they can be measured from laboratory tests or fitted by some empirical equations. For example, SWCC and HCF in the previous studies are often represented by van Genuchten's (1980) and Mualem's (1976) models, respectively.

There are also different choices for boundary conditions at the slope surface and bottom and initial conditions. For example, the slope may have a constant pressure or a free drainage boundary condition. Initial conditions of PWP and ground water table (GWT) can be hydrostatic, truncated hydrostatic, and constant. Detailed illustration of these initial conditions of PWP are referred to Liu and Wang (2021).

It is possible to develop more than a dozen of candidate slope hydraulic models with different model settings. Previous studies (Ali et al., 2014; Phoon et al., 2015) reported that these model settings are found important in controlling soil hydraulic responses, and soil hydraulic parameters may have large uncertainties. It is of interest to utilize monitoring data to choose the most suitable model and identify the most appropriate soil hydraulic parameters.

2.2 Bayesian updating with structural reliability methods (BUS)

The selection of the most suitable and identification of parameters can be formulated as a Bayesian updating problem. The problem may not be solved analytically due to nonlinear equations. The problem is often solved by generating random samples satisfy posterior distributions of uncertain parameters or posterior random samples using sampling-based methods, including Markov Chain Monte Carlo simulation (MCMCS) method (Press et al., 2007), ensemble Kalman filter method (Vardon et al., 2016), and BUS (Straub and Papaioannou, 2015; Betz et al., 2018). BUS has the advantage of combining with existing well-developed structural reliability methods, for example, subset simulation (SS) (Au and Beck, 2001) to take the advantage of SS in generating high-dimension random samples. The essence of BUS is to transform the Bayesian updating problem to a structural reliability problem, whose limit state function can be written as

$$g(\boldsymbol{\theta}, \pi) = \ln\pi + d - \ln L(\hat{\mathbf{y}} | \boldsymbol{\theta}, M_k) \tag{1}$$

where $\boldsymbol{\theta}$ is the uncertain parameters considered in the candidate model; ln(·) is the natural logarithm function; π is an additional uniform random variable in [0, 1], and it extends the dimension of parameter domain from $\boldsymbol{\theta}$ to [$\boldsymbol{\theta}$, π]; d is a positive constant that satisfies $d - \ln L(\hat{\mathbf{y}} | \boldsymbol{\theta}, M_k) \geq 0$, and d can be automatically determined by an adaptive strategy (Betz et al., 2018); and $L(\hat{\mathbf{y}} | \boldsymbol{\theta}, M_k)$ is the likelihood function of $\boldsymbol{\theta}$ and k-th candidate model M_k for monitoring data of PWP $\hat{\mathbf{y}}$.

The corresponding failure probability is given by

$$p_\Lambda = P\left[g(\boldsymbol{\theta}, \pi) \leq 0\right] \tag{2}$$

Let $\mathbf{y}$ denote the simulated responses from M_k. Because model assumptions and simplification, there are inevitable residuals between $\mathbf{y}$ and $\hat{\mathbf{y}}$ that may be modelled by random variables. The likelihood function can be written as

$$L(\hat{\mathbf{y}} | \boldsymbol{\theta}, M_k) = f_{\boldsymbol{\varepsilon}}(\mathbf{y} - \hat{\mathbf{y}}) \tag{3}$$

where $f_{\boldsymbol{\varepsilon}}(\cdot)$ denotes the probability density function (PDF) of $\mathbf{y} - \hat{\mathbf{y}}$, which may be represented by a zero-mean jointly independent Gaussian distribution.

According to Bayesian model selection method, the most suitable model is selected as the candidate model with the largest evidence, where evidence is a quantitative indicator that incorporates the prior distribution of $\boldsymbol{\theta}$, likelihood function, and monitoring data. Under the framework of BUS, the model evidence can be calculated by

$$e_k = p_\Lambda \cdot \exp(d) \tag{4}$$

where e_k denotes the evidence of the k-th candidate model.

Table 1. Summary of different settings for establishing candidate slope hydraulic models

Model setting	Category	Possible choices
Governing equations	Transient seepage flow with rainfall infiltration	1.Green-Ampt model (e.g., Chen and Young, 2006) 2. Analytical solution of Richards equation (e.g., Srivastava and Yeh, 1991) 3. Numerical solution of Richards equation (e.g., HYDRUS-1D software)
	Soil water characteristic curve (SWCC)	1. A simplified exponential model (e.g., Srivastava and Yeh, 1991) 2. Gardner model (Gardner, 1958) 3. van Genuchten model (van Genuchten, 1980) 4. Fredlund and Xing model (Fredlund and Xing, 1994)
	Hydraulic conductivity function (HCF)	1. A simplified exponential model (e.g., Srivastava and Yeh, 1991) 2. Gardner model (Gardner, 1958) 3. Mualem model (Mualem, 1976)
Boundary conditions	Surface boundary condition	1. Infiltration with ponding 2. Infiltration without ponding
	Boundary condition at slope bottom	1. Impermeable 2. Constant pressure head 3. Free drainage 4. Prescribed flux
Initial conditions	Initial profile of PWP and ground water table	1. Hydrostatic 2. Truncated hydrostatic 3. Constant

The abovementioned structural reliability problem can be easily solved using SS to obtain the model evidence. The failure samples that satisfy $g \leq 0$ are exactly posterior samples of concern. Detailed implementation procedure of BUS can be found in the previous studies (Straub and Papaioannou, 2015; Betz et al., 2018; Liu and Wang, 2021).

3 Application to a real slope example with monitoring data

The proposed method is applied to a real slope with in situ monitoring data. The slope is located in Tung Chung east, Lantau Island of Hong Kong. Preliminary investigation (Evans and Lam, 2003) showed that the slope may have a typical shallow failure induced by rainfall. A monitoring project was initiated by Geotechnical Engineering Office of Hong Kong to monitor rainfall, PWP, and displacement at the site. Figure 2 shows the monitoring data of PWP and the rainfall observed at the site from 8 – 15 June of 2001. The monitoring data of rainfall and PWP have an equal length of 192h and an equal interval of one hour. The monitoring data of PWP are measured from the piezometer SP3 that is buried 2m below the ground surface. Figure 2 suggests that rainfall infiltration controls the variation of PWP time series. Details of the monitoring project and data can be found in the published report (Evans and Lam, 2003).

The model is modelled a one-dimensional slope hydraulic model. The slope angle and depth are 40° and 5m, respectively. Due to a lack of subsurface information, three candidate slope hydraulic models are developed for probabilistic back analysis. Table 2 shows that the three models are different in initial conditions of PWP, bottom boundary condition, SWCC, and HCF. Models 1 and 2 are developed using the open-source software HYDRUS-1D, while Model 3 is solved by the analytical solution (Srivastava and Yeh, 1991; Zhang et al., 2013) for comparison. The uncertain parameters considered in the Models 1 and 2 include saturated hydraulic conductivity K_s, saturated volumetric water content θ_s, SWCC parameter α, SWCC parameter n, the depth of initial ground water table d_w, and parameters for defining the matric suction in the shallow soil (Liu and Wang, 2021). Prior distributions of the soil parameters are taken as the those of loam reported in Carsel and Parrish (1988) based on the soil texture. In addition, the slope is divided into 50 parallel soil layers with an equal thickness to model the spatial variability of K_s using a random field. The uncertain parameters considered in Model 3 and their prior distributions are taken from Zhang et al. (2013). The residuals of PWP are assumed to follow an independent Gaussian distribution with zero mean and an equal standard deviation (SD) of 0.05m.

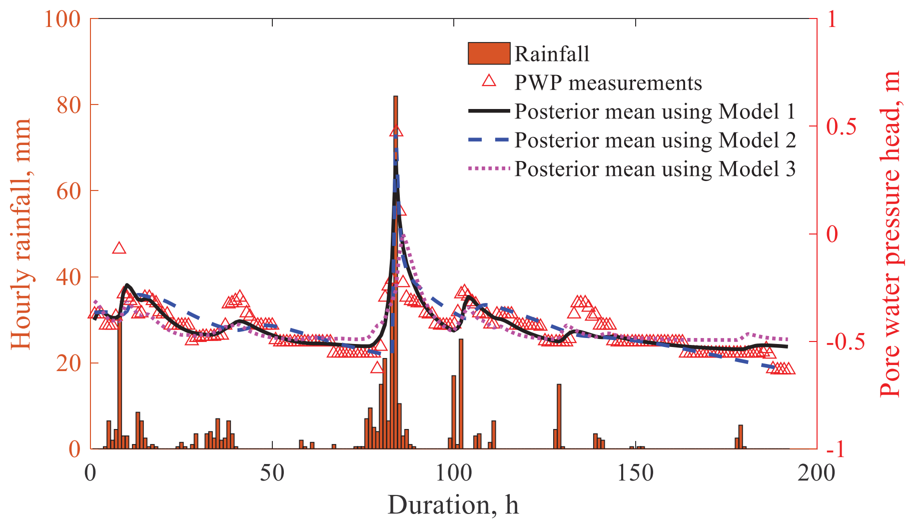


Figure 2. Monitoring data and posterior mean of PWP during a real rainfall

Table 2. Comparison of candidate slope hydraulic models

Candidate model	Initial condition of PWP	Bottom boundary condition	SWCC	HCF	Maximum logarithmic likelihood	Model evidence
Model 1*	Truncated hydrostatic	Constant pressure	van Genuchten's	Mualem's	237.4	8.91×10^{92}
Model 2	Constant	Free drainage	van Genuchten's	Mualem's	214.7	5.39×10^{79}
Model 3	Steady	Constant pressure	Exponential	Exponential	185.6	1.81×10^{70}

*The most suitable slope hydraulic model selected by the proposed method.

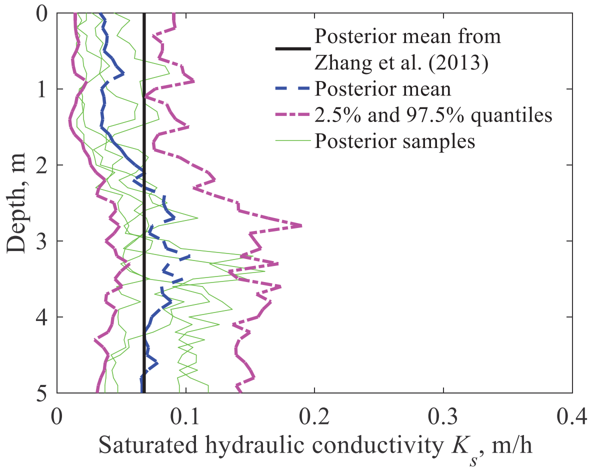
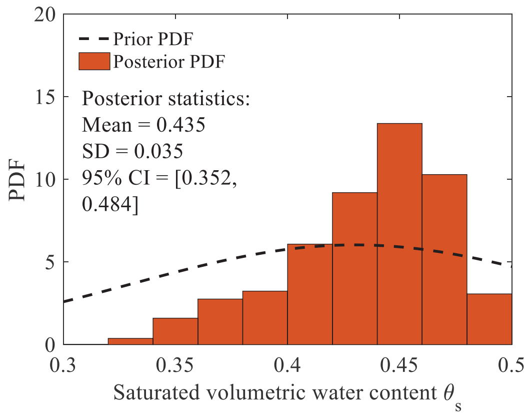
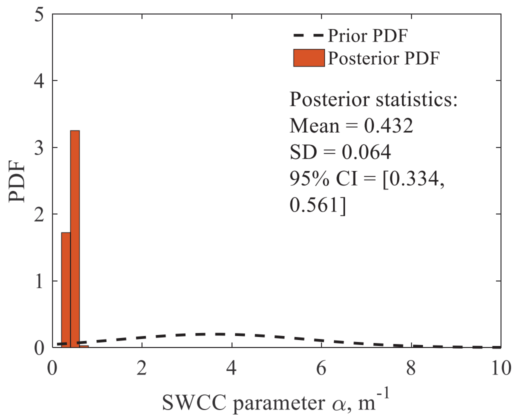


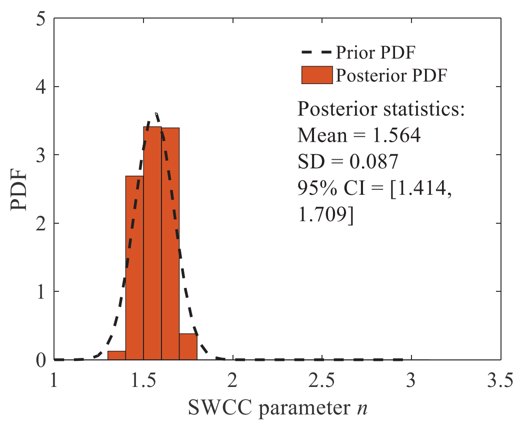
Figure 3. Posterior distribution of spatially varying saturated hydraulic conductivity



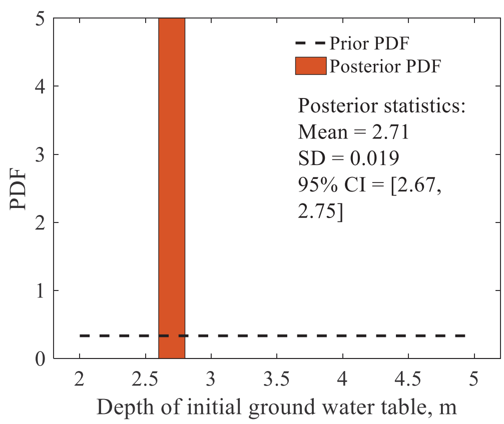
(a) Saturated volumetric water content θ_s



(b) SWCC parameter α



(c) SWCC parameter n



(d) Depth of Initial GWT d_w

Figure 4. Posterior distributions of uncertain soil hydraulic parameters

Then, the three candidate models were respectively used to perform probability back analysis using BUS. The number of samples generated in each simulation level of SS was taken as 10,000. The number of posterior samples was taken as 50,000. Table 2 provides their maximum likelihood and model evidence. Model 1 has the largest evidence of 8.91×10^{92} among the three candidate models, and it is selected as the most suitable model given the monitoring data. Figure 2 shows the posterior mean of PWP obtained from three models. All the three models seem to produce PWP responses comparable to the monitoring data, but Model 1 is the most accurate. This suggests that BUS enables a monitoring data-driven way for numerical modeling of slope hydraulic analysis.

Posterior results obtained from only Model 1 are presented in the following. Figure 3 illustrates the posterior results of K_s along the depth. K_s slightly increses with the depth. The mean of K_s is consistent with that obtianed from Zhang et al. (2013), in which homogenous K_s is assumed and represented by a random variable. Figure 4 (a-d) show the prior and posterior distributions of θ_s, α, n, and d_w, respectively. As shown by Figure 4(b), the prior distribution of α is very flat, but the posteiror distribution becomes highly concentrated between 0

– 1. The SD of α decreases from 2.1 to 0.064, indicating a significant reduction in coefficient of variation from 0.58 to 0.15. Similar phenomena can also be observed from other subfigues of Figure 4. This indicates that probabilistic back analysis with monitoring data effectively reduces the uncertainties of uncertain soil hydraulic parameters, leading to a more accurate and less uncertain numeircal model for slope hydraulic analysis.

In addition, Figure 5 shows PWP responses of a future rainfall from 22 – 30 June of 2001 predicted using three candidate models with the corresponding posterior mean parameters. Among the three candiate models, the PWP responses obtained from Model 1, i.e., the most suitable model, are the most consistent with measurements. This implies that the most suitable model can outperform other models in predicting soil hydraulic responses under future rainfalls.

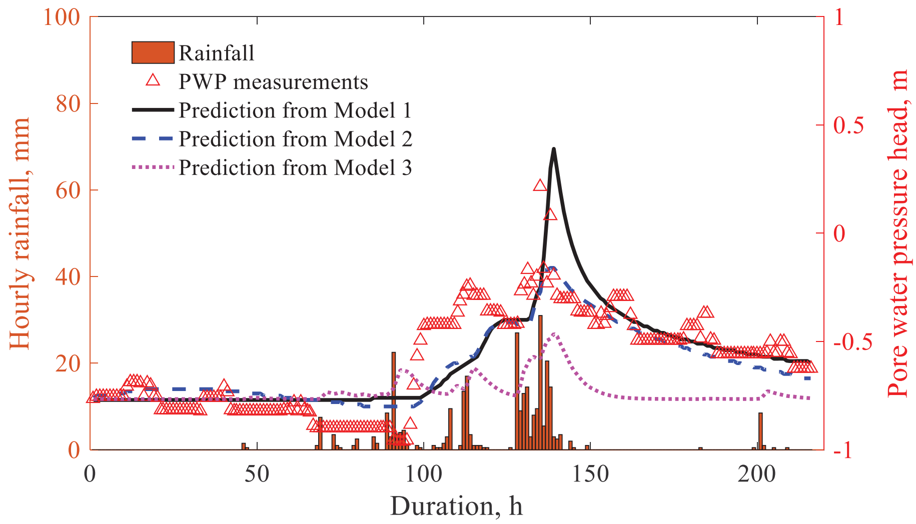


Figure 5. PWP responses of a future rainfall predicted using three candidate models

4 Conclusions

This study developed a probabilistic back analysis method based on BUS that utilizes existing monitoring data to select the most suitable numerical model and identify parameters for slope hydraulic analysis. The proposed method was illustrated by a real slope with in situ monitoring data of rainfall and PWP. Results showed that the most suitable model can produce responses consistent with the monitoring data. The monitoring data can be used to determine model settings, including governing equation, boundary condition, and initial condition. The most suitable slope hydraulic model not only improves quantification of uncertainties in soil hydraulic parameters, but also accurately predicts soil hydraulic responses under future rainfalls The proposed method enables a monitoring data-driven way for numerical modeling of slope hydraulic analysis. It may be extended to two-dimensional (2D) and three-dimensional (3D) slopes for future studies.

Acknowledgments

The work described in this paper was supported by a grant from the Research Grant Council of Hong Kong Special Administrative Region, China (Project no. C6006-20G) and a grant from City University of Hong Kong (Project no. 7005551). The financial support is gratefully acknowledged.

References

Ali, A., Huang, J., Lyamin, A. V., Sloan, S. W., and Cassidy, M. J. (2014). Boundary effects of rainfall-induced landslides. *Computers and Geotechnics*, 61, 341–354.

Au, S.K., and Beck, J. L. (2001). Estimation of small failure probabilities in high dimensions by subset simulation. *Probabilistic Engineering Mechanics*, 16(4), 263–277.

Betz, W., Papaioannou, I., Beck, J. L., and Straub, D. (2018). Bayesian inference with Subset Simulation: Strategies and improvements. *Computer Methods in Applied Mechanics and Engineering*, 331, 72–93.

Carsel, R. F., and Parrish, R. S. (1988). Developing joint probability distributions of soil water retention characteristics. *Water Resources Research*, 24(5), 755–769.

Chen, L., and Young, M. H. (2006). Green-Ampt infiltration model for sloping surfaces. *Water Resources Research*, 42(7), 1–9.

Evans, N. C., and Lam, J. S. (2003). Tung Chung East Natural Terrain Study Area Ground Movement and Groundwater Monitoring Equipment and Preliminary Results *(GEO Report No. 142)*. Hong Kong, China.

Fredlund, D. G., and Xing, A. (1994). Equations for the soil-water characteristic curve. *Canadian Geotechnical Journal*, 31(4), 521–532.

Gardner, W. R. (1958). Some steady-state solutions of the unsaturated moisture flow equation with application to evaporation from a water table. *Soil Science*, 85(4), 228–232.

Liu, X., and Wang, Y. (2021). Bayesian selection of slope hydraulic model and identification of model parameters using monitoring data and subset simulation. *Computers and Geotechnics*, 139, 104428.

Lu, N., and Godt, J. (2013). Hillslope Hydrology and Stability. *Cambridge University Press, Cambridge, UK*.

Mualem, Y. (1976). A new model for predicting the hydraulic conductivity of unsaturated porous media. *Water Resources Research*, 12(3), 513–522.

Phoon, K.K., Santoso, A., and Quek, S.T. (2010). Probabilistic Analysis of Soil-Water Characteristic Curves. *Journal of Geotechnical and Geoenvironmental Engineering*, 136(3), 445–455.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P. (2007). Numerical Recipes: The Art of Scientific Computing (3rd ed.). *Cambridge University Press, Cambridge, UK.*

Srivastava, R., and Yeh, T.C. J. (1991). Analytical solutions for one-dimensional, transient infiltration toward the water table in homogeneous and layered soils. *Water Resources Research*, 27(5), 753–762.

Straub, D., and Papaioannou, I. (2015). Bayesian updating with structural reliability methods. *Journal of Engineering Mechanics*, 141(3), 04014134.

van Genuchten, M. T. (1980). A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal*, 44(5), 892.

Vardon, P. J., Liu, K., and Hicks, M. A. (2016). Reduction of slope stability uncertainty based on hydraulic measurement via inverse analysis. *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, 10(3), 223–240.

Zhang, L. L., Zuo, Z. B., Ye, G. L., Jeng, D. S., and Wang, J. H. (2013). Probabilistic parameter estimation and predictive uncertainty based on field measurements for unsaturated soil slope. *Computers and Geotechnics*, 48, 72–81.