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Equivalence between Safety Factor-Based and Reliability-Based Design Requirements for Gravity Retaining Wall Design

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Abstract: Traditional deterministic design approaches, e.g., safety factor-based design (SFBD), were prevailing in retaining wall design practice due to its simplicity. With SFBD, the safety factors (*FS*) is taken as the measure of safety margin that does not explicitly consider uncertainties. However, there are inevitably many uncertainties in the process of retaining wall, such as inherent variability, model uncertainty, etc. The reliability level of feasible designs with the same *FS* values can vary significantly using SFBD. As a result, SFBD results might be unsatisfactory from a perspective of reliability-based design (RBD). RBD can reasonably consider various uncertainties, for which the probability of failure (P_f) is taken as the measure of safety margin. Although a number of RBD codes of retaining wall design have been developed, the reliability level corresponding to the allowable factor of safety (FS_a) used in SFBD is unknown, and the final design obtained using RBD may be significantly different from that obtained using SFBD. This poses a question that whether the equivalence between SFBD and RBD can be achieved for retaining wall design. If that is the case, RBD gives the feasible design domain similar to that obtained using SFBD. In general, the equivalence between SFBD and RBD does not always exist. This study discussed sufficient conditions for the equivalence between SFBD and RBD and their application to retaining wall design. When the sufficient conditions are satisfied, the target probability of failure (P_T) can be calibrated from FS_a adopted in SFBD. The proposed approach is illustrated using a gravity retaining wall design example. Results show that the gravity retaining wall design example satisfies the sufficient conditions, and the feasible design domain of RBD based on the calibrated P_T is consistent with that of the SFBD based on the FS_a adopted in design codes.

Keywords: target probability of failure; allowable factor of safety; equivalence; feasible designs; retaining wall.

1 Introduction

Uncertainties are unavoidable in retaining wall design, such as inherent variability, model uncertainty, etc. (Baecher and Christian 2003). Such uncertainties cannot be explicitly considered in the traditional design of retaining wall because the traditional design methods are usually based on factor of safety (*FS*) and the design process is deterministic in nature (e.g., Ching 2009). Recently, the reliability-based design (RBD) methods that deal, rationally, with uncertainties in geotechnical design have been attracting more and more attentions (e.g., Phoon and Retief 2016, Ji et al., 2019). Although geotechnical RBD approaches have been successfully applied in retaining wall, geotechnical practitioners are hesitated to adopt RBD in design practice. Such hesitation arises from, at least partially, the fact that the final design obtained from RBD may be significantly different from that obtained using safety factor-based design (SFBD).

For example, the allowable factor of safety, FS_a, is taken as equal to 1.35 for sliding failure mode of retaining wall with Grade 1, and the target reliability(β_T) corresponding to retaining wall with Grade 1 is taken as 3.7 according to MWRC (2007) and MHURC (2013). The final design obtained from β_T = 3.7 may be significantly different from that obtained using FS_a = 1.35. Such a difference might be expected because SFBD and RBD use different design requirements. However, this has complicated the transition to RBD, because engineers are more familiar with SFBD and like to compare results of SFBD and RBD as “reality check”. To address this issue, it is useful to investigate reliability levels of SFBD feasible designs, and to establish a target reliability level corresponding to FS_a, with which feasible designs of RBD are identical to those from SFBD, i.e., achieving the equivalence between SFBD and RBD requirements of retaining wall.

This study verifies the sufficient conditions for equivalence between SFBD and RBD of gravity retaining wall design. Based on the sufficient conditions, the target reliability level (target reliability index, β_T, or P_T) is calibrated from FS_a adopted in SFBD so that the same feasible design domains are obtained based on β_T in RBD and FS_a in SFBD. This is illustrated using a gravity retaining wall design example.

2 Safety Factor-based Design (SFBD) and Reliability-based Design (RBD) Methods

Consider, for example, gravity retaining wall design have design parameters $\boldsymbol{D_P}$ (e.g., the height, top width, and bottom width) and uncertain parameters $\boldsymbol{U_P}$ (e.g., the density of wall, surcharge loading, dry weight density and the effective friction angle of backfill soils, and the interface friction between backfill and wall). For SFBD, the safety margin of a design is measured by a nominal factor of safety, $FS_n = FS(\boldsymbol{U_P} = \boldsymbol{U_{Pn}}, \boldsymbol{D_P})$, where $\boldsymbol{U_{Pn}}$ are nominal values of the uncertain parameters, $\boldsymbol{U_P}$. Then, the feasible designs of SFBD are those with FS_n greater than or equal to FS_a, i.e.,

$$FS_n / FS_a = FS\left(\boldsymbol{U_P} = \boldsymbol{U_{Pn}}, \boldsymbol{D_P}\right) / FS_a \geq 1 \tag{1}$$

The whole set of feasible designs of SFBD constitute its feasible design domain, Ω_D, in design space.

In contrast, the reliability-based design approaches require that the failure probability, P_f, value is less than the prescribed target probability of failure P_T:

$$P_f = P\left[FS\left(\boldsymbol{U_P}, \boldsymbol{D_P}\right) - 1 < 0\right] = \int I\left[FS\left(\boldsymbol{U_P}, \boldsymbol{D_P}\right) < 1\right] p\left(\boldsymbol{U_P}\right) d\boldsymbol{U_P} \leq P_T \tag{2}$$

where $p(\boldsymbol{U_P})$ is the probability density function (PDF) of $\boldsymbol{U_P}$ prescribed by a design scenario; $I[\bullet]$ is an indicator function, and it is equal to one if the $FS(\boldsymbol{U_P}, \boldsymbol{D_P})$ is less than 1 or zero otherwise.

In addition, the feasible designs of RBD satisfying Eq. (2) are also able to use the probabilistic sufficiency factor, *PSF*, in literature (Qu and Haftka 2004; Zhou et al. 2022), using the cumulative distribution function, CDF_{FS}, of *FS*, i.e.,

$$PSF = CDF_{FS}^{-1}\left(P_T\right) \tag{3}$$

where CDF_{FS}^{-1} is the inverse cumulative distribution function (CDF) of *FS*. Eq. (3) defines the *PSF* from a perspective of the CDF. $PSF \geq 1.0$ implies that the design is feasible according to the RBD requirement (i.e., $P_f \leq P_T$). Then, the whole set of feasible designs of RBD constitute its feasible design domain, Ω_R, in design space.

In current codes of retaining wall, the P_T is specified separately from FS_a used in existing SFBD codes. The feasible domains Ω_D and Ω_R obtained using SFBD and RBD might be different from each other. In the cases where Ω_D and Ω_R are identical, SFBD and RBD are considered equivalent, and such an equivalence can only occur when some conditions are satisfied.

3 Sufficient conditions and its verification for the equivalence between SFBD and RBD based on PSF

3.1 Sufficient conditions for the equivalence between SFBD and RBD based on *PSF*

This subsection briefly introduces the sufficient conditions proposed by Zhou et al. (2022), under which a SFBD requirement is equivalent to a RBD requirement in the sense that the two design methods demarcate the same domain of feasible designs. For this purpose, SFBD and RBD requirements of retaining wall are compared as $FS_n/FS_a \geq 1$ and $PSF \geq 1$, respectively. Take gravity retaining wall for example, the two design requirements are equivalent for a given FS_a if the following two conditions are, simultaneously, satisfied:

Condition 1 (denoted as **C-1):** There exists a critical design (i.e., $\boldsymbol{D_P} = \boldsymbol{D_{Pc}}$) specified by a set of design parameters in the design space, Ω_S, satisfying

$$PSF\left(\boldsymbol{U_P}, \boldsymbol{D_P} = \boldsymbol{D_{Pc}}\right) = FS\left(\boldsymbol{U_P} = \boldsymbol{U_{Pn}}, \boldsymbol{D_P} = \boldsymbol{D_{Pc}}\right) / FS_a = 1, \quad \exists \boldsymbol{D_{Pc}} \in \Omega_S \tag{4}$$

where $PSF(\boldsymbol{U_P}, \boldsymbol{D_P} = \boldsymbol{D_{Pc}})$ and $FS(\boldsymbol{U_P} = \boldsymbol{U_{Pn}}, \boldsymbol{D_P} = \boldsymbol{D_{Pc}})$ are, respectively, the *PSF* and FS_n of the critical design $\boldsymbol{D_{Pc}}$;

Condition 2 (denoted as **C-2):** FS_n/FS_a (or FS_n) is monotonically increasing with the *PSF* in the design space, i.e.,

$$FS\left(\boldsymbol{U_P} = \boldsymbol{U_{Pn}}, \boldsymbol{D_P}\right) / FS_a = F\left[PSF\left(\boldsymbol{U_P}, \boldsymbol{D_P}\right)\right], \quad \forall \boldsymbol{D_P} \in \Omega_S \tag{5}$$

where $F(\bullet)$ denotes a monotonically increasing function.

Figure 1 schematically shows the two conditions. **C-2** means that *PSF* and FS_n of each possible design in the design space is monotonous. The propose of **C-2** is to ensures that when the *PSF* of one possible design is greater than that of the other designs, the FS_n of this design is also greater than that of the other designs. Combining the critical design whose $FS_n = FS_a$ (FS_a is the allowable factor of safety) and $PSF = 1$, the feasible design domains obtained by SFBD and RBD are the same.

The **C-1** can be automatically satisfied by assigning the design with the FS_n equal to FS_a as the critical design $\boldsymbol{D_{Pc}}$. Then, the *PSF* of the critical design is determined as unity, i.e., $PSF(\boldsymbol{U_P}, \boldsymbol{D_P} = \boldsymbol{D_{Pc}}) = 1$, by taking its P_f as the target probability of failure, P_{TE}, that allows achieving the equivalency between SFBD and RBD

Herein, the "*E*" is added in the subscript to differentiate it from the target failure probability, i.e., P_T, that is adopted in RBD codes and is assigned separately from FS_a. Nevertheless, the **C-2** might not be satisfied because there exist multiple possible designs with $FS_n = FS_a$, and, in general, these designs may have different P_f values. The next subsection discusses the method to verify the **C-2**.

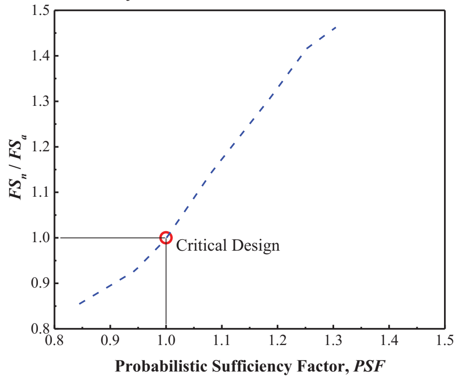


Figure 1. Illustration of the proposed sufficient conditions (After Zhou et al., 2022).

3.2 Verification of the sufficient conditions with MCS

In general, MCS-based stochastic analysis can be used to evaluate *PSF* values of all possible designs in Ω_S for a given target failure probability (e.g., P_f of critical designs determined for **C-1**, i.e., P_{TE}) to verify the relationship between *PSF* and FS_n (or FS_n/FS_a) of different designs in a systematic and quantifiable way. The verification process is: (1) select one target failure probability (e.g., P_{TE}) and calculate the *PSF* of all possible designs; (2) calculate the FS_n of all possible designs; (3) compare *PSF* and FS_n to verify monotonicity.

3.3 Verification of the sufficient conditions for simple cases

In some cases, the **C-2** can be verified using some simple approaches considering functional form and distribution types of *FS* such as separable, normally and lognormally distributed inseparable performance function (Zhou et al., 2022). For example, when *FS* is distributed lognormally, the **C-2** is satisfied if the coefficient of variation of *FS* (COV_{FS}) and μ_{FS}/FS_n are invariant over the design space. The verification process considering distribution of *FS* is divided into two parts. Firstly, verify the distribution type of *FS*. Then, calculate COV_{FS} and μ_{FS}/FS_n to verify its variability. Detailed validation methods for other simple cases can be referred to Zhou et al. (2022), which is not provided here for the sake of conciseness.

4 Calibration of Target Failure Probability of retaining wall under Sufficient Conditions

Based on the sufficient conditions descripted in the previous section, the reliability level corresponding to a FS_a value of retaining wall can be uniquely determined provided that the proposed sufficient conditions (**C-1** and **C-2**) are satisfied. The reliability level corresponding to a given FS_a can be determined in 6 steps:

(1) Develop the performance function and deterministic calculation model of retaining wall;

(2) Determine uncertain parameters and their probability distributions, and specify the nominal values of uncertain parameters and the design space;

(3) Verify the sufficient conditions **C-1** and **C-2**;

(4) Identify the critical design with $FS_n = FS_a$;

(5) Evaluate the P_f (or β) of the critical design, which is then taken as the P_{TE} (or β_{TE}) in RBD;

(6) Determine respective feasible design domains, i.e., Ω_D and Ω_R, of SFBD and RBD according to FS_a and its corresponding P_{TE} determined in Step (5). If the feasible design domains of SFBD and RBD are identical, the equivalence between SFBD and RBD is achieved.

5 Illustration Example

5.1 Deterministic Model and Parameters

This section illustrates the proposed sufficient conditions using a gravity retaining wall example, which is adopted from Eurocode 7 and Gao et al. (2019). As shown in Figure 2, the gravity retaining wall has a symmetrical cross section with a top width *a* and a heel width b_h (or equivalently a base width $b = a + 2b_h$), and it has a fixed height H = 4m for retaining the granular backfill that is assumed to be dry in this example. The wall is made of concrete with a density (γ_c) of 24kN/m^3. The uncertain parameters include the surcharge loading (*q*), the dry weight density of the backfill soil (γ), the effective friction angle of the backfill soil (φ), the interface friction between backfill and wall (δ), and the effective friction angle of shear resistance of the rock beneath the wall (φ_{fdn}). These uncertain parameters are assumed lognormally distributed and the φ and δ are positively correlated with a correlation coefficient of 0.8. Table 1 gives the statistics of uncertain parameters, which are adopted from Gao (2019). The ground surface of the backfill is inclined upwards at an angle, α = 14° from the horizontal. In addition, the interface friction angle (δ_{fdn}) is taken as $0.8\varphi_{fdn}$ in this example.

Design parameters considered in this example include the top width and bottom heel width (i.e., a and b_h, respectively) of the retaining wall. The design space is defined by $a \in [0.5, 1.0]$m and $b_h \in [0.1, 0.5]$m, The possible value of a varies from 0.5m to 1.0m with an increment of 0.05m, and b_h is considered varying from 0.1m to 0.5m with an increment of 0.05m, yielding a total of 99 possible design in the design space.

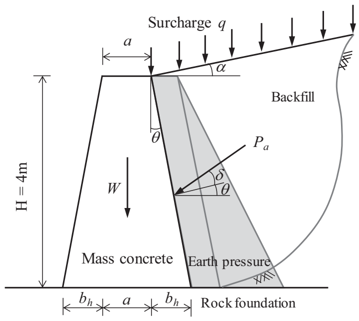


Figure 2. Illustration of the retaining wall example (After Gao, 2019).

Table 1. Statistics of uncertain parameters in the retaining wall example (After Gao, 2019).

Random variables	Statistics		Nominal value	Distribution type	Correlation coefficient
	Mean value	COV			
q	7.36kPa	0.2	10kPa	Lognormal	
γ	16.2kN/m^3	0.1	19kN/m^3	Lognormal	
φ	38°	0.1	36°	Lognormal	0.8
δ	31.7°	0.1	30°	Lognormal	
φ_{fdn}	42.3°	0.1	40°	Lognormal	

In this example, the sliding and overturning failure modes of the wall are considered. The safety factors (FS_S and FS_O) of sliding and overturning modes are calculated as:

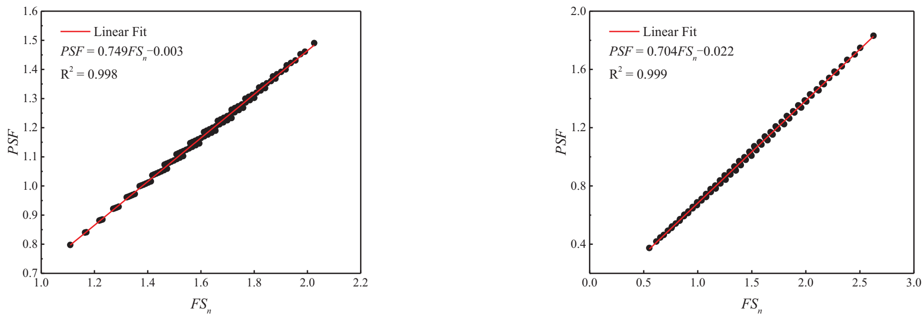
$$FS_S = \frac{H_R}{H_E} \tag{6}$$

$$FS_O = \frac{M_{Stb}}{M_{Dst}} \tag{7}$$

where H_R and H_E are the horizontal resistance and action effect along the base of the wall, respectively; M_{Stb} and M_{Dst} are the stabilizing and destabilizing moment acting about the toe of the wall, respectively. Their calculation methods refer to Gao et al. (2019) and Bond and Harris (2008) omitted here for simplicity.

5.2 Verification of sufficient conditions

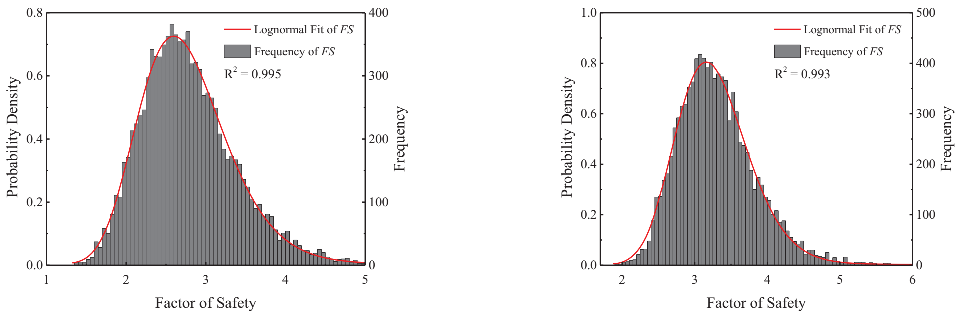
This subsection uses the two method mentioned in Subsection 3.2 and Subsection 3.3 to verify the sufficient conditions of the gravity retaining wall. For sliding and overturning failure modes, 10^{-3} and 10^{-5} are selected as P_T to calculate PSF, respectively. The FS_n is monotonically increasing with the PSF, as shown in Figure 3.



(a) Sliding failure mode (b) Overturning failure mode

Figure 3. Comparison between PSF and FS_n for verifying sufficient conditions

In addition, as previously mentioned in Subsection 3.3, for lognormally distributed performance functions, **C-2** can be accomplished by examining the distribution type, COV_{FS} and μ_{FS}/FS_n. In this example, based on statistics shown in Table 1, it is found that the *FS* of each possible design is lognormally distributed in this example for sliding and overturning failure modes. Figure 4(a) shows, for example, the histogram of *FS* of sliding failure mode of a possible design with a = 0.75m and b_h = 0.3m constructed based on 10000 direct MCS samples and the best-fit of the lognormal distribution. Similar observations are also obtained for other possible designs, and their results are not presented herein for conciseness. In addition, the *FS* of overturning failure mode of the design with a = 0.75m and b_h = 0.3m similarly follows the lognormal distribution, as shown in Figure 4(b).



(a) Sliding failure mode (b) Overturning failure mode

Figure 4. Probability distribution of *FS* for the design with a = 0.75m and b_h = 0.3m.

Moreover, the COV_{FS} of each possible design is calculated. It is found that the COV_{FS} values of sliding and overturning failure modes remain at around 0.223 and 0.167, respectively, and they are almost unchanged over the design space. Similarly, the μ_{FS}/FS_n values of sliding and overturning failure modes remain at around 1.383 and 1.257, respectively. The variations of COV_{FS} and μ_{FS}/FS_n over design space are effectively uniform. Hence, it is ready to reason that the proposed sufficient conditions **C-2** is satisfied in this example.

5.3 Reliability level corresponding to the FS_a adopted in SFBD

Consider, for example, FS_a equals to two different values (i.e., 1.35 and 1.30) and two different values (i.e., 1.60 and 1.50) for sliding and overturning failure modes, respectively, which are corresponding to the retaining wall Grades 1 and 2 according to MWRC (2007) and MHURC (2013). For each given FS_a, the critical design with $FS_n = FS_a$ is identified from possible designs by changing one design parameter (e.g., b_h) while fixing the other parameter (e.g., a), and it is used to determine the reliability level corresponding to the given FS_a. As shown in Table 2, it is noted that the reliability levels (i.e., β_{TE}) corresponding to the FS_a in MWRC (2007) of sliding failure mode are lower than that prescribed in MHURC (2013). Inversely, the reliability levels corresponding to the FS_a of overturning failure mode are higher than that prescribed in the code.

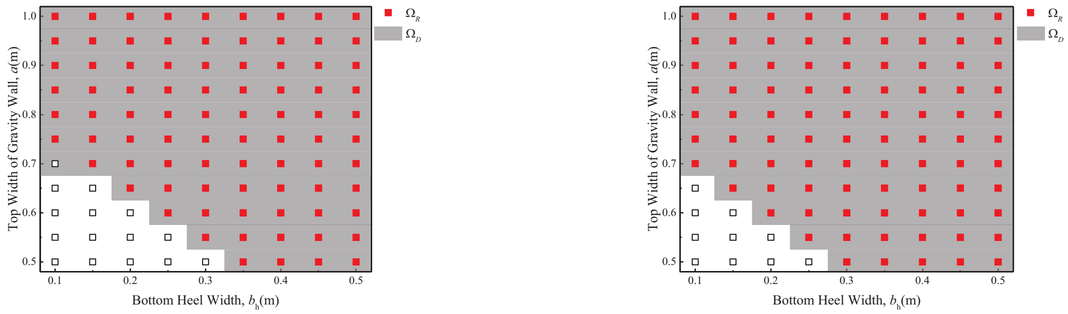
Taking the volume of retaining wall to evaluate the construction cost of RBD, which is proportional to the summation of the top width and bottom width, i.e., $2a+2b_h$. The minimum value (i.e., $\min(2a+2b_h)$) of $2a+2b_h$ of the final design is 1.7m (a = 0.5m and b_h = 0.35m) for sliding failure mode given β_{TE} = 3 but it is 2.1m (a = 0.55m and b_h = 0.5m) given β_T = 3.7 in code. In addition, the minimum value of $2a+2b_h$ of the final design is 2.1m (a = 0.55m and b_h = 0.5m) for overturning failure mode given β_{TE} = 5.15 but it is 1.9 m (a = 0.5m and b_h = 0.45m) given β_T = 3.7 in code. The final designs obtained using β_{TE} and β_T in code are not consistent.

Table 2. Reliability levels corresponding to different values of FS_a.

Failure modes	Retaining wall grades	FS_a	Critical designs		FS_k	$P_f = P_{TE}$	β_{TE}	β_T in code
			a	b_h				
Sliding	1	1.35	0.5	0.330	1.350	1.34×10^{-3}	**3.00**	**3.7**
	2	1.30	0.5	0.280	1.301	2.74×10^{-3}	**2.78**	**3.2**
Overturning	1	1.60	0.75	0.357	1.602	1.30×10^{-7}	**5.15**	**3.7**
	2	1.50	0.75	0.322	1.500	2.66×10^{-6}	**4.55**	**3.2**

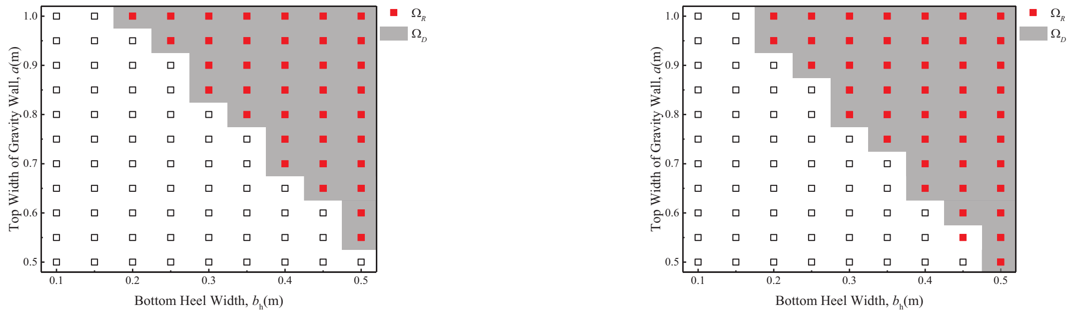
5.4 Comparison of feasible designs from SFBD and RBD

Similarly, the equivalence between SFBD and RBD given the proposed sufficient conditions can be verified by comparing their feasible design domains obtained using FS_a and its corresponding P_{TE} value. The Ω_D of SFBD are determined as those designs with $FS_n \geq FS_a$, which are shown in Figure 5 and Figure 6 by the shadow. The Ω_R of RBD are also determined by comparing those designs with $P_f \leq P_{TE}$, as shown by solid squares in Figure 5 and Figure 6. It is shown that, for a given FS_a, Ω_D is in good agreement with Ω_R. Hence, using SFBD with a given FS_a and RBD based on the P_{TE} determined from FS_a give the same feasible designs. The equivalence of the two design methodologies is achieved in this example given the proposed sufficient conditions.



(a) $FS_a = 1.35$ (b) $FS_a = 1.30$

Figure 5. Comparison of feasible design domains of SFBD and RBD of sliding failure mode.



(a) $FS_a = 1.60$ (b) $FS_a = 1.50$

Figure 6. Comparison of feasible design domains of SFBD and RBD of overturning failure mode.

6 Summary and Conclusions

This paper presented an attempt to understand the reliability level of retaining wall implied by the allowable safety factor, FS_a, adopted in safety-factor based design (SFBD) so that reliability-based design (RBD) and SFBD can provide the same feasible designs based on their respective design requirements. For this purpose, the sufficient conditions and its verification methods were adopted for the existence of a unique mapping from FS_a to a target failure probability level (i.e., P_{TE}). A gravity retaining wall example is used to obtain the P_{TE} corresponding to FS_a. It was shown that using the FS_a in SFBD and its corresponding P_{TE} in RBD give the same feasible designs, achieving the equivalence between SFBD and RBD. Moreover, it is found that the reliability level of sliding failure mode prescribed in MHURC (2013) is conservative compared with the reliability level corresponding to FS_a = 1.3 and 1.35 in MWRC (2007). Inversely, the reliability level of overturning failure mode prescribed in MHURC (2013) is lower than that corresponding to FS_a = 1.5 and 1.6. This results in inconsistent final designs by using FS_a and P_T adopted in design codes.

Acknowledgments

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