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Reliability Analysis of Pile Considering Spatial Variability of Soil Properties

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Abstract: To investigate the influence of spatial variability on the bearing characteristics and reliability of pile under vertical compression, a numerical model of pile in spatially various soil is developed. The KL-FORM is employed to analyze the reliability of pile for the ultimate limit state of bearing capacity. The influence of spatial variability of soil's undrained shear strength and elastic modulus on the reliability index of the pile is analyzed. Moreover, the influence of spatial variability of soil properties on the pile-soil interface parameters are investigated. Results show that the cohesive strength and the elastic modulus of pile-soil interface should be considered as variables which vary with the discrete values of random fields of soil properties. The spatial variability of elastic modulus influences the shape of load-displacement curve, but it does not influence the bearing capacity and the reliability index of the pile. The reliability index of the pile decreases with increasing coefficient of variation of undrained shear strength and vertical autocorrelation distance.

Keyword: Pile; Spatial variability; Random field; Karhunen-Loève expansion; Reliability index

1 Introduction

In recent years, the effects of spatial variability of soil properties on the reliability of piles are getting more and more attention. Generally, spatial variability can be described by the random field which is composed of infinite correlated random variables. To consider the spatial variability of soil properties, discreting a continuous random field into a finite number of random variables is required. The random field discretization methods include midpoint method (Der Kiureghian and Ke 1988), local average subdivision method (Fenton and Vanmarcke 1990), Karhunen-Loève series expansion (KL expansion) (Phoon et al. 2002), etc. Currently, random field theory and Monte Carlo simulation (MCS) is generally employed to perform reliability analysis considering spatial variability, in which the computational efficiency is a significant challenge. Fei et al. (2021) and Tan et al. (2020) proposed a KL-FORM algorithm to improve the computational efficiency. In the proposed KL-FORM, the KL expansion is used for discretizing random fields, and first-order reliability method (FORM) is employed for reliability analysis. The KL FORM provides high computational efficiency and accuracy.

Pile is commonly buried in different soil layers, and soil properties along the pile depth have a large impact on the side and tip resistances of the pile. Currently, the effects of spatial variability of soil properties on pile reliability is extensively studied. Haldar and Babu(2008a) investigated the impact of spatial variability of undrained shear strength S_u on pile reliability for both the ultimate limit state (ULS) and the service limit state (SLS), and concluded that reasonable design loads can be obtained by considering the impact of soil spatial variability on the pile. Leung et al. (2018) developed a probabilistic analysis method to consider large pile groups and pile rafts on three-dimensional spatially variable soil, revealing the importance of the interaction between the superstructure and soil's spatial variability. Naghibi et al. (2014 and 2016) examined the influence of spatial variability of soil's elastic modulus on the failure probability of a vertical compressive pile. Hence, it is necessary to consider soil's spatial variability especially along the pile depth for the bearing characteristics and reliability analysis of vertical compressive pile.

In this study, the KL-FORM is employed to discretize random fields and to calculate reliability of pile. Meanwhile, the influence of spatial variability of the undrained shear strength S_u and elastic modulus E of soil on the bearing characteristics and reliability of pile are discussed.

2 Random Field Realization of Soil Properties and Reliability Calculation Method

2.1 Spatial variability of soil property

Spatial variability is an inherent uncertainty of soil properties. Spatial variability is also known as spatial autocorrelation, which means that soil property at one point is correlated to those at nearby points. The spatial variability of soil property can be indicated by the autocorrelation function (ACF) (Vanmarcke 1977). In this study, squared exponential (SQX) model is adopt to represent the ACF of soil properties:

$$\rho(\boldsymbol{x}_1, \boldsymbol{x}_2) = \exp\left[-((x_{11} - x_{21})/L_h)^2 - ((x_{12} - x_{22})/L_v)^2\right] \tag{1}$$

where $\rho(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the ACF; x_{i1} and x_{i2} are the coordinate components of points $\boldsymbol{x}_i$ along the horizontal and vertical directions, respectively; and L_h and L_v are autocorrelation distances (ACDs) along the horizontal and vertical directions, respectively (Srivastava and Babu 2011).

2.2 First-order reliability analysis method based on KL expansion (KL-FORM)

2.2.1 KL expansion

The KL expansion can be used for simulation of both homogeneous and non-homogeneous random field. For a log-normal (LN) random field, the discrete result for the original random field $H(\boldsymbol{x})$ is as follows (Phoon and Kulhuwy 1999):

$$\hat{H}(\boldsymbol{x}) = \exp\left(\mu_{\ln X_i} + \sigma_{\ln X_i} \sum_{i=1}^{M} \sqrt{\lambda_i} f_i(\boldsymbol{x}) \xi_i\right) \tag{2}$$

where $\hat{H}(\boldsymbol{x})$ is the final discrete result at coordinate $\boldsymbol{x}$; $\mu_{\ln Xi}$ and $\sigma_{\ln Xi}$ are the mean and standard deviation of the log-normal random field; M is the number of series expansion terms; λ_i and $f_i(\boldsymbol{x})$ are the eigenvalues and eigenfunctions, respectively; ξ_i are independent standard Gaussian random variables in the Gaussian random field (i = 1, 2, …, M). The λ_i and $f_i(\boldsymbol{x})$ of the ACF are solutions of the Fredholm integral equation of the second kind (Fei et al. 2021).

2.2.2 Determination of number of KL expansion

The number of series expansion terms M in Eq. (2) should theoretically be infinite. To reduce the computation effort of random field discretization and reliability analysis, M is generally assumed to be a small value. As suggested by Sudret and Kiureghian (2000), the mean value of point-wise discrete errors can be used to indicate the global accuracy of the discretization. The formula for the mean discrete error ($\bar{\varepsilon}$) is as follows:

$$\bar{\varepsilon} = \frac{1}{|\Omega_v|} \int_{\Omega_v} \varepsilon(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \approx \frac{1}{|\Omega_v|} \sum_{j=1}^{n_e} \left| 1 - \sum_{i=1}^{M} \lambda_i f_i^2(\boldsymbol{x}) \right| S_j \tag{3}$$

where $|\Omega_v| = \int_{\Omega_v} \mathrm{d}\boldsymbol{x}$, $|\Omega_v|$ is the spatial domain corresponding to the valid random field; S_j is the area of the random field grid (j = 1, 2, …, n_e; and n_e is the number of discrete grids).

The value of M determines the discrete accuracy of a random field, the calculation accuracy of the reliability index, and the computational cost of the reliability analysis. Therefore, it is important to determine the optimal number of series expansion terms M_0. It is obvious from Eq. (3) that $\bar{\varepsilon}$ decreases with the increase of M. In this paper, the M corresponding to $\bar{\varepsilon}$=10% is considered as M_0.

2.2.3 Main steps of the KL-FORM

For specific projects, the mean, standard deviation, eigenvalues and eigenfunctions of a random field are constants. In Eq. (2), the variables are only the random vector matrix $\boldsymbol{\xi}$ in the Gaussian random field of size M. Thus, these random variables are considered as the basic variables in reliability analysis, and FORM is employed for reliability analysis. The basic variables in reliability analysis are $\boldsymbol{X}$ = [X_1, X_2, …, X_n], and X_i = ξ_i (i = 1, 2, …, n; and n = M_0). Reliability index solution of KL-FORM is the same as the traditional FORM. The solution process is described in detail in Fei et al. (2021) and Tan et al. (2020).

3 Numerical Simulation of Pile's Bearing Characteristics Under Vertical Load

3.1 Numerical model

A pile of length L_P =10 m, diameter D_P =1.0 m is located in undrained clay. A constant load of P = 387.6 kN is acting at the top of the pile (Figure 1). The material parameters of soil and pile are shown in Table 1, where k_{cs} and k_{cb} of pile-soil interface are related to S_u based on Eq. (4) (Haldar and Babu 2008b):

$$k_{cs} = \pi \cdot D_P \cdot S_u, \quad k_{cb} = 9 \cdot D_P \cdot S_u \tag{4}$$

The undrained shear strength S_u and the elastic modulus E of soil are lognormal random fields, whose mean values are listed in Table 1. The *COV*s of S_u and E are both 0.5, and the L_v is 1.0 m (the baseline condition). Because the pile length is much longer than the pile diameter, the influence of soil's vertical spatial variability is much greater than that of soil's horizontal spatial variability. Therefore, a large value of L_h = 4500 m is assumed for the soil, which represents that the horizontal spatial variability is not considered. To investigate the influence of soil's variability on pile's reliability, six values of L_v (1.0, 1.5, 2.0, 2.5, 3.0 and 5.0 m) and three values of *COV* (0.2, 0.3 and 0.5) are assumed. These values are within the general range of ACDs and *COV*s reported in Phoon and Kulhuwy (1999).

According to the results of Haldar and Babu (2012), a model of width $L_x = 20$ m and height $L_y = 2L_P = 20$ m is considered. The FLAC software was applied to simulate the bearing characteristics of the pile under vertical compression load. The pile is simulated by pile element, which is discretized into 21 equal segments. The soil was discretized into 900 grids. At the bottom plane of the grid, all movements were restrained. The lateral sides of the grids were free to move in *Y*-direction but not in the *X*-direction. A FISH function was written for automatically performing the numerical simulation. The compression and displacement process of the pile under vertical load is simulated by applying a gradually increasing displacement downward at the top of the pile. Based on the numerical model, the relationship between the load (Q) and displacement (s) at the top of the pile can be obtained.

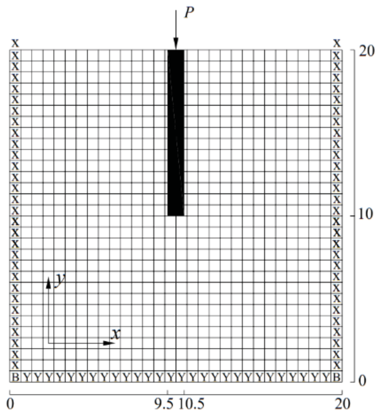


Figure 1. Numerical model.

Table 1. Material parameters of soil and pile.

Material	Parameters	Value
Soil	Undrained shear strength of soil, S_u (kPa)	20.0
	Unit weight of soil, γ (kN/m³)	18.0
	Elastic modulus of soil, E (kPa)	3600.0
	Poisson of soil, v	0.4
Pile	Pile elastic modulus, E_P (kPa)	2.24×10^7
Pile-Soil interface	Interface spring shear stiffness, k_n and k_s (kN/m/m)	1.15×10^5
	Stiffness of the end bearing spring, k_b (kN/m/m)	7.76×10^7
	Cohesive strength of shear coupling spring, k_{cs}	62.8
	Limiting strength of end bearing spring, k_{cb} (kN/m)	180.0

Note that the pile-soil interface parameters depend on the values of soil properties. Hence, the variation of soil properties in the vertical direction will lead to the variation of pile-soil interface parameters. The influence of pile-soil interface parameters can be investigated by the Q–s relationship of the pile. In Figure 3, three groups of typical Q–s relationship corresponding three realizations of random field S_u are shown. The solid curves (denoted as C-RF) represents that the pile-soil interface parameters (k_{cs} and k_{cb}) vary with S_u based on Eq. (4), and the dashed curves (denoted as N-RF) represents k_{cs} and k_{cb} are deemed as constant whose values are listed in Table 1. For each Q–s curve, the load corresponding to the inflection point of the curve can be used to estimate pile's bearing capacity. As shown in Figure 3, the differences between the Q–s curves of the C-RF and the N-RF are apparent. Consequently, the differences of bearing capacities estimated from the simulated Q–s curves of the C-RF and the N-RF are also apparent. This indicates that the pile-soil interface parameters of k_{cs} and k_{cb} should vary with the variation of S_u.

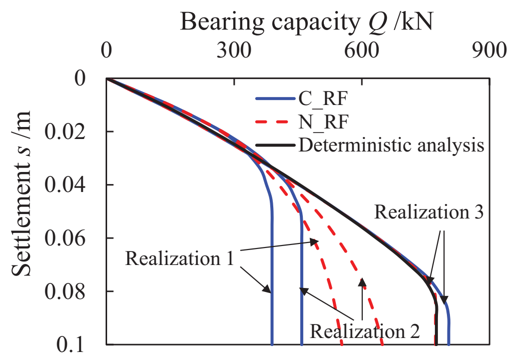


Figure 3. Q-s curves for three realizations.

Table 2. Comparison of bearing capacities.

Results	Q_u (kN)	Q_{us} (kN)	Q_{ub} (kN)
This Paper	775.25	632.46	142.79
Poulos and Davis (1980)	769.62	628.32	141.37
Haldar and Babu (2012)	763.00	620.00	143.00

3.2 Validation of the numerical model

To verify the correctness of the numerical model, the numerical simulations are compared with the analytical calculations (Poulos and Davis 1980) and the numerical simulations of Haldar and Babu (2012) in Table 2, where Q_u is the ultimate bearing capacity of pile; Q_{us} is the ultimate side resistance; and Q_{ub} is the ultimate tip resistance. The Q_u, Q_{us}, and Q_{ub} calculated in this paper are very close to those of Poulos and Davis (1980) and Haldar and Babu (2012). The results confirm the correctness of the numerical model.

4 Reliability of Pile in Spatially Variable Soil

Taking the ULS as an example, the performance function (Z) of the pile is usually expressed as follows：

$$Z = g(\boldsymbol{X}) = Q_u(\boldsymbol{X}) - P \tag{5}$$

where $Q_u(\boldsymbol{X})$ is the ultimate bearing capacity of the pile, which is a function of basic variable $\boldsymbol{X}$. As descripted in Section 2.2.3, the independent standard Gaussian random variables ξ_i in Eq. (2) of the KL expansion of random fields S_u and E are deemed as the basic variables in the FORM algorithm.

For the ULS of a pile, the main influence factor of pile's reliability is the shear strength parameter of S_u. It is reported that there is a linear relationship between S_u and E (Hu and Randolph 1998; Li et al. 2017). As shown in Table 1, $E = 180\ S_u$. Hence, to investigate the effects of the spatial variability of E and S_u on pile's bearing capacity, three types of reliability analyses are performed: 1) Only the spatial variability of S_u is considered and E is constant; 2) Both the spatial variabilities of S_u and E are considered with the relationship of $E = 180\ S_u$; 3) Both the spatial variability of S_u and E are considered, but S_u and E are independent.

4.1 Optimal number of KL expansion terms

After discretizing random fields using KL expansion, the $\bar{\varepsilon}$–M curve can be obtained. As shown in Figure 4, the discrete error $\bar{\varepsilon}$ decreases with the increase of M and L_v. The optimal number of KL expansion terms (M_0) determined by the cross-points of $\bar{\varepsilon}$-M curves and the horizontal line of $\bar{\varepsilon}$ = 10% are 15, 10, 7, 6, 5 and 3. These values of M_0 are employed in the following reliability analysis.

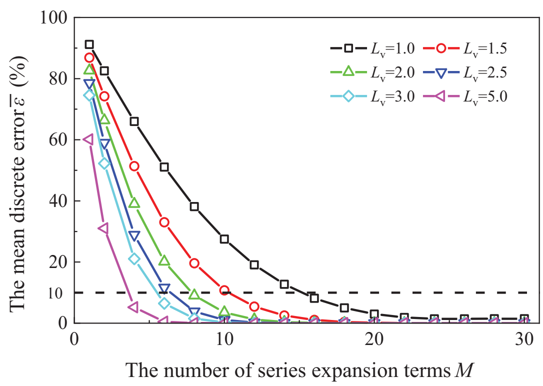


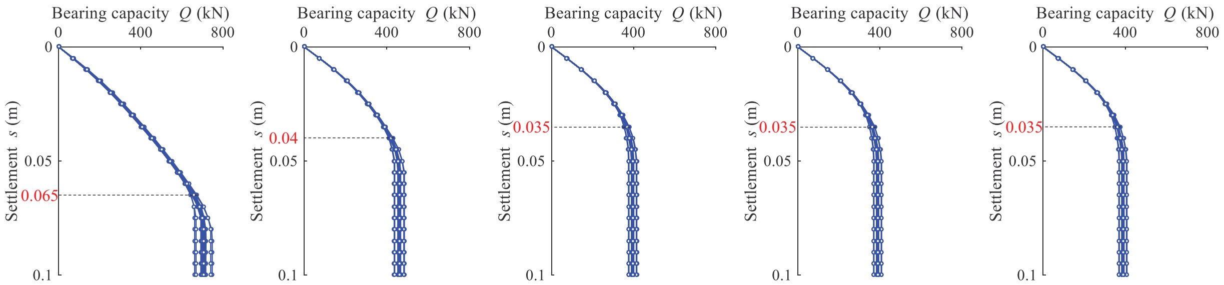
Figure 4. $\bar{\varepsilon}$–M curves with different ACDs.

4.2 Iterative computation of reliability index

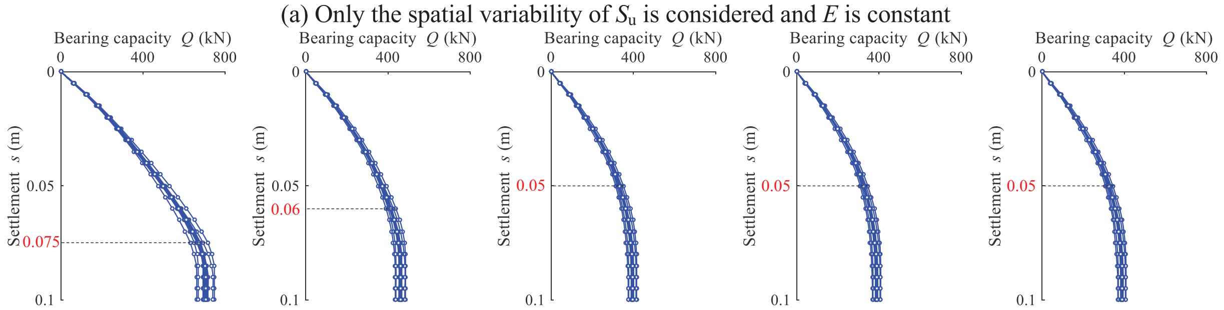
Computation of reliability index in the KL-FORM requires iterative computations of reliability index (Fei et al. 2021; Tan et al. 2020). Under the baseline condition of L_v = 1.0 m and COV = 0.5, the computations show that five iterations are required to obtain the reliability index of the pile. For each iterative computation of reliability index, a total of $N_s = 2M_0+1 = 2\times15 + 1 = 31$ sample points are required and 31 Q–s curves are produced. Figures 5 (a), (b), and (c) shows the Q–s curves produces in each iterative process for the three computational cases. For each computation case, the Q–s curves converge gradually with the increase of the number of iterations, which proves the convergence and correctness of the KL-FORM.

Comparisons of Figures 5(a), (b) and (c) shows that bearing capacities of the three computation cases are close to each other, which means that whether considering the spatial variability of E does not influence the bearing capacity of the pile. And consequently, whether considering the spatial variability of E does not influence the reliability for the ULS. For example, the reliability indexes of the pile for the ULS under the three computation cases are $\beta_1 = 3.75$, $\beta_2 = 3.77$, and $\beta_3 = 3.75$, respectively. The maximum difference of the three reliability indexes is only 0.02, which is very small and can be neglected.

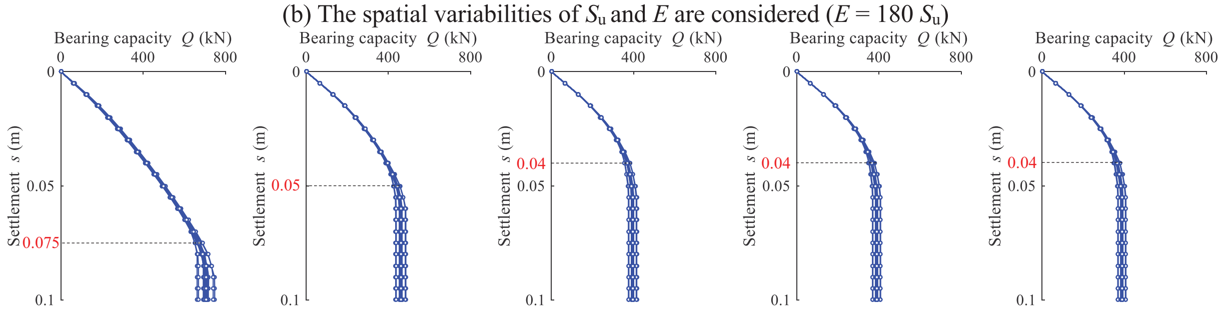
Although the bearing capacities of the three computation cases are close to each other, the shapes of Q–s curves of the three computation cases are different. The Q–s curves in Figure 5 (a) is steeper than those in Figure 5(b); and although the shapes of Q–s curves in Figures 5 (a) and (c) are similar, the displacements of the inflection points in Figure 5(c) are relatively larger than the corresponding points in Figure 5(a). The reason for the differences of the Q–s curves of the three computation cases is that E is a deformation parameter and it mainly affects the deformation characteristics of the soil rather than the strength characteristics. And furthermore, because S_u and E are correlated, not considering the correlation between S_u and E will under-estimate the displacement at the inflection points of the Q–s curves, which will further influence the reliability index for the SLS of pile.



(a) Only the spatial variability of S_u is considered and E is constant



(b) The spatial variabilities of S_u and E are considered (E = 180 S_u)



(c) The spatial variabilities of S_u and E are considered independently

Figure 5. *Q*–*s* curves for each iteration of reliability index (Left to right are the results of the 1st to 5th iterations).

4.3 Influence of ACD and *COV* on the reliability analysis

Benefit from the high computational efficiency of the KL-FORM, reliability indexes of the pile with different ACDs and *COV*s of soil can be evaluated. Figure 6 shows the reliability index decreases with the increase of vertical ACDs and *COV*s. Assuming a large value of *COV* or not considering the spatial variability of soil parameter (i.e., assuming the vertical ACD is infinite) will under-estimate the reliability index of the pile, which will lead to conservative design of piles. Reasonable determination of the ACD and *COV* of soil properties is very important in geotechnical engineering.

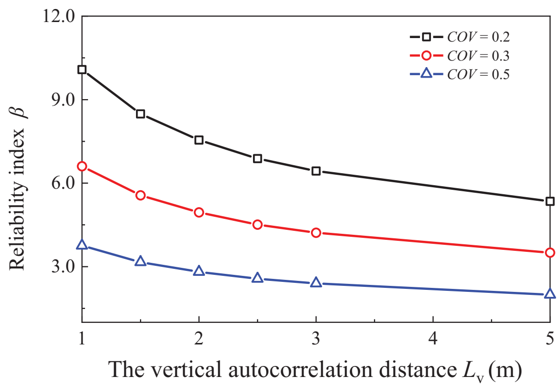


Figure 6. β–L_v curves with different *COV*s.

5 Conclusions

A numerical model for the bearing capacity of vertical compressive pile embedded in spatially various soil was developed, and the KL-FORM was employed to analyze the reliability of the pile for the ULS. The reliability index and its influences were investigated. It is found that the pile-soil interface parameters should vary with the variation of soil properties. The spatial variation of soil's elastic modulus affects the shapes of *Q–s* curves but it has minor influence on pile's bearing capacity. The reliability indexes of the pile for the ULS decrease with the increase of ACD and *COV* of soil parameters. Not considering the spatial variability will lead to conservative design of pile.

Acknowledgments

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